TECHNICAL APPENDIX: CAPITAL CONTROLS AND OPTIMAL CHINESE MONETARY POLICY

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Abstract. This appendix presents a summary of equilibrium conditions in the model of Chang et al. (2015) (CLS).

Appendix A. Summary of stationary equilibrium conditions

A.1. **Stationary equilibrium.** We focus on a stationary equilibrium with balanced growth. On a balanced growth path, output, intermediate inputs, consumption, real money balances, current account balances, real domestic debt, real foreign asset holdings, and the real wage rate all grow at a constant rate. In our model, aggregate productivity Z_t grows at a constant rate λ_z. To obtain balanced growth, we make the stationary transformations

$$\tilde{Y}_t = \frac{Y_t}{Z_t}, \quad \tilde{\Gamma}_{ht} = \frac{\Gamma_{ht}}{Z_t}, \quad \tilde{\Gamma}_{ft} = \frac{\Gamma_{ft}}{Z_t}, \quad \tilde{C}_t = \frac{C_t}{Z_t}, \quad \tilde{ca}_t = \frac{ca_t}{Z_t}, \quad \tilde{w}_t = \frac{w_t}{Z_t},$$

$$\tilde{m}_t = \frac{M_t}{\bar{P}_t Z_t}, \quad \tilde{b}_t = \frac{B_t}{P_t Z_t}, \quad \tilde{b}^*_t = \frac{B^*_t}{P^*_t Z_t}, \quad \tilde{b}^*_{pt} = \frac{B^*_{pt}}{P^*_t Z_t}, \quad \tilde{b}^*_{gt} = \frac{B^*_{gt}}{P^*_t Z_t}, \quad \tilde{\Lambda}_t = \Lambda_t Z_t.$$

We summarize the stationary equilibrium conditions below, in the same order as in the dynare code.[1]

$$\frac{\Phi_l L_t^\eta}{\tilde{\Lambda}_t} = \tilde{w}_t \tag{A1}$$

$$\frac{\Phi_m}{\tilde{m}_t \tilde{\Lambda}_t} = 1 - \mathrm{E}_t \frac{\beta \tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t \lambda_z} \frac{1}{\pi_{t+1}}, \tag{A2}$$

$$\mathrm{E}_t \frac{\beta \tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t \lambda_z} \frac{R_t}{\pi_{t+1}} = 1 + \frac{\Omega_b}{2}(\psi_t - \bar{\psi})^2 + \Omega_b(\psi_t - \bar{\psi})(1 - \psi_t), \tag{A3}$$

$$\Omega_b(\psi_t - \bar{\psi}) = \mathrm{E}_t \frac{\beta \tilde{\Lambda}_{t+1}}{\tilde{\Lambda}_t \lambda_z} \frac{1}{\pi_{t+1}} \left[R_t - R^*_t \frac{e_{t+1}}{e_t} \right], \tag{A4}$$

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[1] In our dynare code, all variables are expressed in terms of log-deviations from steady state. In the equilibrium conditions listed here, however, the variables are expressed in levels. For example, for the level variable X_t, the log-deviation from its steady-state value of X is $\hat{x}_t \equiv log(X_t/X)$.

$$\tilde{\Lambda}_t = \frac{1}{\tilde{C}_t}, \tag{A5}$$

$$\psi_t = \frac{\tilde{b}_t}{\tilde{b}_t + q_t \tilde{b}^*_{pt}}, \tag{A6}$$

$$\upsilon_t = \tilde{\phi} q^{\phi}_{mt} \tilde{w}^{1-\phi}_t, \tag{A7}$$

$$\tilde{w}_t = (1-\phi)\frac{\upsilon_t \tilde{Y}_t}{L_t}, \tag{A8}$$

$$\tilde{\Gamma}_t = \tilde{\Gamma}^{\alpha}_{ht}\tilde{\Gamma}^{1-\alpha}_{ft}, \tag{A9}$$

$$q_{mt} = \tilde{\alpha} q^{1-\alpha}_t, \tag{A10}$$

$$q_t = \frac{1-\alpha}{\alpha}\frac{\tilde{\Gamma}_{ht}}{\tilde{\Gamma}_{ft}}, \tag{A11}$$

$$\frac{q_t}{q_{t-1}} = \gamma_{et}\frac{\pi^*}{\pi_t}, \tag{A12}$$

$$\frac{\theta_p - 1}{\theta_p} = \upsilon_t - \frac{\Omega_p}{\theta_p}\frac{\tilde{C}_t}{\tilde{Y}_t}\left[\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} - \beta \mathrm{E_t}\left(\frac{\pi_{\mathrm{t+1}}}{\pi} - 1\right)\frac{\pi_{\mathrm{t+1}}}{\pi}\right], \tag{A13}$$

$$\tilde{ca}_t = \tilde{X}_t - q_t\tilde{\Gamma}_{ft} + \frac{q_t}{\pi^*\lambda_z}(R^*_{t-1} - 1)\tilde{b}^*_{t-1}, \tag{A14}$$

$$\tilde{X}_t = q^{\theta}_t \tilde{X}^*_t, \tag{A15}$$

$$\tilde{ca}_t = q_t\left[\tilde{b}^*_t - \frac{\tilde{b}^*_{t-1}}{\pi^*\lambda_z}\right], \tag{A16}$$

$$q_t\left[\tilde{b}^*_{gt} - \frac{\tilde{b}^*_{g,t-1}R^*_{t-1}}{\pi^*\lambda_z}\right] = \tilde{b}_t - \frac{\tilde{b}_{t-1}R_{t-1}}{\pi_t\lambda_z} + \tilde{m}_t - \frac{\tilde{m}_{t-1}}{\pi_t\lambda_z}, \tag{A17}$$

$$\tilde{Y}_t = \tilde{C}_t + \tilde{\Gamma}_{ht} + \tilde{X}_t + \frac{\Omega_p}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2\tilde{C}_t + \frac{\Omega_b}{2}\left(\psi_t - \bar{\psi}\right)^2(\tilde{b}_t + q_t\tilde{b}^*_{pt}), \tag{A18}$$

$$\tilde{b}^*_t = \tilde{b}^*_{pt} + \tilde{b}^*_{gt}, \tag{A19}$$

$$\tilde{Y}_t = \tilde{\Gamma}^{\phi}_t L^{1-\phi}_t, \tag{A20}$$

$$G\tilde{D}P_t = \tilde{C}_t + \tilde{X}_t - q_t\tilde{\Gamma}_{ft}, \tag{A21}$$

$$\frac{\tilde{m}_t}{\tilde{m}_{t-1}} = \frac{\mu_t}{\pi_t\lambda_z}, \tag{A22}$$

$$\gamma_{et} = \bar{\gamma}_e, \tag{A23}$$

where the term $\gamma_{et} \equiv \frac{e_t}{e_{t-1}}$ denotes the growth rate of the nominal exchange rate, which is assumed to be constant under a pegged exchange rate regime. The model is closed by assuming that the central bank (or the Ramsey planner) chooses the appropriate policy instrument (in particular, the nominal interest rate R_t) to maximize the representative household's welfare, subject to the equilibrium conditions (A1)-(A23) and shocks to the foreign interest rate R_t^* and export demand $\tilde{X}_t^*$.

The foreign interest rate R_t^* follows the stationary stochastic process

$$\ln R_t^* = (1-\rho_r)\ln R^* + \rho_r \ln R_{t-1}^* + \sigma_r \varepsilon_{rt}, \tag{A24}$$

where $\rho_r \in (0,1)$ is a persistence parameter, σ_r is the standard deviation of the shock, and ε_{rt} is an i.i.d. standard normal process.

The export demand shock $\tilde{X}_t^*$ follows the stochastic process

$$\ln \tilde{X}_t^* = (1-\rho_x)\ln \tilde{X}^* + \rho_x \ln \tilde{X}_{t-1}^* + \sigma_x \varepsilon_{xt}, \tag{A25}$$

where $\rho_x \in (0,1)$ is a persistence parameter, σ_x is the standard deviation, and ε_{xt} is an i.i.d. standard normal process.

A.2. **The steady state.** In the steady state, all shocks are absent and all stationary variables are constant. We solve the steady-state equilibrium recursively with following the steps.

(1) Use the steady-state pricing decision (A13) to obtain the real marginal cost $v = \frac{\theta_p - 1}{\theta_p} = 0.9$ (corresponding to a markup of 11 percent)

(2) Target import-GDP ratio of 0.20 and export-GDP ratio of 0.23. GDP is related to gross output through

$$GDP = Y - \Gamma_h - q\Gamma_f = (1-\phi v)Y \tag{S1}$$

Imports are related to gross output through $IM = q\Gamma_f = (1-\alpha)q_m\Gamma = (1-\alpha)\phi v Y$. Thus, we have

$$0.2 = \frac{IM}{GDP} = \frac{(1-\alpha)\phi v}{1-\phi v} \tag{S2}$$

which gives a steady-state value of α given the values of $\phi = 0.5$ and $v = 0.9$. The implied value of α is 0.7556.

(3) Target export-GDP ratio of 0.23. If we obtain a solution for Y, then we can obtain a solution for X given by

$$X = 0.23GDP = 0.23(1-\phi v)Y \tag{S3}$$

(4) Target steady-state employment of $L = 0.4$ (40 percent of total time endowment spent on working). Target the real exchange rate $q = 1$, which puts

restrictions on X^* through the export demand function $X = q^\theta X^*$ for given X. We can then solve for Y using the production function:

$$Y = \Gamma^\phi L^{1-\phi} = \left(\frac{\phi v Y}{\tilde{\alpha} q^{1-\alpha}}\right)^\phi L^{1-\phi}, \tag{S4}$$

where the second equality uses the cost-minimizing condition that $q_m \Gamma = \phi v Y$ and that $q_m = \tilde{\alpha} q^{1-\alpha}$.

(5) Given Y, we have $GDP = (1 - \phi v)Y$, $IM = 0.2GDP$, and $X = 0.23GDP$. To target $q = 1$, we have $X^* = X/q^\theta = X$.

(6) Since net export is $NX = X - IM = 0.03GDP$, consumption is then $C = GDP - NX = 0.97GDP$.

(7) Given Y and L, the real wage rate is solved:

$$w = (1 - \phi)vY/L \tag{S5}$$

and the labor supply equation pins down the value of the utility weight parameter $\Phi_l = \frac{w}{CL^\eta}$.

(8) The domestic component of intermediate input is given by $\Gamma_h = \alpha\phi v Y$, the foreign component is $\Gamma_f = (1 - \alpha)\phi v Y/q$, and the composite intermediate input is $\Gamma = \Gamma_h^\alpha \Gamma_f^{1-\alpha}$

(9) The steady-state UIP condition (A4) implies that $R = R^*\gamma_e$ and $\pi = \pi^*\gamma_e$.

(10) The money growth rate is given by $\mu = \pi\lambda_z$

(11) Given R and C, we obtain the real money balance $m = \Phi_m CR/(R - 1)$

(12) Using equations (A14) and (A16) to eliminate ca and obtain

$$qb^*\left(1 - \frac{R^*}{\pi^*\lambda_z}\right) = NX, \tag{S6}$$

where $q = 1$ and $NX = 0.03GDP$ are known.

(13) We now solve for b. The portfolio share equation (A6) implies that

$$qb_p^*\left(1 - \frac{R^*}{\pi^*\lambda_z}\right) = \frac{1-\psi}{\psi} b\left(1 - \frac{R^*}{\pi^*\lambda_z}\right);$$

the government budget constraint (A17) implies that

$$qb_g^*\left(1 - \frac{R^*}{\pi^*\lambda_z}\right) = b\left(1 - \frac{R}{\pi\lambda_z}\right) + m\left(1 - \frac{1}{\mu}\right).$$

Summing these equations up, we obtain

$$qb^*\left(1 - \frac{R^*}{\pi^*\lambda_z}\right) = \frac{1-\psi}{\psi} b\left(1 - \frac{R^*}{\pi^*\lambda_z}\right) + b\left(1 - \frac{R}{\pi\lambda_z}\right) + m\left(1 - \frac{1}{\mu}\right) = NX, \tag{S7}$$

where the second equality follows from equation (S6) and m has been solved above. Further, noting that $\frac{R}{\pi\lambda_z} = \frac{R^*}{\pi^*\lambda_z}$, we get

$$NX = b\left(1 - \frac{R}{\pi\lambda_z}\right)\frac{1}{\psi} + m\left(1 - \frac{1}{\mu}\right)$$

This relation gives the solution for b

$$\frac{b}{\psi}\left(1 - \frac{R}{\pi\lambda_z}\right) = NX - m\left(1 - \frac{1}{\mu}\right). \tag{S8}$$

(14) Once we have b, we solve for b_p^* using the portfolio share equation (and targeting the portfolio share to be $\psi = 0.9$). Specifically, we have

$$b_p^* = \frac{1-\psi}{q\psi}b. \tag{S9}$$

(15) Given NX, we obtain b^* from equation (S7). We then have

$$b_g^* = b^* - b_p^*. \tag{S10}$$

(16) Current account balance is then obtained from (A14) and (S6). In particular, we have

$$ca = NX + \frac{qb^*}{\pi^*\lambda_z}(R^* - 1) = qb^*\left(1 - \frac{1}{\pi^*\lambda_z}\right). \tag{S11}$$

References

Chang, C., Z. Liu, and M. M. Spiegel (2015): "Capital Controls and Optimal Chinese Monetary Policy," *Journal of Monetary Economics*, 74, 1–15.