Equilibrium System

Households:

Euler equation:

$$u'(c_t^e) = \beta \mathbf{E}_t \left(\frac{1+i_{t+1}}{\Pi_{t+1}} \left[e_{t+1} u'(c_{t+1}^e) + u_{t+1} u'(c_{t+1}^u)) \right] \right) \tag{1}$$

Constraints:

$$x_t = \frac{b_{t+1}}{P_t} + (1+i_t)\frac{a_t}{P_t} - (1+i_t)\frac{b_t}{P_t} \tag{2}$$

$$c_t^u = x_t + \tau_t^u \tag{3}$$

$$\frac{a_{t+1}}{P_t} = x_t + (1-\tau_t)[e_t w_t + e_t d_t] + u_t \tau_t^u - [e_t c_t^e + u_t c_t^u] \tag{4}$$

Labor Market:

Job finding rate:

$$f_t^s = \alpha_m (\frac{v_t}{s_t})^{1-\alpha} \tag{5}$$

Job filling rate:

$$f_t^v = \alpha_m (\frac{v_t}{s_t})^{-\alpha} \tag{6}$$

Employment accumulation:

$$e_t = \rho_t e_{t-1} + f_t^s s_t \tag{7}$$

with searchers s_t given by:

$$s_t = 1 - \rho_t e_{t-1} \tag{8}$$

Nash-Bargained Wage:

$$w_t^{NB} = \vartheta \left(q_t z_t + \mathbf{E}_t \Lambda_{t,t+1} \rho_{t+1} f_{t+1}^s \frac{k}{f_{t+1}^v} \right) + (1-\vartheta) \frac{\Upsilon_t}{1-\tau_t} \tag{9}$$

where:

$$\Upsilon_t = \tau_t^u + (c_t^e - c_t^u) + \frac{u(c_t^u) - (u(c_t^e) - \chi)}{u'(c_t^e)} \tag{10}$$

Asset market equilibrium:

$$\frac{b_{t+1}}{P_t} = \frac{a_{t+1}}{P_t} = \bar{b}_t \tag{11}$$

Firms:

Optimal hiring:

$$\frac{k}{f_t^v} = q_t z_t - w_t + \mathbf{E}_t \left\{ \Lambda_{t,t+1} \rho_{t+1} \frac{k}{f_{t+1}^v} \right\} \tag{12}$$

Discount factor:

$$\Lambda_{t,t+1} = \beta \mathbf{E}_t \left\{ \frac{(1-\tau_{t+1}) u'(c_{t+1}^e)}{(1-\tau_t) u'(c_t^e)} \right\} \tag{13}$$

Dividends definition:

$$d_t^w = q_t z_t e_t - w_t e_t - k_t v_t \tag{14}$$

Prices:

$$\frac{p_t^*}{P_t} = \frac{p_t^A}{p_t^B} \tag{15}$$

with:

$$p_t^A = \frac{\varepsilon_p}{\varepsilon_p - 1} q_t Y_t + \mathbf{E}_t \Lambda_{t,t+1}(1-\theta)\pi_{t+1,t}^{\epsilon_p} p_{t+1}^A \tag{16}$$

and:

$$p_t^B = Y_t + \mathbf{E}_t \Lambda_{t,t+1}(1-\theta)\pi_{t+1,t}^{\epsilon_p - 1} p_{t+1}^B \tag{17}$$

$$\tag{18}$$

Inflation:

$$\pi_t = \left(\frac{1-\theta}{1-\theta(\frac{p_t^*}{P_t})^{1-\epsilon_p}} \right)^{\frac{1}{1-\epsilon_p}} \tag{19}$$

Output:

$$\varsigma_t Y_t = z_t e_t \tag{20}$$

Price dispersion:

$$\varsigma_t = \theta(\frac{p_t^*}{P_t})^{-\epsilon_p} + (1-\theta)\pi_t^{\epsilon_p}\varsigma_{t-1} \tag{21}$$

Dividends:

$$D_t = Y_t - q_t z_t e_t + d_t^w \tag{22}$$

Government

Government budget constraint:

$$\tau_t[e_t(w_t + d_t)] = u_t \tau_t^u \tag{23}$$

Unemployment Insurance:

$$\tau_t^u = 0.3 * w_{t-1} + \varepsilon_{\tau,t} \tag{24}$$

Taylor rule:

$$1 + i_t = (1 + \bar{i})(\frac{P_t}{P_{t-1}})^{\psi} e^{\epsilon_{it}}$$

Shocks:

Productivity:

$$log(z_t) = (1 - \rho_z)log(\bar{z}) + \rho_z log(z_{t-1}) + \rho_{\rho z}\sigma_\rho \varepsilon_{\rho,t} + \sigma_z \varepsilon_{z,t} \quad (25)$$

Separation:

$$log(\rho_t) = (1 - \rho_\rho)log(\bar{\rho}) + \rho_\rho log(\rho_{t-1}) + \rho_{\rho z}\sigma_z \varepsilon_{z,t} + \sigma_\rho \varepsilon_{\rho,t} \quad (26)$$

Borrowing:

$$\bar{b}_t = (1 - \rho_b)(\bar{b}) + \rho_b \bar{b}_{t-1} + \sigma_b \varepsilon_{b,t} \quad (27)$$

Monetary policy:

$$\epsilon_{it} = \rho_i \epsilon_{i,t-1} + \sigma_i \varepsilon_{it} \quad (28)$$

Steady-state equations

Euler Equation:

$$u'(c^e) = \beta(1+\bar{i})\left[eu'(c^e) + uu'(c^u))\right] \;\Rightarrow\; \beta = \frac{1}{(1+\bar{i})(e + u\frac{u'(c^u)}{u'(c^e)})}$$

where we target $\bar{i} = 0.003$ and $\frac{c^u}{c^e} = 0.72$.

Constraints:

$$x = \bar{b}$$
$$\bar{b} = c^u - \tau^u,\; with\; c^u = 0.72c^e$$
$$c^e = \frac{(1-\tau)[ew + D] + u\tau^u}{e + 0.72u} \Rightarrow c^e = \frac{ew + D}{e + 0.72u}$$

Employment:

$$e(1-\rho) = f^s(1-\rho e) \Rightarrow f^s = \frac{e(1-\rho)}{1-\rho e},\; where\; e = 1-u,\; and\; target\; u = 0.062$$

Market tightness: $\frac{v}{s} = (\frac{f^s}{\alpha_m})^{\frac{1}{1-\alpha}} \quad \Rightarrow \quad v = s * (\frac{f^s}{\alpha_m})^{\frac{1}{1-\alpha}}$

Searchers and job filling rate:

$$s = 1 - \rho e$$
$$f^v = \alpha_m(\frac{v}{s})^{-\alpha}$$

Discount factor:

$$\Lambda = \beta$$

Hiring cost:

$$k = f^v \frac{q-w}{1-\beta\rho}$$

Nash-Bargained Wage:

$$w^{NB} = \vartheta \left(q + \beta\rho f^s \frac{k}{f^v} \right) + (1-\vartheta)\frac{\Upsilon}{1-\tau}$$

which after substituting the expressions for c^e, k, τ gives:

$$w = \frac{1-\beta\rho}{1-\beta\rho+\vartheta\beta\rho f^s}\left(\vartheta q(\frac{1-\beta\rho+\beta\rho f^s}{1-\beta\rho}) + (1-\vartheta)\Upsilon\frac{(1-\beta\rho)e - vf^v q + vf^v w}{(1-\beta\rho)e - vf^v q + vf^v w - 0.3uw(1-\beta\rho)}\right)$$

The opportunity cost of work, Υ, is given by:

$$\Upsilon = (c^e - x) + \frac{u(c^u) - (u(c^e) - \chi)}{u'(c^e)} = c^e(0.28 + log(0.72) + \chi) + \tau^u =$$
$$= w\left(0.3 + \frac{vf^v(0.28 + log(0.72) + \chi)}{(1-\beta\rho)(e+0.72u)}\right) + (0.28 + log(0.72) + \chi)\left(\frac{e}{e+0.72u} - \frac{vf^v q}{(1-\beta\rho)(e+0.72u)}\right)$$

Unemployment benefits are calibrated to $\tau^u = 0.3w$.

Dividends:

$$d^w = (q-w)e - kv$$
$$D = Y - qe + d^w = (1-q)e + d^w = (1-w)e - kv$$

Government budget constraint:

$$\tau = \frac{u\tau^u}{ew + D}$$

Prices:

$$q = \frac{\varepsilon_p - 1}{\varepsilon_p}$$

with:

$$p^A = \frac{\varepsilon_p}{\varepsilon_p - 1}(\frac{qY}{1-\beta(1-\theta)})$$
$$p^B = \frac{Y}{1-\beta(1-\theta)}$$