RBC Model with Matching Frictions

1 Household Problem

The maximization problem of the household is as follows:

$$V(K_{-1}; E_{-1}, A) = \max_{K,C} \log(C) + \beta \mathbb{E}[V(K; E, A')]$$

$$s.t.$$

$$C + K = wE_{-1}(1-\tau) + b(1-E_{-1}) + T + \pi + (1 + r - \delta)K_{-1} \tag{1}$$

The first order HH condition for K' and C imply:

$$\frac{1}{C} = \beta E[\frac{1}{C'}(1 + r' - \delta)] \tag{2}$$

2 Government BC

$$T + b(1 - E_{-1}) = \tau w E_{-1} \tag{3}$$

3 Firm

The firm solves the following maximization problem:

$$J(E_{-1}; K_{-1}, A) = \max_{k,V} Ak^{\alpha} E_{-1}^{1-\alpha} - wE_{-1} - rk - \kappa V + \beta \mathbb{E}[\frac{C}{C'} J(E; K, A)]$$

$$s.t.$$

$$E = E_{-1}(1-s) + hV$$

Notice that $K_{-1} = k$ by market clearing. Then, substituting that in, the first order conditions imply:

$$r = \alpha A K_{-1}^{\alpha-1} E_{-1}^{1-\alpha} \tag{4}$$

$$\frac{\kappa}{h} = \beta \mathbb{E}[\frac{C}{C'} J_E]$$

$$J_{E_{-1}} = (1-\alpha) A K_{-1}^{\alpha} E_{-1}^{-\alpha} - w + \beta \mathbb{E}[\frac{C}{C'} J_E (1-s)]$$

The last two equations together imply:

$$\frac{\kappa}{h} = \beta \mathbb{E}[\frac{C}{C'}((1-\alpha) A' K^{\alpha} E^{-\alpha} - w' + \frac{\kappa}{h'}(1-s))] \tag{5}$$

We can also derive the firm profit as:

$$\pi = A K_{-1}^{\alpha} E_{-1}^{1-\alpha} - w E_{-1} - r K_{-1} - \kappa V \tag{6}$$

4 Labor Market

The law of motion of employment is given as:

$$E = (1-s) E_{-1} + m \tag{7}$$

The matching function:

$$m = \mu V^{\xi} (1 - E_{-1})^{1-\xi} \tag{8}$$

Hiring rate:

$$h = \frac{m}{V} \tag{9}$$

Unemployment Rate:

$$U = 1 - E_{-1} \tag{10}$$

5 Wage Determination

I used two alternate specifications for wages:

$$w = \omega A^{\gamma} \tag{11}$$

or Nash Bargaining:

$$w(1-\tau) - b(1-B) = B((1-\alpha) A K_{-1}^{\alpha} E_{-1}^{-\alpha} + \kappa \frac{V}{1-E_{-1}}) \tag{12}$$

where B is the bargaining weight.

6 Shock Process

The shock process is as follows:

$$A = (1 - \rho_A)1 + \rho_A A_{-1} + \epsilon_A \tag{13}$$

where $\epsilon_A \sim N(0, \sigma_A)$