Net export to output ratio at time t (NX_t) is defined as below:

$$NX_t = \frac{\hat{b}_t - q_t \hat{b}_{t+1} (e^{g_{a,t}})^{\frac{1}{1-\alpha}} (e^{g_{v,t}})^{\frac{\alpha}{1-\alpha}}}{\hat{y}_t}$$

Linearization of net exports to output ratio and log-linearization of R.H.S result the following:

In the steady state, $g_a = \mu_{g_a}$ and $g_v = \mu_{g_v}$

$$\begin{aligned} NX + (NX_t - NX) \\ &= \frac{b - qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} + \frac{1}{y} b\tilde{b}_t - \frac{b(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} q\tilde{q}_t \\ &- \frac{q(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} b\tilde{b}_{t+1} - \frac{qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} \frac{1}{1-\alpha} (g_{a,t} - \mu_{g_a}) \\ &- \frac{qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} \frac{\alpha}{1-\alpha} (g_{v,t} - \mu_{g_v}) \\ &- \frac{b - qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y^2} y\tilde{y}_t \end{aligned}$$

Letting, $\widetilde{NX}_t = NX_t - NX$ and discarding steady state relationships

$$\begin{aligned} \widetilde{NX}_t = \frac{1}{y} b\tilde{b}_t - \frac{b(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} q\tilde{q}_t - \frac{q(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} b\tilde{b}_{t+1} \\ - \frac{qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} \frac{1}{1-\alpha} (g_{a,t} - \mu_{g_a}) \\ - \frac{qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y} \frac{\alpha}{1-\alpha} (g_{v,t} - \mu_{g_v}) \\ - \frac{b - qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{\mathrm{y}} \tilde{y}_t \end{aligned}$$

Simplifying,

$$\widetilde{NX}_t = \frac{b}{y}\left(\tilde{b}_t - \tilde{y}_t\right) - \frac{qb(e^{\mu_{g_a}})^{\frac{1}{1-\alpha}}(e^{\mu_{g_v}})^{\frac{\alpha}{1-\alpha}}}{y}\left(\tilde{q}_t + \tilde{b}_{t+1} + \frac{1}{1-\alpha}(g_{a,t} - \mu_{g_a}) + \frac{\alpha}{1-\alpha}(g_{v,t} - \mu_{g_v}) - \tilde{y}_t\right)$$

Observation Equation for trade-balance-to-output ratio with **demeaned** series

$$tby_t = \widetilde{NX}_t + measurement\ error$$