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The Macroeconomics of Skills Mismatch in the Presence of Emigration*

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Abstract

Employment in mismatch (low-skill) jobs is a potential factor in the emigration of highly qualified workers. At the same time, high-skilled emigration and emigration of mismatch workers can free up positions for stayers. In bad times, it could also amplify demand losses and the unemployment spell, which in turn affects the mismatch rate. In this paper, we investigate the link between vertical skills mismatch and emigration of both non-mismatch and mismatch workers in a DSGE model. The model features also skill and wealth heterogeneous households, capital-skill complementarity (CSC) and labor frictions. We find that an adverse productivity shock reduces investment and primarily hurts the high-skilled who react by turning to both jobs abroad and mismatch jobs in the domestic labor market. A negative shock to government spending crowds-in investment and primarily hurts the low-skilled who thus turn to jobs abroad. Following the fiscal cut, the high-skilled instead reduce their search for mismatch employment and later they also reduce their search for jobs abroad.

Keywords: vertical skills mismatch, under-employment, emigration, capital-skill complementarity, RBC model.

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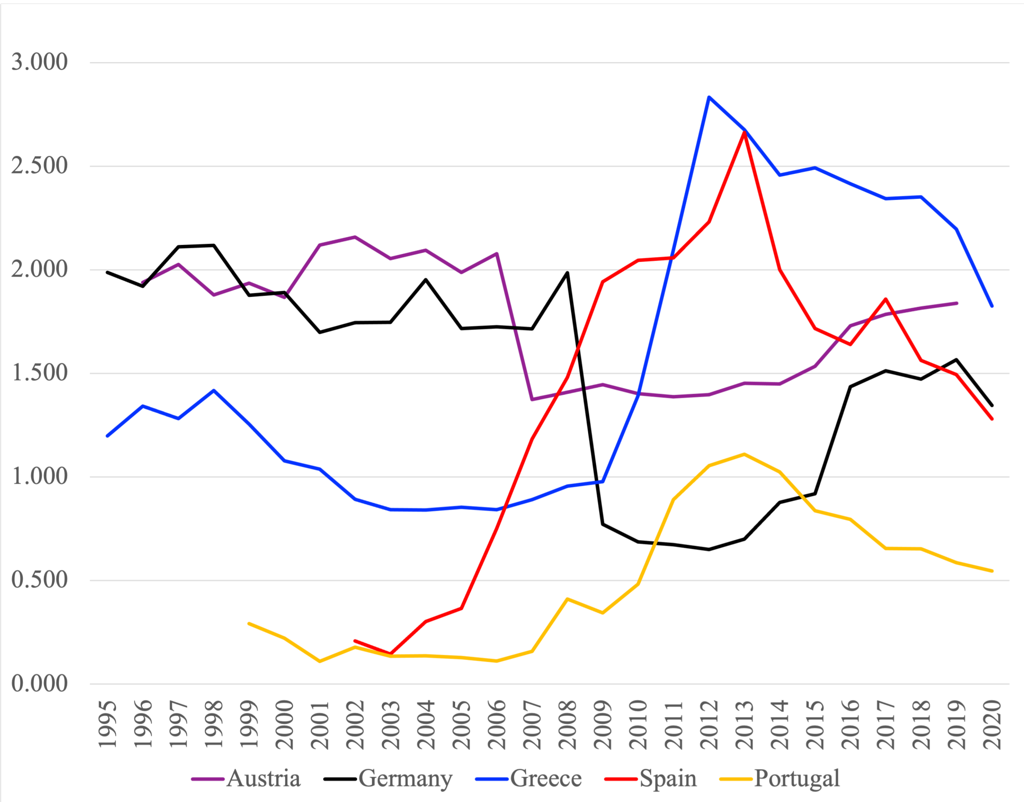
1 Introduction

This paper. During the European debt crisis, there was a significant increase in migration outflows from crisis-hit countries, while the opposite pattern was observed in core countries (see Figure 1). This emigration wave predominantly concerned high-skilled members of the labor force. For instance, according to an ICAP survey conducted from March 27 to May 8, 2019 with 942 participants in 43 countries, nearly 70% percent of recent Greek emigrants hold a postgraduate degree. According to the same survey, 1 out of 5 respondents gave as a reason for emigration the inability to find a job in their field of studies in Greece. This suggests that employment in mismatch (low-skill) jobs is a potential factor in the emigration of highly qualified workers. At the same time, high-skilled emigration and emigration of mismatch workers can free up positions for stayers. In bad times, it could also amplify demand losses and the unemployment spell, which in turn affects the mismatch rate. Surprisingly, a Dynamic General Equilibrium analysis of these issues is still missing. This paper fills this gap by investigating the link between vertical skills mismatch rate (i.e., share of over-qualified workers) and emigration in a DSGE model.

Figure 1: Emigration from Eurozone countries (% of labor force)



Source: Own calculations based on Eurostat data

Theoretical model. We provide a new framework that incorporates skills mismatch and heterogeneous labor with international labor mobility to RBC models. For simplicity, high-skilled households make investment decisions, while low-skilled households are hand-to-mouth consumers. Involuntary unemployment explains the existence of skills mismatch (i.e. employment of a high-skilled worker in a job requiring only low skills) in the model, which arises endogenously from an interplay of households' and firms' decisions. High-skilled households decide the share of their members who search for a low-skill job and firms posting low-skill vacancies decide on a share allocated to high-skilled workers. Mismatched workers continue searching on-the-job to find an upgraded position. In the event of a mismatch, a trade-off arises: the worker is more productive than the non-mismatched counterpart, but receives a higher wage and also the mismatch maybe terminated if she quits to take up a high-skill job via on-the-job search. Two additional key ingredients of the model are capital-skill complementarity (CSC) and search and matching (S&M) frictions. The presence of CSC in the production function is motivated by its empirical plausibility, but we also conduct sensitivity analysis with respect to this specification.[1] S&M frictions are instrumental to model the on-the-job search of mismatched workers and have been extensively used in the mismatch literature.

We find that an adverse productivity shock reduces investment and primarily hurts the high-skilled who react by turning to both jobs abroad and mismatch jobs in the domestic labor market. A negative shock to government spending crowds-in investment and primarily hurts the low-skilled who thus turn to jobs abroad. Following the fiscal cut, the high-skilled instead reduce their search for mismatch employment and later they also reduce their search for jobs abroad.

The main policy implication of our findings is that fiscal consolidation policy makers should take into account the potential implications for skills mismatch and emigration, as well as the feedback effects from the rise of mismatch and emigration on economic activity. In addition, in the face of recessionary shocks that increase the high-skilled emigration, thus contributing to brain drain, additional policies should be put in place to support investment and to incentivize the high-skilled workers to remain in the domestic economy.

Literature. The paper adds to the literature on skills mismatch (see, e.g., Albrecht and Vroman (2002), Dolado et al. (2009), Chassamboulli (2011), Barnichon and Zylberberg (2019), Şahin et al. (2014), Baley et al. (2022)) by documenting the effect of adverse macroeconomic shocks in the presence of skill-specific emigration. The S&M approach, pioneered by Mortensen and Pissarides (1994), has provided a structural approach for the study of mismatch in the labor market. The S&M approach has been also used to study skills mismatch along with the effects of immigration flows (see, e.g. Iftikhar and Zaharieva (2019) and Liu et al. (2017)) in the context of a steady-state analysis.

Second, our paper is related to the literature on the Greek debt crisis using micro-founded macroeconomic models (see, e.g., Gourinchas et al. (2017); de Córdoba et al. (2017); Chodorow-Reich et al. (2023); Economides et al. (2020); Papageorgiou et al. (2021); Bandeira et al. (2022); Oikonomou (2022)). Our paper departs from this research by offering novel evidence on the joint role of skills mismatch and emigration in that context.

Finally, we extend the recent and growing macroeconomics literature investigating the implications of outward migration by incorporating skills mismatch. Alessandria et al. (2020) discuss a feedback loop between emigration and sovereign default with an application to Spain, absent fiscal austerity. Default risk induces emigration, which in turn intensifies default risk by lowering the tax base and investment. Bandeira et al. (2022)

[1] The importance of the CSC relationship has been evidenced by many empirical studies (see, e.g., the seminal paper by Krusell et al. (2000)).

differ by focusing on fiscal tightening during the decade-long Greek crisis and study the implications of emigration for the tax and spending multipliers. Oikonomou (2022) embeds the CSC framework in an open economy RBC model with S&M frictions that features changes in the skill composition of the domestic workforce due to heterogeneous migration outflows. A paper closely related to ours is also Hauser and Seneca (2022), which develops a two-region New Keynesian DSGE model with matching frictions and labor mobility to study optimal monetary policy.

Layout. The paper is organized as follows. Section 2 presents our DSGE model. Section 3 discusses the calibration strategy and Section 4 describes impulse responses from the model. Section 5 concludes.

2 Model with skills mismatch and emigration

In this section, we outline our DSGE model which combines skills mismatch with skill-specific migration.

2.1 Model overview

We build a small open economy model which features two types of households (high and low-skilled), endogenous participation, search and matching frictions and endogenous migration for both skill types. A pecuniary cost to migrate micro-founds migration flows. The production technology is characterized by capital-skill complementarity as in Krusell et al. (2000). The model also features trade links, investment adjustment costs and real rigidities to capture empirically relevant features of the business cycle dynamics. These modelling choices allow us to study the aggregate and distributional impact of emigration in a unified way.

2.2 Population and skill mismatch

The small open economy (SOE) is comprised by two household types. Type $h = 1, ..., N_t^h$, which supplies high-skilled labour, and type $l = 1, ..., N_t^l$, which supplies low-skilled labour. Total population is given by $N_t = N_t^h + N_t^l$, where we denote by $t^h \equiv N_t^h/N_t$ and $t^l \equiv N_t^l/N_t$ the non-equal population shares of high-skilled and low-skilled households, respectively.

Each high-skilled household h consists of members that are employed n_t^h, members that are unemployed, u_t^h, and members that are out of the labour force and enjoy leisure, l_t^h. The employed members can work either at home (H), with a possibility of being vertically mismatched, or abroad (F). Hence, the high-skilled employees, n_t^h, are employed in a high-skill position at home, $n_t^{h,H}$, a low-skill position at home (hereafter, called mismatched employees), $n_t^{hl,H}$, and a high-skill position abroad, $n_t^{h,F}$ (hereafter, called high-skilled emigrants), so that:

$$n_t^h = n_t^{h,H} + n_t^{hl,H} + n_t^{h,F} \tag{1}$$

The members of the high-skilled household are normalized to unity:

$$n_t^{h,H} + n_t^{hl,H} + u_t^h + l_t^h + n_t^{h,F} = 1 \tag{2}$$

where the fraction $1-n_t^{h,F}=n_t^{h,H}+n_t^{hl,H}+u_t^h+l_t^h$ denotes the high-skilled household members that stay at the home country (hereafter, called high-skilled stayers).

The high-skilled household chooses the fractions of its unemployed members that search for a high-skill job abroad, O_t^h, a high-skill job at home, $\left(1-O_t^h\right)s_t$ and a low-skill (mismatch) job at home, $\left(1-O_t^h\right)(1-s_t)$:

$$u_t^h=\left(1-O_t^h\right)s_tu_t^h+\left(1-O_t^h\right)(1-s_t)\,u_t^h+O_t^hu_t^h=u_t^{h,H}+u_t^{hl,H}+u_t^{h,F} \tag{3}$$

where s_t is the fraction of high-skilled unemployed that search for a high-skill job, $u_t^{h,H}\equiv\left(1-O_t^h\right)s_tu_t^h$, $u_t^{hl,H}\equiv\left(1-O_t^h\right)(1-s_t)\,u_t^h$ and $u_t^{h,F}\equiv O_t^hu_t^h$.

Following the literature, we assume that mismatched workers perform on-the-job search in the domestic economy and apply to vacancies with a high-skill requirement. The efficacy of this search, denoted by $\phi(z_t)$, depends positively on the endogenous effort they exert, z_t, while the cost of searching is denoted by $b(z_t)$, with $db(z_t)/dz_t>0$. If the search of mismatched workers is successful, they quit the low-skill position in favor of the high-skill position (see below). Note that all unemployed members search with intensity one and there is no pecuniary cost associated with their search.

Each low-skilled household l consists of members that are employed in a low-skill position, n_t^l, members that are unemployed, u_t^l, and members that enjoy leisure, l_t^l. Unlike the high-skilled employees, there is no possibility of a mismatch as we abstract from over-employment in the model. The low-skilled employees, n_t^l, are employed in a low-skill position at home, $n_t^{l,H}$, and a low-skill position abroad, $n_t^{l,F}$ (hereafter, low-skilled emigrants), so that

$$n_t^{l,H}+u_t^l+l_t^l+n_t^{l,F}=1 \tag{4}$$

where the fraction $1-n_t^{l,F}=n_t^{l,H}+u_t^l+l_t^l$ denotes the low-skilled households that stay at the home country (hereafter, called low-skilled stayers).

The low-skilled household chooses the fraction of its members that search for a a low-skill job abroad O_t^l versus at home $\left(1-O_t^l\right)$:

$$u_t^l=\left(1-O_t^l\right)u_t^l+O_t^lu_t^l=u_t^{l,H}+u_t^{l,F} \tag{5}$$

where $u_t^{l,h}\equiv\left(1-O_t^l\right)u_t^l$ and $u_t^{l,F}\equiv O_t^lu_t^l$.

2.3 Labour market

2.3.1 Labour market frictions and skill mismatch

The model considers three labour sub-markets in the domestic economy, depending both on the workers' skill type and on the position's qualifications. In each sub-market, jobs are created through a matching function. In particular, $M_t^{l,H}$ denotes the aggregate matches in the low-skill labour market, $M_t^{h,H}$ denotes the aggregate matches in the high-skill labour market, whereas $M_t^{hl,H}$ denotes the aggregate mismatches in the low-skill sub-market. The respective functions are given by:

$$M_t^{l,H} = \mu_1 \left(\left(1-x_t\right) V_t^l\right)^{\mu_2} \left(\left(1-O_t^l\right) u_t^l N_t^l\right)^{1-\mu_2} \tag{6}$$

$$M_t^{h,H} = \mu_1 \left(V_t^h\right)^{\mu_2} \left(\left(1-O_t^h\right) s_t u_t^h N_t^h + \phi\left(z_t\right) n_t^{hl,H} N_t^h\right)^{1-\mu_2} \tag{7}$$

$$M_t^{hl,H} = \mu_1 \left(x_t V_t^l\right)^{\mu_2} \left(\left(1-O_t^h\right)\left(1-s_t\right) u_t^h N_t^h\right)^{1-\mu_2} \tag{8}$$

where μ_1 denotes the efficiency of the matching process, μ_2 denotes the elasticity of matches with respect to vacancies, V_t^j denotes the aggregate vacancies posted by firms for skill type $j = h, l$ and x_t is the fraction of low-skill vacancies that are allocated by firms to high-skilled applicants, thus generating a mismatch. In equation (7), total searchers for a high-skill position comprise both the high-skilled unemployed job seekers, $(1-O_t^h)s_t u_t^h N_t^h$, and the mismatched employees who perform on-the-job search, $\phi(z_t)n_t^{hl,H} N_t^h$.

2.3.2 Probabilities and labour market tightness

Next, we define the hiring probabilities as follows:

$$\psi_{H,t}^{l,H} = \frac{M_t^{l,H}}{\left(1-O_t^l\right) u_t^l N_t^l} \quad \psi_{H,t}^{h,H} = \frac{M_t^{h,H}}{\left(1-O_t^h\right) s_t u_t^h N_t^h + \phi\left(z_t\right) n_t^{hl,H} N_t^h} \quad \psi_{H,t}^{hl,H} = \frac{M_t^{hl,H}}{\left(1-O_t^h\right)\left(1-s_t\right) u_t^h N_t^h} \tag{9}$$

We also define the vacancy-filling probabilities:

$$\psi_{F,t}^{l,H} = \frac{M_t^{l,H}}{\left(1-x_t\right) V_t^l} \qquad \psi_{F,t}^{h,H} = \frac{M_t^{h,H}}{V_t^h} \qquad \psi_{F,t}^{hl,H} = \frac{M_t^{hl,H}}{x_t V_t^l} \tag{10}$$

In turn, labour market tightness is given by:

$$\theta_t^{l,H} = \frac{\left(1-x_t\right) V_t^l}{\left(1-O_t^l\right) u_t^l N_t^l} \qquad \theta_t^{h,H} = \frac{V_t^h}{\left(1-O_t^h\right) s_t u_t^h N_t^h + \phi\left(z_t\right) n_t^{hl,H} N_t^h} \qquad \theta_t^{hl,H} = \frac{x_t V_t^l}{\left(1-O_t^h\right)\left(1-s_t\right) u_t^h N_t^h} \tag{11}$$

2.3.3 Employment laws of motion

The law of motion for aggregate *mismatch* employment is given by:

$$N_t^h n_{t+1}^{hl,H} = \left(1-\sigma^l-\phi(z_t)\psi_{H,t}^{h,H}\right) n_t^{hl,H} N_t^h + M_t^{hl,H} \tag{12}$$

where σ^l is the exogenous destruction rate of low-skill positions (whether there is a mismatch or not), and $\phi(z_t)\psi_{H,t}^{h,H}$ is the endogenous destruction rate due to on-the-job search and quits to take up a non-mismatched job. The laws of motion for the various types of aggregate non-mismatch employment in the model are:

$$N_t^l n_{t+1}^{l,H} = \left(1-\sigma^l\right) n_t^{l,H} N_t^l + M_t^{l,H} \tag{13}$$

$$N_t^h n_{t+1}^{h,H} = \left(1-\sigma^h\right) n_t^{h,H} N_t^h + M_t^{h,H} \tag{14}$$

$$N_t^j n_{t+1}^{j,F} = \left(1 - \sigma^{j,F}\right) n_t^{j,F} N_t^j + \psi^{j,F} O_t^j u_t^j N_t^j, \; j = h, l \tag{15}$$

The law of motion of emigrant workers depends on the foreign separation rate, $\sigma^{j,F}$, and the foreign job-finding rate, $\psi^{j,F}$, which the SOE takes as given.[2]

2.4 Households

2.4.1 High-skilled households

The high-skilled household maximizes the following lifetime utility:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left(1 - n_t^{h,F}\right) \left(\frac{\left(c_t^{h,H}\right)^{1-\eta}}{1-\eta} + \Phi^h \frac{\left(l_t^h\right)^{1-\phi^h}}{1-\phi^h} \right) + n_t^{h,F} \frac{\left(c_t^{h,F}\right)^{1-\eta}}{1-\eta} \right\} \tag{16}$$

where $c_t^{h,H}$ and $c_t^{h,F}$ are the household h's consumption quantities at home and abroad respectively, l_t^h is the leisure time, $\eta > 0$ is the inverse of intertemporal elasticity of substitution, Φ^h is the relative preference for leisure and ϕ^h is the inverse of the Frisch elasticity of labour supply.

Each high-skilled household chooses consumption levels $c_t^{h,H}$, $c_t^{h,F}$, leisure l_t^h, and also decides to invest in physical capital, k_t^h, and in an international non-state contingent bond, d_t^h. Thus, it receives an interest income $r_t^k k_t^h$ and $r_t^d d_t^h$ from capital and net foreign assets, where r_t^k and r_t^d denote the respective returns. Additionally, each high-skilled household receives as the firm owner a share of profits $\pi_{d,t}$. Each household h also decides on the fraction s_t of its unemployed members that search for a high-skill position, with the remaining $(1 - s_t)$ searching for a position in the low-skill labour market. The household h's budget constraint is:

$$\begin{aligned} &\left(1 - n_t^{h,F}\right) c_t^{h,H} + e_t n_t^{h,F} c_t^{h,F} + i_t^h + X^h \left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h + e_t r_t^d d_t^h + b\left(z_t\right) n_t^{hl,H} \\ &= w_t^{h,H} n_t^{h,H} + w_t^{hl,H} n_t^{hl,H} + e_t w^{h,F} n_t^{h,F} + r_t^k k_t^h - \tau_t^h + \pi_{d,t} + e_t d_{t+1}^h + \bar{\omega} u_t^h + \bar{g}_t^{t,h} \end{aligned} \tag{17}$$

where e_t is the real exchange rate, τ_t^h are the lump-sum taxes common to all types of households[3], $O_t^h u_t^h$ refers to the new flow of emigrants who pay a pecuniary moving cost $X^h(\tilde{O}_t^h \tilde{u}_t^h)$, $\tilde{O}_t^h$ and $\tilde{u}_t^h$ are the average shares of O_t^h and u_t^h per household[4], $w_t^{h,H}$ is the wage rate of non-mismatched workers at home, $e_t w^{h,F}$ is the exogenous real-exchange-rate-adjusted foreign wage, $w_t^{hl,H}$ is the wage rate of mismatched workers, $\bar{\omega} u_t^h$ is the unemployment benefit, and $\bar{g}_t^{t,h}$ is a lump-sum transfers.

The capital law of motion evolves according to:

$$i_t^h = k_{t+1}^h - (1-\delta) k_t^h + \frac{\Xi}{2} \left(\frac{k_{t+1}^h}{k_t^h} - 1 \right)^2 k_t^h \tag{18}$$

where δ is the depreciation rate and parameter Ξ controls the capital adjustment costs, which are useful to obtain smooth impulse responses.

[2] The SOE has no impact on foreign labour market conditions which are taken as exogenous processes. This is a reasonable assumption even in the presence of large migration outflows if their relative size compared to the destination country labour force is small (e.g., due to dispersed search across several destination countries).

[3] $\tau_t^h = t^h \tau_t$

[4] See, e.g., Bandeira et al. (2022) for a similar specification of the moving cost.

High-skilled household optimization problem

The optimization problem of the high-skilled household is subject to the budget constraint (17), the time constraint (2), the law of motion of capital (18), the laws of motion of the stayers, the mismatched and the emigrants (14, 12 and 15), and the hiring probabilities of the stayers and the mismatched (9).

The first-order conditions with respect to $c_t^{h,H}$, $c_t^{h,F}$, k_{t+1}^h, d_{t+1}^h, $n_{t+1}^{h,H}$, $n_{t+1}^{hl,H}$, $n_{t+1}^{h,F}$, u_t^h, s_t, O_t^h, z_t are:

$[c_t^{h,H}]$

$$\lambda_{c_t^h} = \left(c_t^{h,H}\right)^{-\eta} \tag{19}$$

$[c_t^{h,F}]$

$$e_t \lambda_{c_t^h} = \left(c_t^{h,F}\right)^{-\eta} \tag{20}$$

$[k_{t+1}^h]$

$$\lambda_{c_t^h}\left(1+\Xi\left(\frac{k_{t+1}^h}{k_t^h}-1\right)\right) = \beta \mathbb{E}_t \lambda_{c_{t+1}^h}\left(1+r_{t+1}^k-\delta+\frac{\Xi}{2}\left(\left(\frac{k_{t+2}^h}{k_{t+1}^h}\right)^2-1\right)\right) \tag{21}$$

$[d_{t+1}^h]$

$$\lambda_{c_t^h} e_t = \beta \mathbb{E}_t \lambda_{c_{t+1}^h} e_{t+1} r_{t+1}^d \tag{22}$$

$[n_{t+1}^{h,H}]$

$$\lambda_{n_t^{h,H}} = \beta \mathbb{E}_t \left\{-\left(1-n_{t+1}^{h,F}\right)\Phi^h \left(l_{t+1}^h\right)^{-\phi^h} + \lambda_{c_{t+1}^h} w_{t+1}^{h,H} + \lambda_{n_{t+1}^{h,H}}\left(1-\sigma^h\right)\right\} \tag{23}$$

$[n_{t+1}^{hl,H}]$

$$\begin{aligned}\lambda_{n_t^{hl,H}} = \beta \mathbb{E}_t \Big\{&-\left(1-n_{t+1}^{h,F}\right)\Phi^h \left(l_{t+1}^h\right)^{-\phi^h} + \lambda_{c_{t+1}^h}\left(w_{t+1}^{hl,H}-b(z_{t+1})\right) + \lambda_{n_{t+1}^{hl,H}}\left(1-\sigma^l-\phi(z_{t+1})\psi_{H,t+1}^{h,H}\right) \\ &+ \lambda_{n_{t+1}^{h,H}}\psi_{H,t+1}^{h,H}\phi\left(z_{t+1}\right)\Big\}\end{aligned} \tag{24}$$

$[n_{t+1}^{h,F}]$

$$\begin{aligned}\lambda_{n_t^{h,F}} = \beta \mathbb{E}_t \Bigg\{&\frac{\left(c_{t+1}^{h,F}\right)^{1-\eta}}{1-\eta} - \frac{\left(c_{t+1}^{h,H}\right)^{1-\eta}}{1-\eta} - \Phi^h \frac{\left(l_{t+1}^h\right)^{1-\phi^h}}{1-\phi^h} - \left(1-n_{t+1}^{h,F}\right)\Phi^h\left(l_{t+1}^h\right)^{-\phi^h} \\ &- \lambda_{c_{t+1}^h}\left(e_t c_{t+1}^{h,F} - c_{t+1}^{h,H} - e_{t+1} w^{h,F}\right) + \lambda_{n_{t+1}^{h,F}}\left(1-\sigma^{h,F}\right)\Bigg\}\end{aligned} \tag{25}$$

$[u_t^h]$

$$\begin{aligned}\left(1-n_t^{h,F}\right)\Phi^h\left(l_t^h\right)^{-\phi^h} = &-\lambda_{c_t^h}\left(X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right)O_t^h - \bar{\omega}\right) + \lambda_{n_t^{h,H}}\psi_{H,t}^{h,H}\left(1-O_t^h\right)s_t \\ &+ \lambda_{n_t^{hl,H}}\psi_{H,t}^{hl,H}\left(1-O_t^h\right)\left(1-s_t\right) + \lambda_{n_t^{h,F}}\psi^{h,F}O_t^h\end{aligned} \tag{26}$$

$[s_t]$

$$\lambda_{n_t^{h,H}}\psi_{H,t}^{h,H} = \lambda_{n_t^{hl,H}}\psi_{H,t}^{hl,H} \tag{27}$$

$[O_t^h]$

$$\lambda_{n_t^{h,H}}\psi_{H,t}^{h,H}s_t + \lambda_{n_t^{hl,H}}\psi_{H,t}^{hl,H}(1-s_t) = \lambda_{n_t^{h,F}}\psi^{h,F} - \lambda_{c_t^h}X^h(\tilde{O}_t^h\tilde{u}_t^h) \tag{28}$$

$[z_t]$

$$\lambda_{c_t^h}\frac{b'\left(z_t\right)}{\phi'\left(z_t\right)} = \psi_{H,t}^{h,H}\left(\lambda_{n_t^{h,H}} - \lambda_{n_t^{hl,H}}\right) \tag{29}$$

The last equation determines the optimal level of search intensity for the mismatched workers stating that search intensity increases with the difference between the asset values of a high-skill job and a mismatch job as well as with the probability of finding a high-skill job. The right-hand-side accounts for the marginal cost of the search in consumption units.

2.4.2 Low-skilled households

The low-skilled household maximizes the following lifetime utility:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \left(1 - n_t^{l,F}\right) \left(\frac{\left(c_t^{l,H}\right)^{1-\eta}}{1-\eta} + \Phi^l \frac{(l_t^l)^{1-\phi^l}}{1-\phi^l} \right) + n_t^{l,F} \frac{\left(c_t^{l,F}\right)^{1-\eta}}{1-\eta} \right\} \tag{30}$$

The household $l's$ budget constraint is:

$$\left(1 - n_t^{l,F}\right) c_t^{l,H} + e_t n_t^{l,F} c_t^{l,F} + X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l = w_t^{l,H} n_t^{l,H} + e_t w^{l,F} n_t^{l,F} - \tau_t^l + \bar{\omega} u_t^l + \bar{g}_t^{t,l} \tag{31}$$

The variables and parameters are similar to the high-skilled households.

Low-skilled household optimization problem

The optimization problem of the low-skilled household is subject to the budget constraint (31), the time constraint (4), the laws of motion of the stayers and the emigrants (13 and 15), and the hiring probability of the stayers (9).

The first order conditions with respect to $c_t^{l,H}$, $c_t^{l,F}$, $n_{t+1}^{l,H}$, $n_{t+1}^{l,F}$, O_t^l, u_t^l are:

$[c_t^{l,H}]$

$$\lambda_{c_t^l} = \left(c_t^{l,H}\right)^{-\eta} \tag{32}$$

$[c_t^{l,F}]$

$$e_t \lambda_{c_t^l} = \left(c_t^{l,F}\right)^{-\eta} \tag{33}$$

$[n_{t+1}^{l,H}]$

$$\lambda_{n_t^{l,H}} = \beta \mathbb{E}_t \left\{ -\left(1 - n_{t+1}^{l,F}\right) \Phi^l \left(l_{t+1}^l\right)^{-\phi^l} + \lambda_{c_{t+1}^l} w_{t+1}^{l,H} + \lambda_{n_{t+1}^{l,H}} \left(1 - \sigma^l\right) \right\} \tag{34}$$

$[n_{t+1}^{l,F}]$

$$\begin{aligned} \lambda_{n_t^{l,F}} = \beta \mathbb{E}_t \Bigg\{ & \frac{\left(c_{t+1}^{l,F}\right)^{1-\eta}}{1-\eta} - \frac{\left(c_{t+1}^{l,H}\right)^{1-\eta}}{1-\eta} - \Phi^l \frac{\left(l_{t+1}^l\right)^{1-\phi^l}}{1-\phi^l} - \left(1 - n_{t+1}^{l,F}\right) \Phi^l \left(l_{t+1}^l\right)^{-\phi^l} \\ & - \lambda_{c_{t+1}^l} \left(e_t c_{t+1}^{l,F} - c_{t+1}^{l,H} - e_{t+1} w^{l,F}\right) + \lambda_{n_{t+1}^{l,F}} \left(1 - \sigma^{l,F}\right) \Bigg\} \end{aligned} \tag{35}$$

$[u_t^l]$

$$\left(1 - n_t^{l,F}\right) \Phi^l \left(l_t^l\right)^{-\phi^l} = -\lambda_{c_t^l} \left(X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l - \bar{\omega}\right) + \lambda_{n_t^{l,H}} \psi_{H,t}^{l,H} \left(1 - O_t^l\right) + \lambda_{n_t^{l,F}} \psi^{l,F} O_t^l \tag{36}$$

$[O_t^l]$

$$\lambda_{n_t^{l,H}} \psi_{H,t}^{l,H} = \lambda_{n_t^{l,F}} \psi^{l,F} - \lambda_{c_t^l} X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) \tag{37}$$

2.5 Firms

2.5.1 Economy-wide final good

The representative final good firm aggregates the domestic intermediate good, Y_t^H, and imported aggregate goods, Y_t^F, to produce the economy-wide final good, Y_t, using a CES technology:

$$Y_t = \left(\omega^{\frac{1}{\gamma}} \left(Y_t^H\right)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}} \left(Y_t^F\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{38}$$

where ω denotes the degree of home bias and γ is the elasticity of substitution between home-produced and imported goods. The economy-wide final good firm maximizes profits, $\Pi_t = P_t Y_t - p_t^H Y_t^H - p_t^F Y_t^F$, where p_t^H and p_t^F are the relative prices of the domestic and foreign intermediate goods respectively. This yields the following optimal demand schedules:

$$Y_t^H = \omega \left(\frac{p_t^H}{P_t}\right)^{-\gamma} Y_t \tag{39}$$

$$Y_t^F = (1-\omega) \left(\frac{p_t^F}{P_t}\right)^{-\gamma} Y_t \tag{40}$$

Combining equations (39) and (40) yields:

$$Y_t^H = \frac{\omega}{1-\omega} \left(\frac{p_t^H}{p_t^F}\right)^{-\gamma} Y_t^F \tag{41}$$

The associated price index is given by:

$$P_t = \left(\omega \left(p_t^H\right)^{1-\gamma} + (1-\omega) \left(p_t^F\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{42}$$

where we have assumed that the law of one price holds as follows:

$$p_t^H = e_t p_t^F \tag{43}$$

2.5.2 Intermediate good

Each intermediate good firm $f = 1, 2, ..., N_t^f$ requires capital k_t^f, low-skilled employment $n_t^{l,f}$, and high-skilled employment $n_t^{h,f}$ to produce the domestic intermediate good $y_{i,t}^f$ with a CES technology:

$$y_{i,t}^f = A_t \left(\alpha \left(n_t^{l,f}\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) \left(x_{i,t}^f\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{44}$$

$$x_{i,t}^f = \left(\zeta \left(k_t^f\right)^{\frac{\rho-1}{\rho}} + (1-\zeta) \left(n_t^{h,f}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{45}$$

where A_t denotes the exogenous technology process, $0 \leq \alpha$ and $0 \leq \zeta$ control the income shares, $0 \leq \epsilon$ is the elasticity of substitution both between low-skilled labour and capital, and between low-skilled labour and high-skilled labour, and $0 \leq \rho$ is the elasticity of substitution between capital and high-skilled labour. For capital-skill complementarity, we need $\rho < \epsilon$ (see, e.g. Krusell et al. (2000)).

As in e.g. Iftikhar and Zaharieva (2019), we assume different productivity between the two types of workers employed in low-skill positions. Following Acemoglu (2001), we assume that sub-markets with different skill requirements produce different intermediate goods, which are used in the production. Thus, for every firm f in the intermediate goods sector, the input of low-skill positions is equal to:

$$n_t^{l,f} = n_t^{ll,f} + q^h n_t^{hl,f} \tag{46}$$

where $n_t^{ll,f}$ denotes the labour demand for low-skilled workers and $q^h n_t^{hl,f}$ denotes the labour demand for high-skilled workers in a low-skill position (who are therefore mismatched). The parameter $q^h \geq 1$ reflects the effective productivity of a high-skilled worker in a low skill position.

We denote the marginal productivities of k_t^f, $n_t^{ll,f}$, $n_t^{h,f}$ and $n_t^{hl,f}$ to be:

$$y_{i,t}^{k,f} = \frac{\partial y_{i,t}^f}{\partial k_t^f}, \qquad y_{i,t}^{ll,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{ll,f}}, \qquad y_{i,t}^{h,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}}, \qquad y_{i,t}^{hl,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{hl,f}} \tag{47}$$

The intermediate good, $y_{i,t}^f$, is sold domestically, y_t^H, and abroad, $y_t^{F^*}$. In aggregate terms this is:[5]

$$Y_{i,t} = Y_t^H + Y_t^{F^*} \tag{48}$$

where foreign aggregate demand $Y_t^{F^*}$ is given exogenously by:

$$Y_t^{F^*} = (1 - \omega^*) \left(\frac{p_t^H}{e_t} \right)^{-\gamma^*} Y_t^* \tag{49}$$

We define the real exchange rate as $e_t = \frac{P_t^*}{P_t}$. We also consider the parameters ω^* and γ^* to be the foreign counterparts for the home bias and elasticity of substitution and denote Y_t^* to be the foreign GDP.[6]

Firm's profit maximization problem

Firms post both positions requiring high skills and positions requiring only low skills. Since a fraction of high-skilled searchers apply for low-skill positions, firms consider all applications for such positions and decide not only $v_t^{l,f}$ and $v_t^{h,f}$, but also the fraction of vacancies requiring low skills that will be allocated to high-skilled applicants, x_t. In doing so, the following trade-off arises: in the event of a mismatch, the worker is more productive than the non-mismatched worker, but the mismatch maybe terminated if she quits to take up a non-mismatched job via on-the-job search. Therefore, skills mismatch arises endogenously from an interplay of the households and firms decisions. Additionally, the firm also chooses the amount of capital to demand.[7]

The firm solves the following problem of discounted expected value of future profits:

$$\begin{aligned} Q(n_t^{ll,f}, n_t^{h,f}, n_t^{hl,f}) = \max_{v_t^{l,f}, v_t^{h,f}, k_t^f, x_t} \Big\{ & p_t^H y_{i,t}^f - w_t^{l,H} n_t^{ll,f} - w_t^{hl,H} n_t^{hl,f} - w_t^{h,H} n_t^{h,f} - r_t^k k_t^f - \kappa^l v_t^{l,f} - \kappa^h v_t^{h,f} \\ & + E_t \left[\Lambda_{t,t+1} Q\left(n_{t+1}^{ll,f}, n_{t+1}^{h,f}, n_{t+1}^{hl,f} \right) \right] \Big\} \end{aligned} \tag{50}$$

[5] $Y_{i,t} = \sum_{f=1}^{N_t^f} y_{i,t}^f$, $Y_t^H = \sum_{f=1}^{N_t^f} y_t^H$ and $Y_t^{F^*} = \sum_{f=1}^{N_t^f} y_t^{F^*}$

[6] The structure of the foreign economy is similar to the home economy but due to the small size of the latter, domestic developments have a negligible effect in foreign economy dynamics. As a result, Y_t^* is considered to be exogenous.

[7] Şahin et al. (2014) allow the misallocation of unemployed workers across sectors to also affect the vacancy creation decisions of firms. In that case, they show that the contribution of mismatch to unemployment's higher. This occurs because unemployed workers who search in declining sectors make it easier to fill jobs in those sectors, distorting firms' incentives in the direction of creating inefficiently vacancies.

where κ^l and κ^h denote the marginal cost of posting a vacancy for low and high skill workers respectively, $w_t^{hl,H} n_t^{hl,f}$ denotes the total cost of employing a mismatched worker at the wage rate $w_t^{hl,H}$.
The above problem is subject to the equivalent expressions of the law of motion of employment, equations (12), (13), (14), using the vacancy-filling probabilities:

$$N_t^f n_{t+1}^{hl,f} = \left(1 - \sigma^l - \phi\left(z_t\right)\psi_{H,t}^{h,H}\right) n_t^{hl,f} N_t^f + \psi_{F,t}^{hl,H} x_t v_t^{l,f} N_t^f \tag{51}$$

$$N_t^f n_{t+1}^{ll,f} = \left(1 - \sigma^l\right) n_t^{ll,f} N_t^f + \psi_{F,t}^{l,H}\left(1 - x_t\right) v_t^{l,f} N_t^f \tag{52}$$

$$N_t^f n_{t+1}^{h,f} = \left(1 - \sigma^h\right) n_t^{h,f} N_t^f + \psi_{F,t}^{h,H} v_t^{h,f} N_t^f \tag{53}$$

As the household owns the firm, the term $\Lambda_{t,t+1} = \beta \frac{\partial u_{c_{t+1}}}{\partial u_{c_t}}$ refers to the household's stochastic discount factor in which λ_{c_t} denotes the Lagrange multiplier for the household budget constraint and β is the household's discount factor.

Before presenting the first order conditions of the firm, we rearrange the laws of motion of employment as follows:

1) Solve equation (51) for x_t and substitute into equation (52):

$$v_t^{l,f} = \frac{n_{t+1}^{ll,f} - \left(1 - \sigma^l\right) n_t^{ll,f}}{\psi_{F,t}^{l,H}} + \frac{n_{t+1}^{hl,f} - \left(1 - \sigma^l - \phi\left(z_t\right)\psi_{H,t}^{h,H}\right) n_t^{hl,f}}{\psi_{F,t}^{hl,H}} \tag{54}$$

2) Solve equation (53) for $v_t^{h,f}$:

$$v_t^{h,f} = \frac{n_{t+1}^{h,f} - \left(1 - \sigma^h\right) n_t^{h,f}}{\psi_{F,t}^{h,H}} \tag{55}$$

Therefore, the firm maximizes equation (50), taking into account the production function, equations (44) and (45), firm's labour demand for low skill employment, equation (46), and the law of motion of employment for low and high-skill positions, equations (54) and (55) respectively. Then, using the optimal values for $n_{t+1}^{ll,f}$ and $n_{t+1}^{hl,f}$, x_t will be given residually from equation (51).

The first order conditions are given by:

$[n_{t+1}^{ll,f}]$

$$\frac{\kappa^l}{\psi_{F,t}^{l,H}} = \mathbb{E}_t \Lambda_{t,t+1}\left\{p_{t+1}^H y_{i,t+1}^{ll,f} - w_{t+1}^{l,H} + \kappa^l \frac{\left(1 - \sigma^l\right)}{\psi_{F,t+1}^{l,H}}\right\} \tag{56}$$

$[n_{t+1}^{h,f}]$

$$\frac{\kappa^h}{\psi_{F,t}^{h,H}} = \mathbb{E}_t \Lambda_{t,t+1}\left\{p_{t+1}^H y_{i,t+1}^{h,f} - w_{t+1}^{h,H} + \kappa^h \frac{\left(1 - \sigma^h\right)}{\psi_{F,t+1}^{h,H}}\right\} \tag{57}$$

$[n_{t+1}^{hl,f}]$

$$\frac{\kappa^l}{\psi_{F,t}^{hl,H}} = \mathbb{E}_t \Lambda_{t,t+1}\left\{p_{t+1}^H y_{i,t+1}^{hl,f} - w_{t+1}^{hl,H} + \kappa^l \frac{\left(1 - \sigma^l - \phi\left(z_{t+1}\right)\psi_{H,t+1}^{h,H}\right)}{\psi_{F,t+1}^{hl,H}}\right\} \tag{58}$$

$[k_t^f]$:

$$r_t^k = p_t^H y_{i,t}^{k,f} \tag{59}$$

where x_t is given by:

$$x_t = \frac{n_{t+1}^{hl,f} - \left(1 - \sigma^l - \phi\left(z_t\right)\psi_{H,t}^{h,H}\right) n_t^{hl,f}}{v_t^{l,f}\psi_{F,t}^{hl,H}} \tag{60}$$

In the last equation, we see clearly that the endogenous probability of quitting a mismatch job to take up an upgraded position negatively affects the share of low-skill positions that a firm is willing to allocate to high-skilled applicants.

2.6 Government

The government imposes a uniform lump-sum tax T_t on both household types and uses the revenue to finance aggregate unemployment benefits, $\bar{\omega}U_t^h$ and $\bar{\omega}U_t^l$, lump-sum transfers, $G_t^{t,h}$ and $G_t^{t,l}$, and government consumption, G_t^c. The government budget constraint in aggregate terms follows:[8]

$$\bar{\omega}U_t^h + \bar{\omega}U_t^l + G_t^{t,h} + G_t^{t,l} + G_t^c = T_t \tag{61}$$

The distribution of lump-sum taxes is assumed to be equal, i.e. $\tau_t^h = \tau_t^l = T_t$, so that we have $T_t = t^h\tau_t^h + t^l\tau_t^l = T_t\left(t^h + t^l\right)$. Under a balanced government budget, T_t adjusts to satisfy equation (61).

2.7 Wage bargaining

The Nash bargaining problem is to maximize the weighted sum of log surpluses for each employment status.[9] The wages are thus given as the optimal solution of the following three problems:

$$\max_{w_t^{h,H}} \left\{\left(1 - \theta^h\right)\ln V_{n_t^h}^h + \theta^h \ln V_{n_t^h}^f\right\} \tag{62}$$

$$\max_{w_t^{hl,H}} \left\{\left(1 - \theta^{hl}\right)\ln V_{n_t^{hl}}^h + \theta^{hl} \ln V_{n_t^{hl}}^f\right\} \tag{63}$$

$$\max_{w_t^{l,H}} \left\{\left(1 - \theta^l\right)\ln V_{n_t^l}^h + \theta^l \ln V_{n_t^l}^f\right\} \tag{64}$$

where θ^l, θ^{hl} and θ^h denote the bargaining power on wage setting of firms for low, mismatch and high-skill positions, $V_{n_t^h}^f$, $V_{n_t^{hl}}^f$ and $V_{n_t^l}^f$ are the respective value functions of an additional unit of high-skill, low-skill and mismatch employment to each firm, and $V_{n_t^h}^h$ and ${}^h_{n_t^{hl}}$ are the respective marginal values of a high-skilled household having an additional member employed in a high-skill or mismatch position and $V_{n_t^l}^h$ is the respective marginal value of a low skill household having an additional member employed in a low-skill position. Hence, in line with equations (23), (24) and (34), the value functions of the household are:

$[V_{n_t^h}^h]$

$$V_{n_t^h}^h = -\left(1 - n_t^{h,F}\right)\Phi^h\left(l_t^h\right)^{-\phi^h} + \lambda_{c_t^h} w_t^{h,H} + \lambda_{n_t^{h,H}}\left(1 - \sigma^h\right) \tag{65}$$

[8] $U_t^h = \sum_{h=1}^{N_t^h} u_t^h, U_t^l = \sum_{l=1}^{N_t^l} u_t^l, G_t^{t,h} = \sum_{h=1}^{N_t^h} \bar{g}_t^{t,h}, G_t^{t,l} = \sum_{l=1}^{N_t^l} \bar{g}_t^{t,l}, T_t^h = t^h T_t, T_t^l = t^l T_t, T_t = \left(t^h + t^l\right) T_t$

[9] For a detailed presentation of the Nash bargaining problem, see Appendix A.

$[V^h_{n^{hl}_t}]$

$$V^h_{n^{hl}_t} = -\left(1 - n^{h,F}_t\right)\Phi^h\left(l^h_t\right)^{-\phi^h} + \lambda_{c^h_t}\left(w^{hl,H}_t - b\left(z_t\right)\right) + \lambda_{n^{hl,H}_t}\left(1 - \sigma^l - \phi\left(z_t\right)\psi^{h,H}_{H,t}\right) + \lambda_{n^{h,H}_t}\psi^{h,H}_{H,t}\phi\left(z_t\right) \quad (66)$$

$[V^h_{n^l_t}]$

$$V^h_{n^l_t} = -\left(1 - n^{l,F}_t\right)\Phi^l\left(l^l_t\right)^{-\phi^l} + \lambda_{c^l_t}w^{l,H}_t + \lambda_{n^{l,H}_t}\left(1 - \sigma^l\right) \quad (67)$$

According to equations (57), (58) and (56), the firm's value functions are:

$[V^f_{n^h_t}]$

$$V^f_{n^h_t} = p^H_t y^{h,f}_{i,t} + \kappa^h \frac{\left(1 - \sigma^h\right)}{\psi^{h,H}_{F,t}} \quad (68)$$

$[V^f_{n^{hl}_t}]$

$$V^f_{n^{hl}_t} = p^H_t y^{hl,f}_{i,t} - w^{hl,H}_t + \kappa^l \frac{\left(1 - \sigma^l - \phi\left(z_t\right)\psi^{h,H}_{H,t}\right)}{\psi^{hl,H}_{F,t}} \quad (69)$$

$[V^f_{n^l_t}]$

$$V^f_{n^l_t} = p^H_t y^{ll,f}_{i,t} - w^{l,H}_t + \kappa^l \frac{\left(1 - \sigma^l\right)}{\psi^{l,H}_{F,t}} \quad (70)$$

The wage rate $w^{h,H}_t$ is given by:

$$w^{h,H}_t = \left(1 - \theta^h\right)\left(p^H_t y^{h,f}_{i,t} + \kappa^h \frac{\left(1 - \sigma^h\right)}{\psi^{h,H}_{F,t}}\right) - \frac{\theta^h}{\lambda_{c^h_t}}\left(-\left(1 - n^{h,F}_t\right)\Phi^h\left(l^h_t\right)^{-\phi^h} + \lambda_{n^{h,H}_t}\left(1 - \sigma^h\right)\right) \quad (71)$$

The wage rate $w^{l,H}_t$ is given by:

$$w^{l,H}_t = \left(1 - \theta^l\right)\left(p^H_t y^{ll,f}_{i,t} + \kappa^l \frac{\left(1 - \sigma^l\right)}{\psi^{l,H}_{F,t}}\right) - \frac{\theta^l}{\lambda_{c^l_t}}\left(-\left(1 - n^{l,F}_t\right)\Phi^l\left(l^l_t\right)^{-\phi^l} + \lambda_{n^{l,H}_t}\left(1 - \sigma^l\right)\right) \quad (72)$$

The wage rate $w^{hl,H}_t$ is given by:

$$\begin{aligned} w^{hl,H}_t = {} & \left(1 - \theta^{hl}\right)\left(p^H_t y^{hl,f}_{i,t} + \kappa^l \frac{\left(1 - \sigma^l - \phi\left(z_t\right)\psi^{h,H}_{H,t}\right)}{\psi^{hl,H}_{F,t}}\right) \\ & - \frac{\theta^{hl}}{\lambda_{c^h_t}}\left(-\left(1 - n^{h,F}_t\right)\Phi^h\left(l^h_t\right)^{-\phi^h} - \lambda_{c^h_t}b\left(z_t\right) + \lambda_{n^{hl,H}_t}\left(1 - \sigma^l - \phi\left(z_t\right)\psi^{h,H}_{H,t}\right) + \lambda_{n^{h,H}_t}\psi^{h,H}_{H,t}\phi\left(z_t\right)\right) \end{aligned} \quad (73)$$

In the last equation, the increased likelihood of leaving the firm $\phi(z_t)\psi^{h,H}_{H,t}$ requires mismatch workers to accept a lower wage. On the other hand, the search cost $b(z_t)$ increases the wage that firms need to pay the worker.

2.8 Closing the model

The known issue of non-stationarity that arises in the small open economy models is addressed by assuming the following debt-elastic interest rate:

$$r^d_t = r^*_t + rp_t \quad (74)$$

where r^*_t is the foreign interest rate which the small open economy takes as given and rp_t is the risk-premium it pays:

$$rp_t = \psi^{rp}\left(exp\left(\frac{e_t d_{t+1}}{gdp_t} - \frac{ed}{gdp}\right) - 1\right) + \epsilon_t^{rp} \tag{75}$$

where ϵ_t^{rp} denotes a risk premium shock.

Aggregating the household's budget constraint using the market clearing conditions, the government's budget constraint and aggregate profits, we obtain the law of motion for net foreign assets:

$$e_t\left(r_t^d d_t - d_{t+1}\right) = nx_t \tag{76}$$

where nx_t are total net exports defined as:

$$nx_t = \underbrace{p_t^H y_t^{F^*}}_{exports} - \underbrace{p_t^F y_t^F}_{imports} \tag{77}$$

In turn, real GDP is defined as:

$$gdp_t = y_t + nx_t \tag{78}$$

2.9 Market clearing conditions

Economy-wide final good (resource constraint)[10]

$$\begin{aligned} y_t = {}& (1 - n_t^{h,F})c_t^{h,H} t^h + (1 - n_t^{l,F})c_t^{l,H} t^l + e_t n_t^{h,F}(c_t^{h,F} - w^{h,F})t^h + e_t n_t^{l,F}(c_t^{l,F} - w^{l,F})t^l \\ & + i_t + g_t^c + X^h(\tilde{O}_t^h \tilde{u}_t^h)O_t^h u_t^h t^h + X^l(\tilde{O}_t^l \tilde{u}_t^l)O_t^l u_t^l t^l + b(z_t)n_t^{hl,H} t^h + \kappa^h v_t^h + \kappa^l v_t^l \end{aligned} \tag{79}$$

Intermediate good

$$y_{i,t} = \frac{1}{N_t}\sum_{f=1}^{N_t^f} y_{i,t}^f = \frac{1}{N_t} N_t^f y_{i,t}^f = t^h y_{i,t}^f \tag{80}$$

$$x_{i,t} = \frac{1}{N_t}\sum_{f=1}^{N_t^f} x_{i,t}^f = \frac{1}{N_t} N_t^f = t^h x_{i,t}^f \tag{81}$$

Capital, investment and foreign assets

$$k_t = \frac{1}{N_t}\sum_{h=1}^{N_t^h} k_t^h = \frac{1}{N_t}\sum_{f=1}^{N_t^f} k_t^f \Leftrightarrow k_t = \frac{1}{N_t} N_t^h k_t^h = \frac{1}{N_t} N_t^f k_t^f \Leftrightarrow k_t = t^h k_t^h = t^h k_t^f \tag{82}$$

$$i_t = \frac{1}{N_t}\sum_{h=1}^{N_t^h} i_t^h = \frac{1}{N_t} N_t^h i_t^h = t^h i_t^h \tag{83}$$

$$d_t = \frac{1}{N_t}\sum_{h=1}^{N_t^h} d_t^h = \frac{1}{N_t} N_t^h d_t^h = t^h d_t^h \tag{84}$$

[10] See Appendix B.2 for details on the calculation.

High-skilled unemployed

$$\frac{U_t^h}{N_t} = \frac{1}{N_t}\sum_{h=1}^{N_t^h} u_t^h = t^h u_t^h \tag{85}$$

Low-skilled unemployed

$$\frac{U_t^l}{N_t} = \frac{1}{N_t}\sum_{l=1}^{N_t^l} u_t^l = t^l u_t^l \tag{86}$$

High-skilled vacancies

$$v_t^h = \frac{V_t^h}{N_t} = \frac{1}{N_t}\sum_{f=1}^{N_t^f} v_t^{h,f} = t^h v_t^{h,f} \tag{87}$$

Low-skilled vacancies

$$v_t^l = \frac{V_t^l}{N_t} = \frac{1}{N_t}\sum_{f=1}^{N_t^f} v_t^{l,f} = t^h v_t^{l,f} \tag{88}$$

2.10 Transformations

Total population is given by:

$$N_t = N_t^h + N_t^l \tag{89}$$

we define t^h and t^l to be the shares of high and low-skilled households in the population respectively:

$$t^h = \frac{N_t^h}{N_t^h + N_t^l}, \qquad t^l = \frac{N_t^l}{N_t^h + N_t^l} \tag{90}$$

Since the high-skilled households own the firms, it must hold:

$$N_t^h = N_t^f \tag{91}$$

High-skilled labor, $n_t^{h,H}$

Supply

$$\sum_{h=1}^{N_t^h} n_t^{h,H} = N_t^h n_t^{h,H} \tag{92}$$

Demand

$$\sum_{f=1}^{N_t^f} n_t^{h,f} = N_t^f n_t^{h,f} \tag{93}$$

Supply=Demand

$$N_t^h n_t^{h,H} = N_t^f n_t^{h,f} \Rightarrow n_t^{h,f} = n_t^{h,H} \tag{94}$$

Low-skilled labor, $n_t^{l,H}$

Supply

$$\sum_{l=1}^{N_t^l} n_t^{l,H} = N_t^l n_t^{l,H} \tag{95}$$

Demand

$$\sum_{f=1}^{N_t^f} n_t^{ll,f} = N_t^f n_t^{ll,f} \tag{96}$$

Supply=Demand

$$N_t^l n_t^{l,H} = N_t^f n_t^{ll,f} \Rightarrow n_t^{ll,f} = \frac{N_t^l}{N_t^f} n_t^{l,H} \Rightarrow n_t^{ll,f} = \frac{t^l}{t^h} n_t^{l,H} \tag{97}$$

Mismatched labor, $n_t^{hl,H}$

Supply

$$\sum_{h=1}^{N_t^h} n_t^{hl,H} = N_t^h n_t^{hl,H} \tag{98}$$

Demand

$$\sum_{f=1}^{N_t^f} n_t^{hl,f} = N_t^f n_t^{hl,f} \tag{99}$$

Supply=Demand

$$N_t^h n_t^{hl,H} = N_t^f n_t^{hl,f} \Rightarrow n_t^{hl,f} = n_t^{hl,H} \tag{100}$$

Aggregate low-skilled labor, n_t^l

Supply

$$\sum_{l=1}^{N_t^l} n_t^{l,H} + \sum_{h=1}^{N_t^h} n_t^{hl,H} = N_t^l n_t^{l,H} + N_t^h n_t^{hl,H} \tag{101}$$

Demand

$$\sum_{f=1}^{N_t^f} n_t^{ll,f} + q^h \sum_{f=1}^{N_t^f} n_t^{hl,f} = N_t^f n_t^{ll,f} + q^h N_t^f n_t^{hl,f} \tag{102}$$

The aggregate low-skilled labor n_t^l is defined by firms taking into account equations (94), (97) and (100).

$$N_t^f n_t^{l,f} = N_t^f \frac{t^l}{t^h} n_t^{l,H} + q^h N_t^f n_t^{hl,H} \Rightarrow n_t^l = \frac{t^l}{t^h} n_t^{l,H} + q^h n_t^{hl,H} \tag{103}$$

3 Calibration

In this section, we discuss our parameterization. We calibrate the model at an annual frequency to match salient features of the Greek economy at the onset of the Global Financial Crisis (GFC) around 2008-2009. We present the key parameters of our model in Table 1 and selected targeted steady-state values in Table 2. Online Appendices C and D report the set of equations in the Decentralized Competitive Equilibrium and the Steady State Equilibrium, respectively. Online Appendix E provides all the details about the calibration strategy.

For conventional parameters, we follow the literature. For less conventional parameters, we target related moments of the Greek economy. To match the model to the data, we define output in our model y as the difference between real gross domestic product and net exports (see equation (S.59)). Following usual practice (e.g., Kehoe and Prescott (2002); Conesa et al. (2007)), we define investment in the model as total investment (gross fixed capital formation, private and public) in the data.

Households. For population, we mostly rely on data from Eurostat (2021). Population weights of the two households, t^l and t^h, are set equal to 0.69 and 0.31, respectively, based on population by educational attainment.[11] Household-specific unemployment rates are calibrated on the unemployment rates by educational attainment level, namely tertiary and non-tertiary education levels.[12] We set u^l and u^h equal to 0.12 and 0.07 for low and high-skilled households, respectively. Using data on employment by educational attainment we solve for household-specific employment rates, $n^{l,H} = 0.49$ and $n^h = 0.81$.[13] As expected, high-skilled employment rates are higher than the low-skilled ones. For the employment rate of the mismatched, we employ the International Labour Organization's (ILO) definition for mismatch employment and, according to Eurostat (2021) data, we set $n^{hl,H} = 0.19$.[14] [15] Then, the share of high-skilled workers employed in a high-skilled position, $n^{h,H}$, is calculated residually according to $n^{h,H} = n^h - n^{hl,H} = 0.62$.

Based on data from the Hellenic Statistical Authority (ELSTAT), emigrants amounted to 53 thousands persons in 2010.[16] According to Labrianidis et al. (2016), approximately 65% of them were graduates. Hence, we set $n^{l,F} = 0.0025$ and $n^{h,F} = 0.0048$, for low and high-skilled emigration, respectively. Through the household composition equations (S.18) and (S.31), we can then pin down the fractions of the non-active members as $l^h = 0.12$ and $l^l = 0.39$. This suggests that labor market non-participants represent a higher fraction of the low-skilled household than the high-skilled household.

Using equation (S.58), we derive the interest rate for net foreign assets $r^d = 5.3\%$, a value which corresponds

[11] See online data code: EDAT-LFS-9903. Population share 15-64 with non-tertiary education is defined as the sum of ISCED 2011 classifications 0-2 and 3-4.

[12] See online data code: LFSA-URGAED. Low-skilled unemployment rate is defined as the ratio of low-skilled 15-64 (ISCED 0-4) unemployed to low-skilled 15-64 labor force. High-skilled unemployment rate is defined as the ratio of the high-skilled 15-64 (ISCED 5-8) unemployed to the high-skilled 15-64 labor force.

[13] See online data code: LFSA-ERGAED. the low-skilled employment rate is defined as the ratio of low-skilled 15-64 (ISCED 0-4) employed to the low-skilled 15-64 population. The high-skilled employment rate is defined as the ratio of high-skilled 15-64 (ISCED 5-8) employed to the high-skilled 15-64 population.

[14] https://ilostat.ilo.org/resources/concepts-and-definitions/description-education-and-mismatch-indicators

[15] See online data code: LFSA-EGISED. The formula is: high-skilled (in thousands) 15-64 employed in low-skill positions to high-skilled (in thousands) 15-64.

[16] https://www.statistics.gr/en/statistics/-/publication/SPO15/2008

to a subjective discount factor $\beta = 0.9497$ through equation (S.15). Furthermore, we set the inverse of the Frisch elasticity of labor supply $\phi^h = \phi^l = 1.7$, close to the values commonly used in the literature (see, e.g., Pappa et al. (2015)). For the inverse elasticity of the intertemporal substitution η, much of the literature uses econometric estimates between 0 and 2 (see, e.g., Hansen and Singleton (1983)); we set it equal to 1.01. We calibrate the utility weight of low-skilled households $\Phi^l = 0.4327$, while the respective elasticity for high-skilled workers is calibrated to $\Phi^h = 0.0166$, indicating that low-skilled employees receive higher utility from leisure compared to their high-skilled counterparts. Finally, we calibrate the value of the depreciation rate equal to 0.0456, using equation (S.12), and the ratios of aggregate investment to output and of aggregate capital stock to output based on the data, i.e. $(i/y) = 0.18$ and $(k/y) = 3.95$.

Labor market. For simplicity, we assume that the destruction rates of high and low-skill positions are equal across at home and abroad. Thus, we set $\sigma^h = \sigma^{h,F} = 0.06$ and $\sigma^l = \sigma^{l,F} = 0.08$. The latter is close to the values found for total employment destruction rates in Hobijn and Şahin (2009). We set the low-skill vacancy-filling rate to $\psi_F^{l,H} = 0.45$ and the hiring probabilities $\psi_H^{h,H} = 0.10$ and $\psi_H^{hl,H} = 0.90$, indicating that a high-skilled searcher is more likely to find a low-skill rather than a high-skill position. We set the probability of finding a high-skill job abroad as $\psi^{h,F} = 0.16$, based on Bandeira et al. (2022). We set the probability of finding a low-skill job abroad equal to $\psi^{l,F} = 0.50$.

Matches are obtained from equations (S.15) and (S.29), $m^{h,H} = 0.0115$ and $m^{l,H} = 0.0270$. The share of low-skilled searchers for a job abroad is calculated from (S.30) as $O^l = 0.0033$. We set the high-skill vacancy-filling probability as $\psi_F^{h,H} = 0.7$, following Bandeira et al. (2022). The hiring probability in a low-skilled position is calculated from equation (S.2) as $\psi_H^{l,H} = 0.3277$. We find the share of high-skilled searchers for a job abroad from (S.17) as $O^h = 0.0256$. We use equations (S.15) and (S.16) to calibrate $\phi(z) = 1.8988$ and the share of searchers for a high-skill position, $s = 0.1647$. Mismatches are derived from equation (S.16), $m^{hl,H} = 0.01590$. The per capita high-skill vacancy is equal to $v^h = 0.0165$ using (S.4). We calculate $\mu_1 = 0.4129$ and $\mu_2 = 0.7288$ by solving a system of two equations (S.7 and S.8) and by using equation (S.5) to get $(1-x)v^l = 0.0601$. We obtain values that are common in the literature (e.g., Petrongolo and Pissarides (2001), Oikonomou (2022) for Greece). Then, $xv^l = 0.0515$ is obtained from (S.9) and this system of equations yields the share of low-skill vacancies allocated to high-skilled employees, $x = 0.4612$, and the per capita low-skill vacancies, $v^l = 0.1116$. The mismatch vacancy-filling probability is obtained from (S.6), $\psi_F^{hl,H} = 0.3090$.

Using the resource constraint (S.27), data on the private consumption to output ratio ($c^p/y = 0.71$), data on the aggregate investment to output ratio ($i/y = 0.18$), and by setting the marginal cost of posting a high-skill vacancy as $\kappa^h = 0.10$ and the ratio of total vacancy costs to output equal to 4%, we find the marginal cost of posting a low-skill vacancy, $\kappa^l = 0.3163$.[17] This indicates that it is more costly to post a low-skill vacancy than a high-skill vacancy as in Oikonomou (2022). By solving a system of equations, we calibrate the efficiency of mismatch workers q^h to 1.3316, indicating that they are more productive than low-skilled workers in low-skill type occupations by 33%. Using data on the average annual compensation per employee and the per educational attainment level from the "Survey on the structure and distribution of wages in firms (2006)", we

[17] The value is close to the range reported in 1997 National Employer Survey, https://census.gov/econ/overview/mu2400.html, which shows that 2% – 3% of GDP is dedicated to recruiting.

obtain the wage premia of high-skilled versus low-skilled workers, $w^{h,H}/w^{l,H} = 1.5$ and of mismatched versus low-skilled workers, $w^{hl,H}/w^{l,H} = 1.05$. We then use these wage ratios along with equations (S.44), (S.45) and (S.46) to find the three wages $w^{h,H}, w^{l,H}, w^{hl,H}$. The firms' bargaining power parameters, θ^h, θ^l, θ^{hl}, to 0.0555, 0.7841, 0.7009, satisfy equations (S.37), (S.38), (S.39), respectively. Finally, the solution of the system of equations yields the values of the pecuniary moving costs for the high-and low-skilled, $X^h\left(\tilde{O}^h\tilde{u}^h\right) = 0.6992$ and $X^l\left(\tilde{O}^l\tilde{u}^l\right) = 1.4809$. The marginal cost values, X^h and X^l, are equal to 39.0603 and 376.4257, indicating that it is more costly for low-skilled employees to move abroad for work, compared to the high-skilled employees.

Production. We set the elasticity of substitution between high skill labour and capital ρ to 0.77, a value which is close to Krusell et al. (2000). We also set the weight attached to low skill labour $\alpha = 0.47$ as in Oikonomou (2022) and the elasticity of substitution between low skill labour, capital and high skill labour $\epsilon = 1.46$. By targeting y^{F^*}/y, y^F/y and d/y, we calibrate the home bias parameter $\omega = 0.8142$, the elasticity of substitution between home-produced and imported goods $\gamma = 1.8008$, and the income share of capital $\zeta = 0.7927$, which are close to values commonly used in the literature (Chodorow-Reich et al. (2019)). Capital-skill complementarity requires that $\rho < \epsilon$, which holds in our calibrated values.

By normalizing the price level P to 1 and using equation (S.52) and data on the imports to output ratio (y^F/y=0.25), we calibrate the price of imported goods $p^F = 0.8481$. We use equations (S.51) and (S.53) to calibrate the price of domestic goods $p^H = 1.0417$. We normalize total factor productivity in equation (S.40) to one, $A = 1$. Furthermore, using the production function, (S.51), we pin down the ratio of the intermediate good distributed domestically to output, $y^H/y = 0.7564$. Using equation (S.58), we find the exchange rate to be $e = 0.8481$ and then, using equation (S.60), we find $P^* = 0.8481$. Finally, using the production function, we solve for output, $y = 0.9231$, which pins down y^* from equation (S.50) as equal to 1.5107.

Government. We set the share of government revenues devoted to transfers $s^{g,t}$ equal to 0.08 and the share of government revenues devoted to wasteful spending $s^{g,c}$ equal to 0.05 as in Bandeira et al. (2022). Using the budget constraints of the high-skilled household, (S.10), and the government budget constraint, (S.54), we pin down the lump-sum tax $\tau = 0.1874$ and the unemployment benefit $\bar{\omega} = 0.6444$. Our solution implies that the unemployment benefit corresponds to the 83% of the net low-skill wage.

Table 1: Parameterization

A.Data/Simulation targets		**Value**	**Source**
t^l, t^h	Population weights of households	0.6900, 0.3100	Eurostat (2021)
ϵ	EoS (l-labor, capital, h-labor)	1.4600	Data on k/y, y^F/y, y^{F^*}/y, d/y
s^c	Consumption (output share)	0.7100	Eurostat data
$s^{g,c}$	Wasteful spending (output share)	0.0500	Eurostat data
$s^{g,t}$	Government transfers (output share)	0.0800	Eurostat data
κ^h	High-skill vacancy cost	0.1000	total vacancy costs: 4% of GDP
x_2^h	Moving cost: high-skilled	0.1000	emigration flows: 0.7% of working-age pop & h-to-l emigrants ratio: 2/3
x_2^l	Moving cost: low-skilled	0.1000	emigration flows: 0.7% of working-age pop & h-to-l emigrants ratio: 2/3
B. Steady-state equations		**Value**	**Rationale**
β	Discount factor	0.9497	Derived from (S.14), annual interest rate = 0.0735
δ	Depreciation rate	0.0456	Derived from (S.12) and $k/y = 3.95$, $i/y = 0.20$
γ	EoS (home produced, imported goods)	1.8004	System of 43 equations in 43 unknowns (pg. 30-33)
ζ	Weight attached to capital	0.7927	System of 43 equations in 43 unknowns (pg. 30-33)
ω	Home bias	0.8142	System of 43 equations in 43 unknowns (pg. 30-33)
q^h	Effective productivity of mismatched workers	1.3316	System of 43 equations in 43 unknowns (pg. 30-33)
Φ^h	Relative disutility for high-skilled labor	0.0166	System of 43 equations in 43 unknowns (pg. 30-33)
Φ^l	Relative disutility for low-skilled labor	0.4327	System of 43 equations in 43 unknowns (pg. 30-33)
κ^l	Low-skill vacancy cost	0.3163	System of 43 equations in 43 unknowns (pg. 30-33)
$\bar{\omega}$	Unemployment benefits	0.6444	System of 43 equations in 43 unknowns (pg. 30-33)
μ_1	Matching efficiency	0.4129	Derived from (S.7) and (S.8)
μ_2	Matching elasticity	0.7288	Derived from (S.7) and (S.8)
b_1	On-the-job search cost	0.1249	Derived from (S.22), (S.61), (S.62)
ϕ_2	Efficiency of on-the-job search	1.4636	Derived from (S.22), (S.61), (S.62)
x_1^h	Moving cost: high-skilled	1.3161	Derived from (S.63)
x_1^l	Moving cost: low-skilled	3.2438	Derived from (S.64)
$\theta^h, \theta^{hl}, \theta^l$	Firms' bargaining power	0.0555, 0.7841, 0.7009	Derived from (S.37), (S.38), (S.39)
C. Literature		**Value**	**Source**
ρ	EoS (h-labor, capital)	0.7700	Krusell et al. (2000)
α	Weight attached to l-labor	0.4700	Oikonomou (2022)
$\phi^h = \phi^l$	Inverse Frisch elasticity	1.7000	Common value in the literature
η	Inverse elasticity of intertemporal substitution	1.0100	Hansen and Singleton (1983)
$\sigma^h = \sigma^{h,F}$	High-skill job destruction rates	0.0600	Close to Hobijn and Şahin (2009)
$\sigma^l = \sigma^{l,F}$	Low-skill job destruction rates	0.0800	Close to Hobijn and Şahin (2009)
Ξ	Capital adjustment costs	4.0000	Dolado Juan J. (2021)
D. Other		**Value**	**Rationale**
ϕ_1	Efficacy of on-the-job search cost to end mismatch	1.0000	Normalization
b_2	On-the-job search cost to end mismatch	2.0000	Quadratic form

Table 2: Steady-state variables

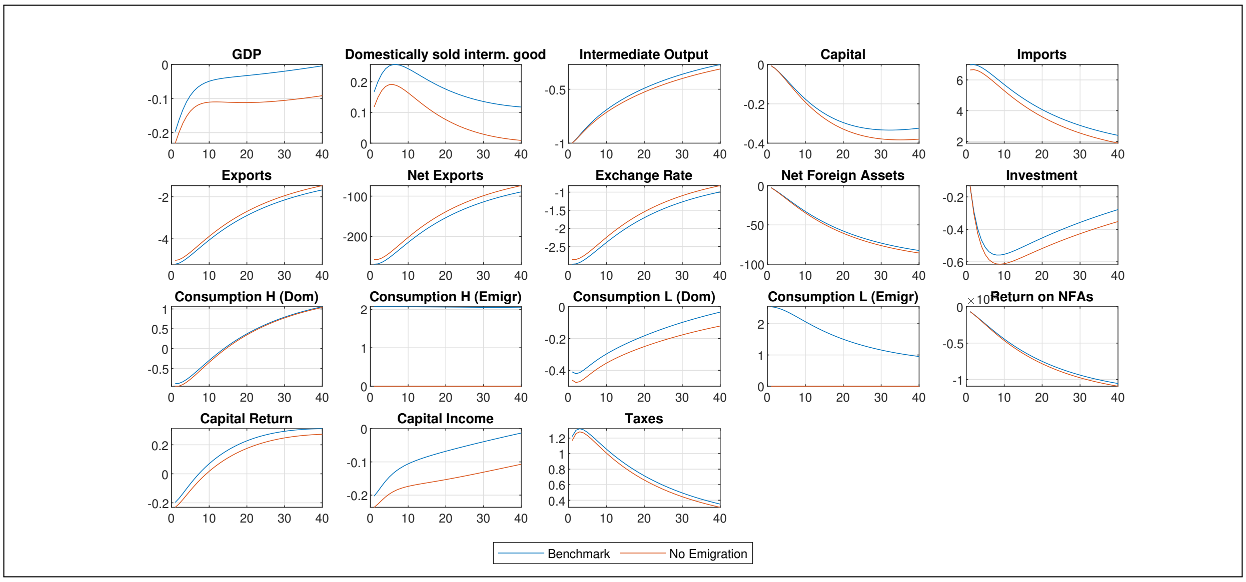
Variable	**Description**	**Value**
y	Output	0.9231
y^{F^*}/y, y^F/y	Exports, imports (output shares)	0.21, 0.25
y^H, y^F	Domestic demand, imports	0.6983, 0.2308
y_i	Intermediate good demand	0.8922
x_i	Capital and high-skilled labor input	1.5463
$c^{h,H}$, $c^{l,H}$	Domestic consumption of high- and low-skilled	1.3570, 0.3445
$c^{h,F}$, $c^{l,F}$	Foreign consumption of high- and low-skilled	1.5975, 0.4055
i/y, k/y	Investment, capital (output shares)	0.18, 3.95
d/y	Net foreign assets (output share)	0.10
$n^{h,H}$, $n^{l,H}$	Employment rates: high-skill and low-skill jobs	0.62, 0.49
$n^{hl,H}$	Employment rate: mismatch jobs	0.19
$n^{h,F}$	Employment rate abroad: high-skill jobs	0.0048
$n^{l,F}$	Employment rate abroad: low-skill jobs	0.0025
l^h, l^l	Non-participants: high- and low-skilled	0.1152, 0.3875
u^h, u^l	Unemployed: high- and low-skilled	0.07, 0.12
r^k	Return on capital	0.0986
$w_H^{h,H}$, $w_H^{l,H}$	High- and low-skill wages	1.1619, 0.7746
$w_H^{hl,H}$	Mismatch wage	0.8133
$m^{h,H}$, $m^{l,H}$	High- and low-skill matches	0.0115, 0.0270
$m^{hl,H}$	Mismatches	0.0159
v^h, v^l	High- and low-skill vacancies	0.0165, 0.1116
$\psi_H^{h,H}$, $\psi_H^{l,H}$	High- and low-skill hiring probabilities	0.10, 0.3277
$\psi_H^{hl,H}$	Mismatch hiring probability	0.90
$\psi_F^{h,H}$, $\psi_F^{l,H}$	High- and low-skill vacancy-filling probabilities	0.70, 0.45
$\psi_F^{hl,H}$	Mismatch vacancy-filling probability	0.3090
p^H, p^F	Domestic and foreign good prices	1.0417, 0.8481
e	Exchange rate	0.8481
r^d, r^*	Gross domestic and world interest rate	1.0530
z	Search effort to end mismatch	1.5498
$b(z)$	Cost of search effort to end mismatch	0.3001
$\phi(z)$	Efficacy of search to end mismatch	1.8988
$X^h\left(O^h u^h\right)$, $X^l\left(O^l u^l\right)$	Moving costs of high- and low-skilled	0.6992, 1.4809
τ	Lump-sum tax	0.1874
x	Fraction of low-skill positions given to high-skilled	0.4612
$1-s$	Fraction of high-skilled searchers for mismatch job	0.8353
O^h, O^l	Fraction of high- and low-skilled searchers for job abroad	0.0256, 0.0033

4 Results

In this section, we investigate how skills mismatch and skill-specific emigration react to standard macroeconomic shocks, namely a negative shock to total factor productivity and a negative shock to government spending. We present impulse responses from our DSGE model under our baseline calibration with emigration and also from an alternative variant of the model where we shut down emigration.

4.1 Negative TFP Shock

Figure 2: Responses to a 1% negative TFP shock



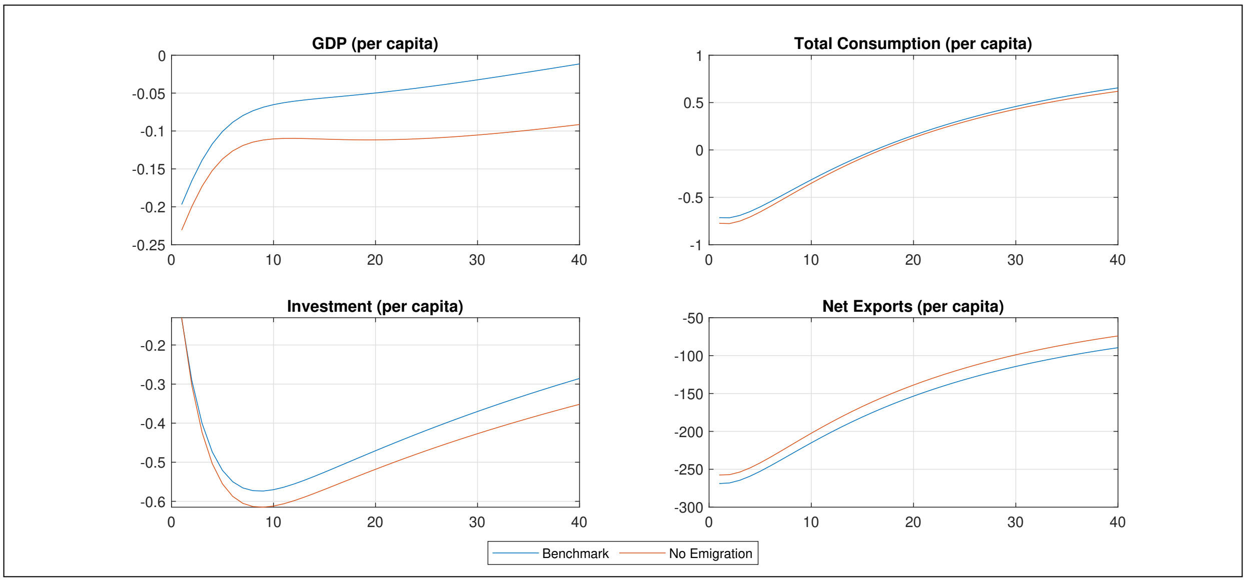
Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

First, we investigate how skills mismatch and emigration react to a negative shock to total factor productivity. In Figure 2, we see that the shock causes the intermediate output and GDP to fall. Investment and capital accumulation also decline. The real exchange rate appreciates, which induces a substantial fall in net exports. Consumption of the domestic good by the low-skilled household falls, while in the case of the high-skilled household the consumption decline is more short-lived and its response turns positive after the tenth period.

In Figure 3, we see that the responses of per capita GDP, per capita investment and per capita net exports have the negative sign mentioned earlier. It is important to confirm that for per capita variables as in our model economy the population is mobile and takes migration decisions. In terms of per capita total consumption, we see a short-lived decline, followed by a rise above the steady state.

In Figure 4, we take a look at the main labor market and emigration variables. In terms of labor demand, vacancies drop for both skill types with the effect being stronger for those positions requiring high skills. Employment in the domestic labor market falls for the high-skilled (non-mismatched) workers and also for the low-skilled workers, but in the latter case the recovery is fast and the response turns positive after the tenth period. The adverse productivity shock also induces mismatch employment to rise substantially. In Figure 5

Figure 3: Responses to a 1% negative TFP shock (continued)



Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years.

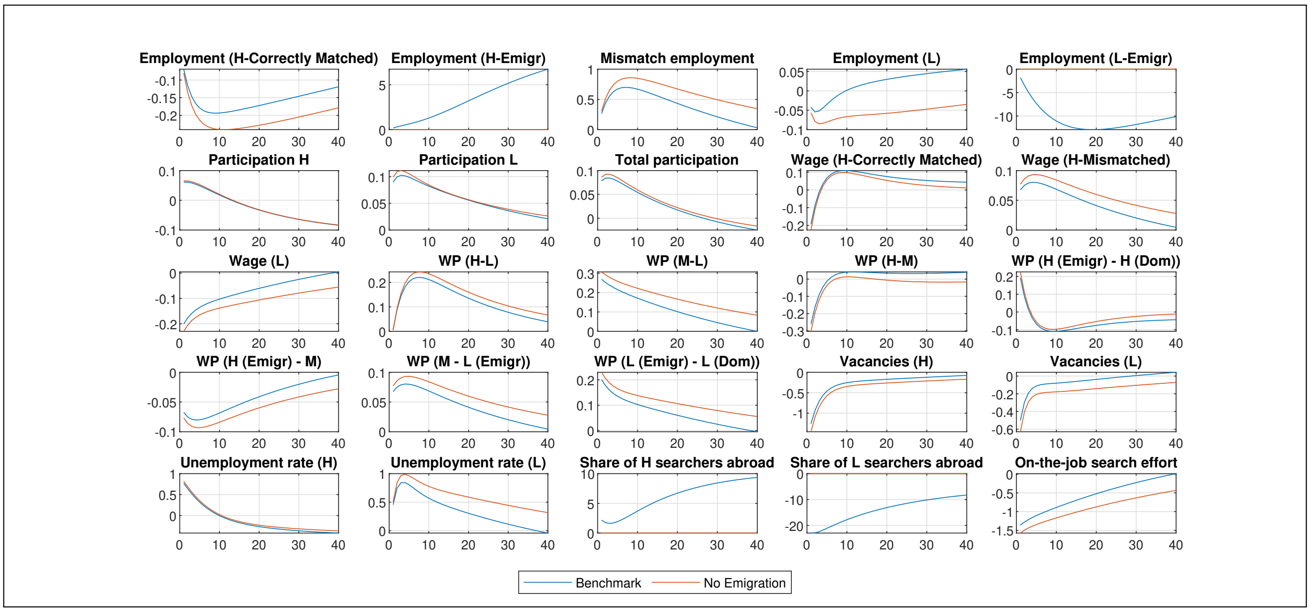
we can analyze the driving forces behind the rise in the mismatch employment rate as follows: (i) firms increase the share of vacancies requiring low skills that go to high-skilled (mismatched) applicants, (ii) high-skilled households increase, after the fifth period, the share of their unemployed members searching for a mismatch position, and (iii) quits from mismatch jobs decline following the sharp fall in high skill vacancies which could provide them with upgraded jobs.

In terms of emigration, we see back in Figure 4 that there is a significant rise in the case of the high-skilled household but a decline in the case of the low-skilled household. Labor market participation rises for both skill types, but the response turns negative after some time for the high-skilled household and drives the response of total participation. The responses of the unemployment rates for the two skill types follow those of the participation rates. When we shut down emigration, the recession is magnified. This is driven by both a stronger decline in investment compared to the baseline calibration and by a reinforced drop in the consumption of the domestic good by the low-skilled household (see Figure 2). Basically, what happens is that we no longer have the return migration of the low-skilled, which meant a demand boost for the domestic economy.

What about wages? Again, in Figure 4 we have that the standard skill wage premium of high-skilled workers versus their low-skill counterparts declines. For the mismatched workers, we see that their wage position improves relative to their low-skilled counterparts and also relative to their high-skilled counterparts after an initial deterioration. Regarding wages abroad, we see that the foreign wage premium for high-skilled workers rises temporarily, but then falls. It also falls persistently in relation to mismatched workers. The mismatch wage rises versus the low-skilled emigrant counterpart, which explains why low-skilled emigration falls, despite the fact that the wage premium of low-skilled emigrants versus low-skilled domestic workers clearly rises.

In sum, the main finding is that a negative TFP shock leads to a) an increase in mismatch employment and b) a decrease in unskilled emigration and an increase in skilled emigration, associated with a fall in investment

Figure 4: Responses to a 1% negative TFP shock (continued)



Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. M denotes mismatch. WP denotes the wage premium.

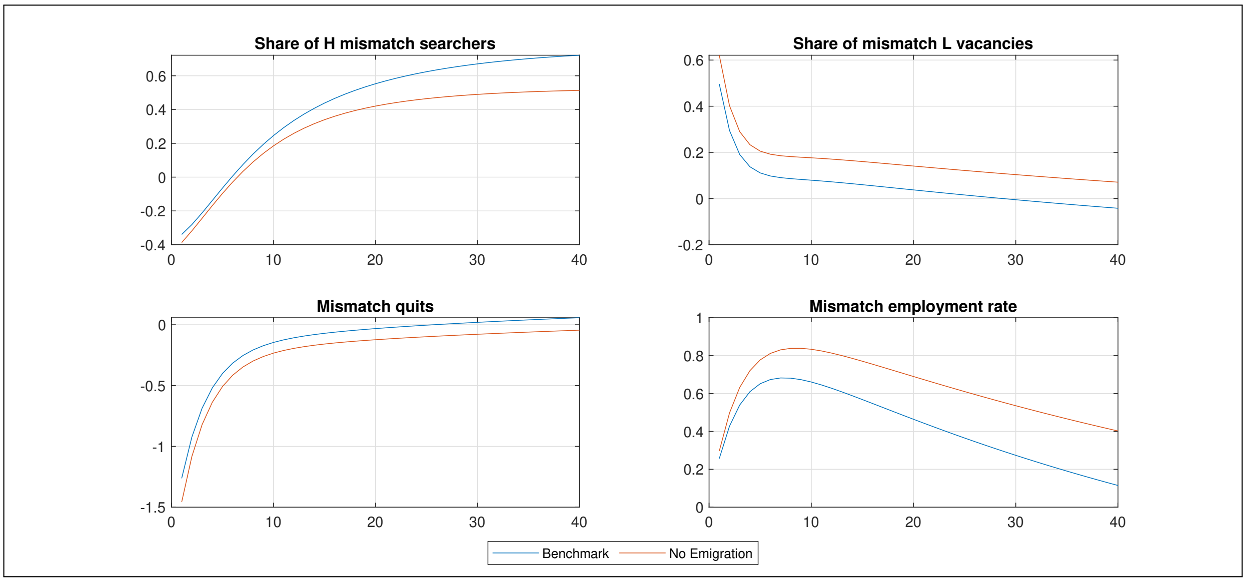
taking into account the CSC relationship. Note here that we have assumed in our calibration that moving costs are higher for the low-skilled to replicate the case of predominantly skilled emigration observed during the decade-long crisis. Recall also that, as mentioned above, vacancies fall much more in response to the shock for high than low skills. In other words, the negative TFP shock reduces investment and primarily hurts the high-skilled who react by turning to both jobs abroad and mismatch jobs in the domestic labor market. Our findings relate to the recent paper by Deng et al. (2021) where, facing an adverse productivity shock, a government has incentives to lower tax progressivity to encourage labor supply and reduce high-income workforce outflows.

4.2 Negative Shock to Government Spending

In this section, we investigate how skills mismatch and emigration react to a negative shock to government spending, where the latter is modelled as a waste in the economy. In Figure 6, we see that the shock causes the intermediate output and GDP to fall during the first half of the time horizon despite an investment crowd-in. The real exchange rate depreciates, which induces a rise in net exports. Consumption of the domestic good rises for both household types as a result of the standard positive wealth effect of this shock. In Figure 7, we see that the responses of per capita GDP, per capita investment, per capita net exports and per capita total consumption have the same sign as the corresponding aggregate variables mentioned above. It is important to confirm that for per capita variables as in our model economy the population is mobile and takes migration decisions.

In Figure 8, we take a look at the main labor market and emigration variables. In terms of labor demand, vacancies requiring high skills slightly rise in line with the investment crowding-in (recall the CSC relationship), while vacancies requiring low skills drop. Employment in the domestic labor market rises for the high-skilled

Figure 5: Responses to a 1% negative TFP shock (continued)



Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. The mismatch employment rate refers to the share of mismatch employees in the total number of the high-skilled household's employed members, $n_t^{h,l}/(n_t^{h,l} + n_t^{h,h})$. H and L refer to high and low skills, respectively.

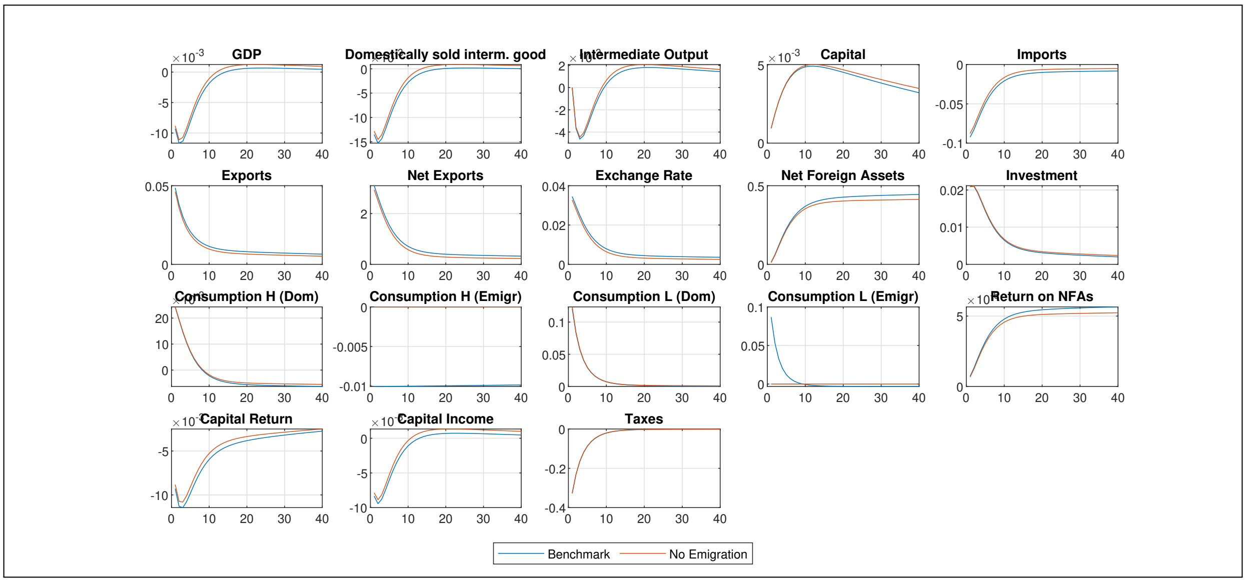
(non-mismatched) workers while it falls for the low-skilled workers. The negative spending shock also induces mismatch employment to decline, which is in line with the fall in vacancies requiring low skills. In Figure 9 we can analyze the driving forces behind the decrease in the mismatch employment rate as follows: (i) quits from mismatch jobs increase following the rise in high skill vacancies which could provide them with upgraded jobs, (ii) firms decrease the share of vacancies requiring low skills that go to high-skilled (mismatched) applicants after the initial periods, and (iii) high-skilled households decrease, after the initial periods, the share of their unemployed members searching for a mismatch position.

In terms of emigration, we see back in Figure 8 that there is a temporary rise in the case of the high-skilled household, which is reversed after the fifteenth period, and a much more persistent rise in the case of the low-skilled household. Labor market participation falls for both skill types, as the result of the positive wealth effect of the shock. The responses of the unemployment rates for the two skill types follow those of the participation rates. When we shut down emigration, the recession is mitigated by the difference is very small. This seems to be driven by a smaller rise in net exports compared to the baseline calibration.

Looking at wages, again in Figure 8, we have that the standard skill wage premium of high-skilled workers versus their low-skill counterparts declines. For the mismatched workers, we see that their wage position deteriorates relative to both their low-skilled and high-skilled counterparts. Regarding wages abroad, we see that the foreign wage premium for high-skilled workers rises temporarily. The mismatch wage falls versus the low-skilled emigrant counterpart and the wage premium of low-skilled emigrants versus low-skilled domestic workers rises on impact, which jointly explains why low-skilled emigration.

In sum, the main finding is that a negative government spending shock leads to a) a decrease in mismatch

Figure 6: Responses to a 1% negative G shock



Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

employment and b) an increase in both unskilled and skilled emigration, but the effect quickly changes sign for the latter. Recall also that, as mentioned above, vacancies fall much more in response to the shock for the low skill type only. In other words, the negative government spending shock here primarily hurts the low-skilled who react by turning to foreign jobs. The high-skilled instead turn less towards mismatch employment and in later periods they also turn less towards foreign jobs, despite the fact that we have assumed in our calibration that moving costs are higher for the low-skilled to replicate the case of predominantly skilled emigration observed during the decade-long crisis.

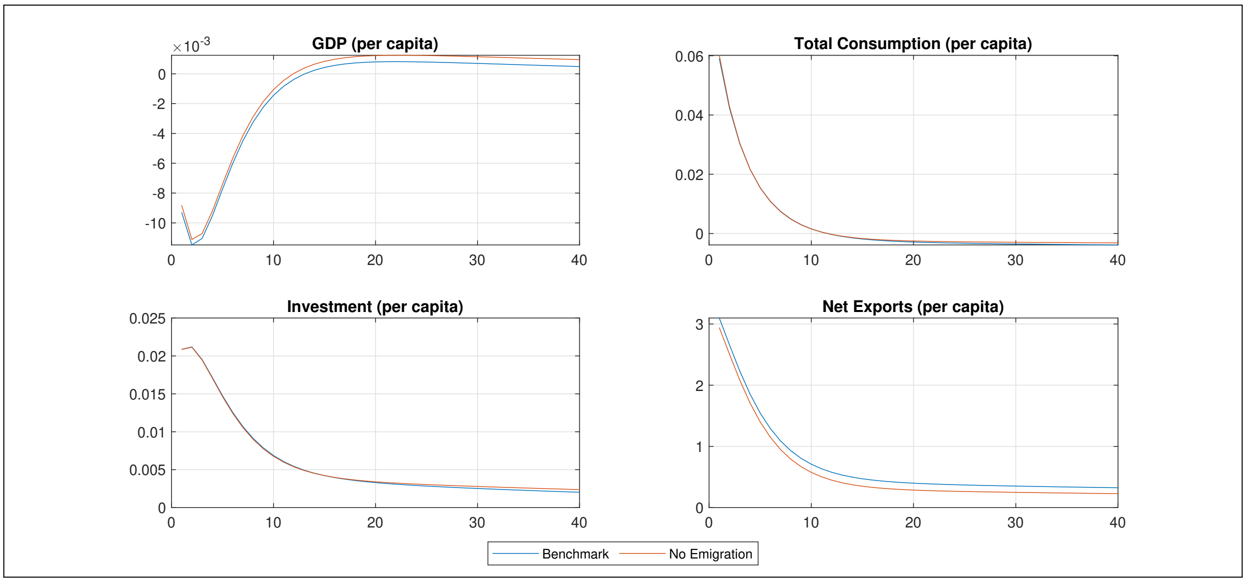
Fiscal multipliers. Let us now examine the output multipliers implied by our DSGE model. Spending multipliers at horizon h are computed by dividing the present-value cumulative response of GDP, IRF_j^{gdp}, by the present-value cumulative response of government spending, IRF_j^{g}, after the shock, and then dividing by the steady-state ratio of government spending to GDP, $\overline{g}/\overline{gdp}$:

$$\text{Present-value multiplier (h)} = \frac{\sum_{j=0}^{h}(1+r)^{-j}IRF_j^{gdp}}{\sum_{j=0}^{h}(1+r)^{-j}IRF_j^{g}}(\overline{g}/\overline{gdp})^{-1},$$

Therefore, the government spending multipliers measure the change in the value of output (in currency units, e.g., euros) due to a one currency-unit increase in government spending. Figure 10 reports the results for government consumption spending, where we see that the value of the multiplier is higher than 0.6 but lower than 0.7. The values implied by our model are plausible and in line with the literature. The fact that multipliers for wasteful spending are smaller than one implies that cuts in this component of the government's budget generates little incentive to emigrate.

Nevertheless, the magnitude of the multiplier is amplified in the presence of international labor mobility.

Figure 7: Responses to a 1% negative G shock (continued)



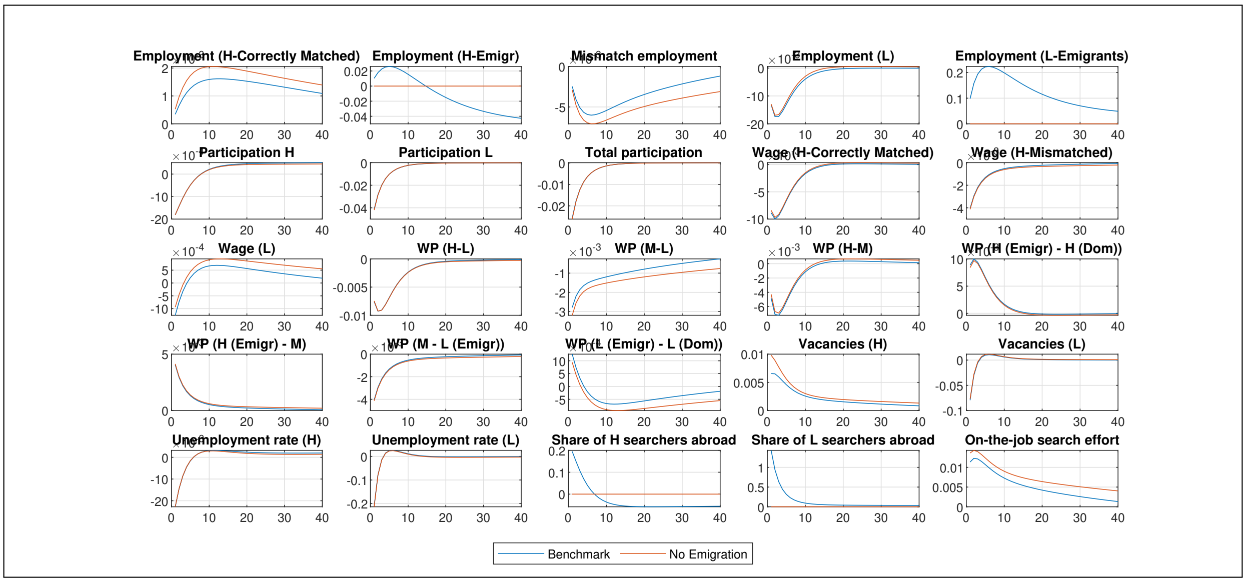
Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years.

This result is opposite from Bandeira et al. (2022), where there is only one type of households. The result there holds after the initial periods, when return migration is observed and this boosts internal demand. By contrast, in our model with heterogeneous households, we find that low-skilled emigration is quite persistent while the medium-term return migration of how-skilled workers is rather weak.

5 Conclusion

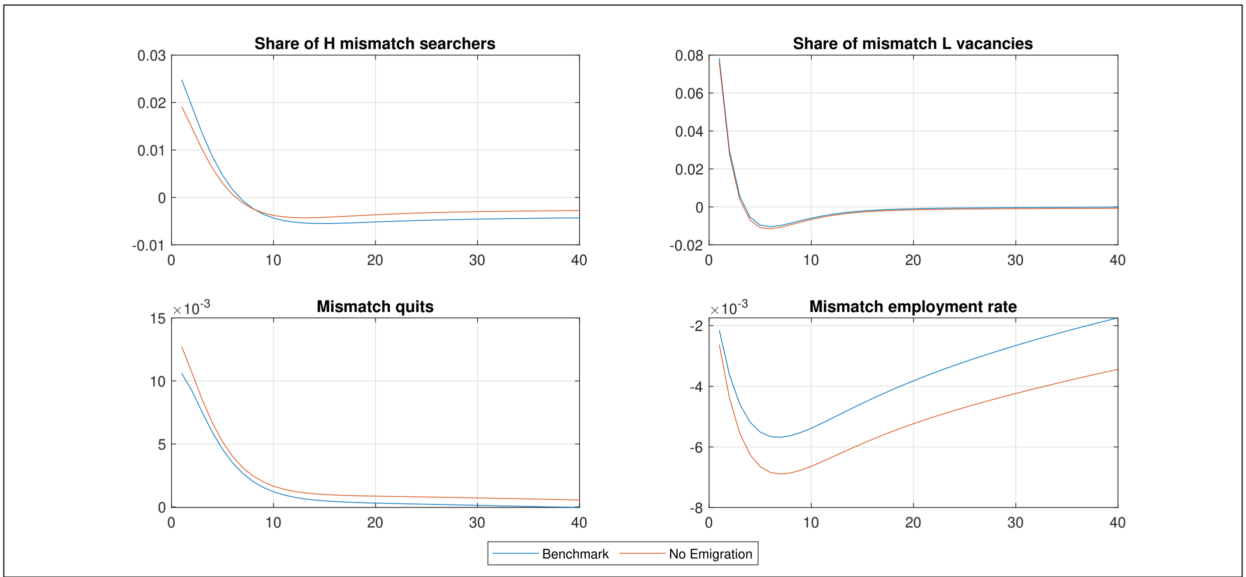
This paper provides a new framework that incorporates skills mismatch and heterogeneous labor with labor mobility to RBC models. We find that a bad productivity shock reduces investment and primarily hurts the high-skilled who react by turning to both jobs abroad and mismatch jobs in the domestic labor market. A negative shock to government spending crowds-in investment and primarily hurts the low-skilled who thus turn to jobs abroad. Following the fiscal cut, the high-skilled instead reduce their search for mismatch employment and later they also reduce their search for jobs abroad.

Figure 8: Responses to a 1% negative G shock (continued)



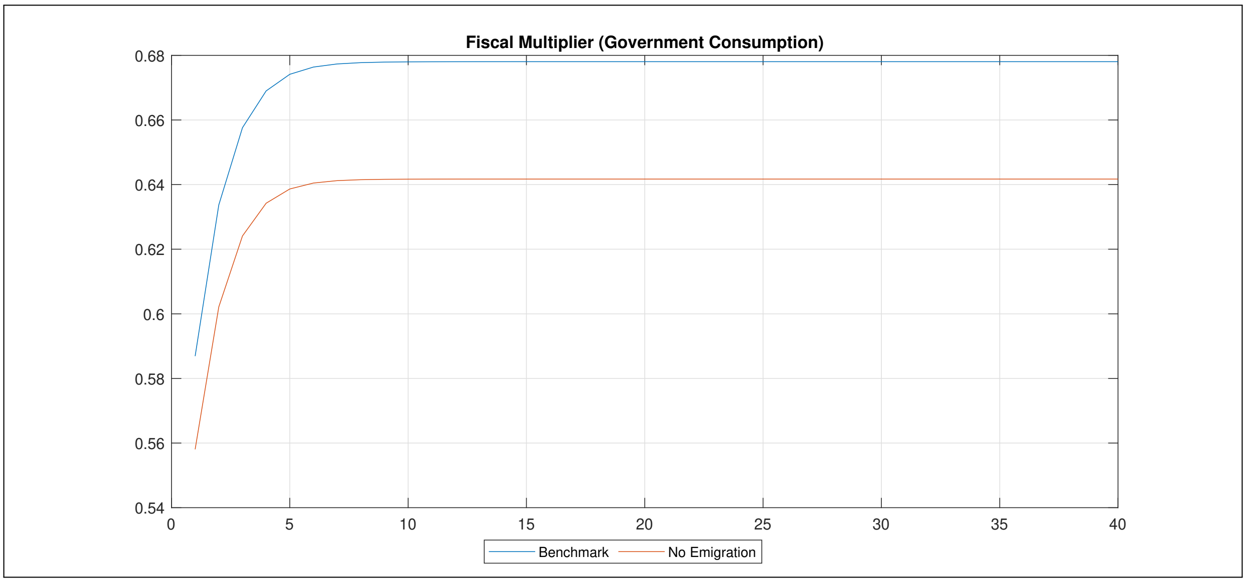
Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively. M denotes mismatch. WP denotes the wage premium.

Figure 9: Responses to a 1% negative G shock (continued)



Notes: Responses are in percent deviations from the steady state. The horizontal axis depicts years. H and L refer to high and low skills, respectively.

Figure 10: Present-value cumulative output multipliers



Notes: The horizontal axis depicts years.

References

Daron Acemoglu. Good jobs versus bad jobs. *Journal of Labor Economics*, 19(1):1–21, 2001. URL https://EconPapers.repec.org/RePEc:ucp:jlabec:v:19:y:2001:i:1:p:1-21.

James Albrecht and Susan Vroman. A Matching Model with Endogenous Skill Requirements. *International Economic Review*, 43:283–305, 2002.

George Alessandria, Yan Bai, and Minjie Deng. Migration and sovereign default risk. *Journal of Monetary Economics*, 113:1–22, 2020.

Isaac Baley, Ana Figueiredo, and Robert Ulbricht. Mismatch cycles. *Journal of Political Economy*, 130(11): 000–000, 2022.

Guilherme Bandeira, Jordi Caballé, and Eugenia Vella. Emigration and fiscal austerity in a depression. *Journal of Economic Dynamics and Control*, 144:104539, 2022.

Regis Barnichon and Yanos Zylberberg. Under-Employment and the Trickle-Down of Unemployment. *American Economic Journal: Macroeconomics*, 11:40–78, 2019.

Andri Chassamboulli. Cyclical upgrading of labor and employment differences across skill groups. *B.E. Journal of Macroeconomics*, 11(1):1–46, 2011. ISSN 19351690. doi: 10.2202/1935-1690.2194.

Gabriel Chodorow-Reich, Loukas Karabarbounis, and Rohan Kekre. The Macroeconomics of the Greek Depression. Working Paper 25900, National Bureau of Economic Research, May 2019. URL https://www.nber.org/papers/w25900.

Gabriel Chodorow-Reich, Loukas Karabarbounis, and Rohan Kekre. The macroeconomics of the greek depression. 2023.

Juan Carlos Conesa, Timothy J Kehoe, and Kim J Ruhl. Modeling great depressions: The depression in finland in the 1990s. Working Paper 13591, National Bureau of Economic Research, November 2007. URL http://www.nber.org/papers/w13591.

Ayşegül Şahin, Joseph Song, Giorgio Topa, and Giovanni L. Violante. Mismatch unemployment. *American Economic Review*, 104(11):3529–64, November 2014. doi: 10.1257/aer.104.11.3529. URL https://www.aeaweb.org/articles?id=10.1257/aer.104.11.3529.

Gonzalo F de Córdoba, Pau S Pujolas, and José L Torres. Fiscal discipline and defaults. *Review of Economic Dynamics*, 24:1–13, 2017.

Minjie Deng et al. Inequality, taxation, and sovereign default risk. *Discussion Papers, Simon Fraser University*, 2021.

Juan José Dolado, Jansen Marcel, and Juan F Jimeno. On-the-Job Search in a Matching Model with Heterogenous Jobs and Workers. *Economic Journal*, 119:200–228, 2009.

Pappa Evi Dolado Juan J., Motyovszki Gergő. Monetary policy and inequality under labor market frictions and capital-skill complementarity. *American Economic Journal: Macroeconomics*, 13(2):292–332, 2021.

George Economides, Dmitris Papageorgiou, and Apostolis Philippopoulos. Macroeconomic policy lessons from Greece. 2020.

Eurostat. European commission. *Eurostat database*, 2021.

Pierre-Olivier Gourinchas, Thomas Philippon, and Dimitri Vayanos. The analytics of the Greek crisis. *NBER* *macroeconomics Annual*, 31(1):1–81, 2017.

Lars Peter Hansen and Kenneth J Singleton. Stochastic consumption, risk aversion, and the temporal behavior of asset returns. *Journal of Political Economy*, 91:249–265, 1983.

Daniela Hauser and Martin Seneca. Labor mobility in a monetary union. *Journal of International Economics*, 137:103600, 2022.

Bart Hobijn and Ayşegül Şahin. Job-finding and separation rates in the oecd. *Economics Letters*, 104(3): 107–111, 2009. ISSN 0165-1765. doi: https://doi.org/10.1016/j.econlet.2009.04.013. URL https://www.sciencedirect.com/science/article/pii/S0165176509001359.

Zainab Iftikhar and Anna Zaharieva. General equilibrium effects of immigration in Germany: Search and matching approach. *Review of Economic Dynamics*, 31:245–276, 2019.

Timothy Kehoe and Edward Prescott. Great depressions of the twentieth century. *Review of Economic Dynamics*, 5(1):1–18, 2002. URL https://EconPapers.repec.org/RePEc:red:issued:v:5:y:2002:i:1:p:1-18.

Per Krusell, Lee E Ohanian, José-Víctor Ríos-Rull, and Giovanni L Violante. Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica*, 68(5):1029–1053, 2000.

Lois Labrianidis, Manolis Pratsinakis, et al. Greece's new emigration at times of crisis. Technical report, Hellenic Observatory, LSE, 2016.

Xiangbo Liu, Theodore Palivos, and Xiaomeng Zhang. Immigration , Skill Heterogeneity and Qualification Mismatch. *Economic Inquiry*, 55:1231–1264, 2017.

Dale T. Mortensen and Christopher A. Pissarides. Job creation and job destruction in the theory of unemployment. *Review of Economic Studies*, 61(3):397–415, 1994. ISSN 1467937X. doi: 10.2307/2297896.

Myrto Oikonomou. Migration, search and skill heterogeneity. *mimeo*, 2022.

Dimitris Papageorgiou, Stylianos Tsiaras, et al. The Greek Great Depression from a neoclassical perspective. Technical report, Bank of Greece, 2021.

Evi Pappa, Rana Sajedi, and Eugenia Vella. Fiscal consolidation with tax evasion and corruption. *Journal of International Economics*, 96:S56–S75, 2015.

Barbara Petrongolo and Christopher A. Pissarides. Looking into the black box: A survey of the matching function. *Journal of Economic Literature*, 39(2):390–431, 2001.

ONLINE APPENDIX

The Macroeconomics of Skills Mismatch in the Presence of Emigration

George Liontos Konstantinos Mavrigiannakis Eugenia Vella

A Wage bargaining

First, let us define the following that will be used later. In our derivations the marginal productivities of k_t^f, $n_t^{ll,f}$, $n_t^{h,f}$ and $n_t^{hl,f}$ are calculated as follows:

$$y_{i,t}^{k,f} \equiv \frac{\partial y_{i,t}^f}{\partial k_t^f} = \zeta (1-\alpha) A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}^f}{x_{i,t}^f}\right)^{\frac{1}{\epsilon}} \left(\frac{x_{i,t}^f}{k_t^f}\right)^{\frac{1}{\rho}} \tag{A.1}$$

$$y_{i,t}^{ll,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{ll,f}} = \alpha A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}^f}{n_t^{l,f}}\right)^{\frac{1}{\epsilon}} \tag{A.2}$$

$$y_{i,t}^{h,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{h,f}} = (1-\zeta)(1-\alpha) A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}^f}{x_{i,t}^f}\right)^{\frac{1}{\epsilon}} \left(\frac{x_{i,t}^f}{n_t^{h,f}}\right)^{\frac{1}{\rho}} \tag{A.3}$$

$$y_{i,t}^{hl,f} \equiv \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{hl,f}} = q_h \alpha A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}^f}{n_t^{l,f}}\right)^{\frac{1}{\epsilon}} \tag{A.4}$$

The Nash bargaining problem is to maximize the weighted sum of log surpluses for each employment status. The wages are given as the optimal solution of the following three problems:

$$\max_{w_t^{h,H}} \left\{ \left(1-\theta^h\right) \ln V_{n_t^h}^h + \theta^h \ln V_{n_t^h}^f \right\} \tag{A.5}$$

$$\max_{w_t^{hl,H}} \left\{ \left(1-\theta^{hl}\right) \ln V_{n_t^{hl}}^h + \theta^{hl} \ln V_{n_t^{hl}}^f \right\} \tag{A.6}$$

$$\max_{w_t^{l,H}} \left\{ \left(1-\theta^l\right) \ln V_{n_t^l}^h + \theta^l \ln V_{n_t^l}^f \right\} \tag{A.7}$$

where θ_l^f and θ_h^f denote the bargaining power on wage setting of firms for low and high skill positions, $V_{n_t^h}^f$, $V_{n_t^{hl}}^f$ and $V_{n_t^l}^f$ are the respective value functions of an additional unit of high skill, low skill and mismatched employment to each firm, and $V_{n_t^h}^h$ and $V_{n_t^{hl}}^h$ are the respective marginal values of a high skill household having an additional member employed in a high skill or mismatched position and $V_{n_t^l}^h$ is the respective marginal value of a low skill household having an additional member employed in a low skill position.

a) Derivation of the high-skill wage, $w_t^{h,H}$:

$$\max_{w_t^{h,H}} \left\{ \left(1-\theta^h\right) \ln V_{n_t^h}^h + \theta^h \ln V_{n_t^h}^f \right\}, \text{ subject to} \tag{A.8}$$

$$V_{n_t^h}^h = -\left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{c_t^h} w_t^{h,H} + \lambda_{n_t^{h,H}} \left(1-\sigma^h\right) - \lambda_{l_t^h} \tag{A.9}$$

$$V_{n_t^h}^f = p_t^H y_{i,t}^{h,f} - w_t^{h,H} + \kappa^h \frac{\left(1-\sigma^h\right)}{\psi_{F,t}^{h,H}} \tag{A.10}$$

Substituting the constraints yields:

$$\begin{aligned} \max_{w_t^{h,H}} \Bigg\{ & \left(1-\theta^h\right) \ln \left(-\left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{c_t^h} w_t^{h,H} + \lambda_{n_t^{h,H}} \left(1-\sigma^h\right) - \lambda_{l_t^h} \right) \\ & + \theta^h \ln \left(p_t^H y_{i,t}^{h,f} - w_t^{h,H} + \kappa^h \frac{\left(1-\sigma^h\right)}{\psi_{F,t}^{h,H}} \right) \Bigg\} \end{aligned} \tag{A.11}$$

Thus, the wage rate $w_t^{h,H}$ is given by:

$$w_t^{h,H} = \left(1-\theta^h\right) \left(p_t^H y_{i,t}^{h,f} + \kappa^h \frac{\left(1-\sigma^h\right)}{\psi_{F,t}^{h,H}} \right) - \frac{\theta^h}{\lambda_{c_t^h}} \left(-\left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{n_t^{h,H}} \left(1-\sigma^h\right) - \lambda_{l_t^h} \right) \tag{A.12}$$

b) Derivation of the mismatched wage, $w_t^{hl,H}$:

$$\max_{w_t^{hl,H}} \left\{ \left(1-\theta^{hl}\right) \ln V_{n_t^{hl}}^h + \theta^{hl} \ln V_{n_t^{hl}}^f \right\}, \text{ subject to} \tag{A.13}$$

$$V_{n_t^{hl}}^h = -\left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{c_t^h} \left(w_t^{hl,H} - b\left(z_t\right)\right) + \lambda_{n_t^{hl,H}} \left(1-\sigma^l - \phi\left(z_t\right) \psi_{H,t}^{h,H}\right) + \lambda_{n_t^{h,H}} \psi_{H,t}^{h,H} \phi\left(z_t\right) - \lambda_{l_t^h} \tag{A.14}$$

$$V_{n_t^{hl}}^f = p_t^H y_{i,t}^{hl,f} - w_t^{hl,H} + \kappa^l \frac{\left(1-\sigma^l - \phi\left(z_t\right) \psi_{H,t}^{h,H}\right)}{\psi_{F,t}^{hl,H}} \tag{A.15}$$

Substituting the constraints yields:

$$\begin{aligned} \max_{w_t^{hl,H}} \Bigg\{ & \left(1-\theta^{hl}\right) \ln \left(-\left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{c_t^h} \left(w_t^{hl,H} - b\left(z_t\right)\right) + \lambda_{n_t^{hl,H}} \left(1-\sigma^l - \phi\left(z_t\right) \psi_{H,t}^{h,H}\right) \right. \\ & \left. + \lambda_{n_t^{h,H}} \psi_{H,t}^{h,H} \phi\left(z_t\right) - \lambda_{l_t^h} \right) + \theta^{hl} \ln \left(p_t^H \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{hl,f}} - w_t^{hl,H} + \kappa^l \frac{\left(1-\sigma^l - \phi\left(z_t\right) \psi_{H,t}^{h,H}\right)}{\psi_{F,t}^{hl,H}} \right) \Bigg\} \end{aligned} \tag{A.16}$$

Thus, the wage rate $w_t^{hl,H}$ is given by:

$$w_t^{hl,H} = \left(1-\theta^{hl}\right)\left(p_t^H y_{i,t}^{hl,f} + \kappa^l \frac{\left(1-\sigma^l-\phi\left(z_t\right)\psi_{H,t}^{h,H}\right)}{\psi_{F,t}^{hl,H}}\right) - \frac{\theta^{hl}}{\lambda_{c_t^h}}\left(-\left(1-n_t^{h,F}\right)\Phi^h\left(l_t^h\right)^{-\phi^h} - \lambda_{c_t^h} b\left(z_t\right)\right.$$
$$\left. + \lambda_{n_t^{hl,H}}\left(1-\sigma^l-\phi\left(z_t\right)\psi_{H,t}^{h,H}\right) + \lambda_{n_t^{h,H}}\psi_{H,t}^{h,H}\phi\left(z_t\right) - \lambda_{l_t^h}\right) \tag{A.17}$$

c) Derivation of the low-skill wage, $w_t^{l,H}$:

$$\max_{w_t^{l,H}}\left\{\left(1-\theta^l\right)\ln V_{n_t^l}^h + \theta^l \ln V_{n_t^l}^f\right\}, \text{ subject to} \tag{A.18}$$

$$V_{n_t^l}^h = -\left(1-n_t^{l,F}\right)\Phi^l\left(l_t^l\right)^{-\phi^l} + \lambda_{c_t^l} w_t^{l,H} + \lambda_{n_t^{l,H}}(1-\sigma^l) - \lambda_{l_t^l} \tag{A.19}$$

$$V_{n_t^l}^f = p_t^H y_{i,t}^{ll,f} - w_t^{l,H} + \kappa^l \frac{\left(1-\sigma^l\right)}{\psi_{F,t}^{l,H}} \tag{A.20}$$

Substituting the constraints yields:

$$\max_{w_t^{l,H}}\left\{\left(1-\theta^l\right)\ln\left(-\left(1-n_t^{l,F}\right)\Phi^l\left(l_t^l\right)^{-\phi^l} + \lambda_{c_t^l} w_t^{l,H} + \lambda_{n_t^{l,H}}\left(1-\sigma^l\right) - \lambda_{l_t^l}\right)\right.$$
$$\left. + \theta^l \ln\left(p_t^H \frac{\vartheta y_{i,t}^f}{\vartheta n_t^{ll,f}} - w_t^{l,H} + \kappa^l \frac{\left(1-\sigma^l\right)}{\psi_{F,t}^{l,H}}\right)\right\} \tag{A.21}$$

Thus, the wage rate $w_t^{l,H}$ is given by:

$$w_t^{l,H} = \left(1-\theta^l\right)\left(p_t^H y_{i,t}^{ll,f} + \kappa^l \frac{\left(1-\sigma^l\right)}{\psi_{F,t}^{l,H}}\right) - \frac{\theta^l}{\lambda_{c_t^l}}\left(-\left(1-n_t^{l,F}\right)\Phi^l\left(l_t^l\right)^{-\phi^l} + \lambda_{n_t^{l,H}}\left(1-\sigma^l\right) - \lambda_{l_t^l}\right) \tag{A.22}$$

B Market clearing conditions

B.1 Profits

a) Intermediate good

Taking into account equations (80), (82), (87), (88), (94), (97), (100), the profits of the intermediate good firm are zero and the profit function of intermediate good firm is now given by:

$$p_t^H y_{i,t} = w_t^{l,H} t^l n_t^{l,H} + w_t^{hl,H} t^h n_t^{hl,H} + w_t^{h,H} t^h n_t^{h,H} + r_t^k k_t + \kappa^l t^h v_t^{l,f} + \kappa^h t^h v_t^{h,f} \tag{A.23}$$

b) Economy-wide final good

The profits of the economy-wide final good firm are zero. Hence, the profit function is now given by:

$$Y_t = p_t^H Y_t^H + p_t^F Y_t^F \Rightarrow \frac{1}{N_t} Y_t = p_t^H \frac{1}{N_t} Y_t^H + p_t^F \frac{1}{N_t} Y_t^F \Rightarrow y_t = p_t^H y_t^H + p_t^F y_t^F \tag{A.24}$$

Net foreign assets law of motion

$$e_t\left(r_t^d d_t - d_{t+1}\right) = p_t^H y_t^{F^*} - p_t^F y_t^F \tag{A.25}$$

Intermediate good distribution

$$\begin{aligned} Y_{i,t} = Y_t^H + Y_t^{F^*} &\Leftrightarrow \sum_{f=1}^{N_t^f} y_{i,t}^f = \sum_{f=1}^{N_t^f} y_t^H + \sum_{f=1}^{N_t^f} y_t^{F^*} \\ &\Leftrightarrow N_t^f y_{i,t}^f = N_t^f y_t^H + N_t^f y_t^{F^*} \\ &\Rightarrow y_{i,t}^f = y_t^H + y_t^{F^*} \xRightarrow{p_t^H} p_t^H y_{i,t}^f = p_t^H y_t^H + p_t^H y_t^{F^*} \end{aligned} \tag{A.26}$$

B.2 Resource constraint

High-skilled household budget constraint (in per capita terms)

$$\begin{aligned} &\left(1 - n_t^{h,F}\right) c_t^{h,H} t^h + e_t n_t^{h,F} c_t^{h,F} t^h + i_t + X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + e_t r_t^d d_t + b\left(z_t\right) n_t^{hl,H} t^h \\ &= \left(w_t^{h,H} n_t^{h,H} + w_t^{hl,H} n_t^{hl,H} + e_t w^{h,F} n_t^{h,F}\right) t^h + r_t^k k_t - \tau_t^h t^h + e_t d_{t+1} + \bar{\omega} u_t^h t^h + \bar{g}_t^{t,h} t^h \end{aligned} \tag{A.27}$$

Low-skilled household budget constraint (in per capita terms)

$$\left(1 - n_t^{l,F}\right) c_t^{l,H} t^l + e_t n_t^{l,F} c_t^{l,F} t^l + X^l\left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l = \left(w_t^{l,H} n_t^{l,H} + e_t w^{l,F} n_t^{l,F}\right) t^l - \tau_t^l t^l + \bar{\omega} u_t^l t^l + \bar{g}_t^{t,l} t^l \tag{A.28}$$

Government budget constraint (in per capita terms)

$$\bar{\omega} t^h u_t^h + \bar{\omega} t^l u_t^l + t^h \bar{g}_t^{t,h} + t^l \bar{g}_t^{t,l} + g_t^c = \tau_t \tag{A.29}$$

To get the resource constraint of the economy we work in steps:

Step 1: Add the budget constraints of the high-skilled and the low-skilled households, equations (A.27) and (A.28):

$$\begin{aligned} &\left(1 - n_t^{h,F}\right) c_t^{h,H} t^h + e_t n_t^{h,F} c_t^{h,F} t^h + i_t + X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + e_t r_t^d d_t + b\left(z_t\right) n_t^{hl,H} t^h + \left(1 - n_t^{l,F}\right) c_t^{l,H} t^l \\ &\quad + e_t n_t^{l,F} c_t^{l,F} t^l + X^l\left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l = \left(w_t^{h,H} n_t^{h,H} + w_t^{hl,H} n_t^{hl,H} + e_t w^{h,F} n_t^{h,F}\right) t^h + r_t^k k_t \\ &\qquad - \tau_t^h t^h + e_t d_{t+1} + \bar{\omega} u_t^h t^h + \bar{g}_t^{t,h} t^h + \left(w_t^{l,H} n_t^{l,H} + e_t w^{l,F} n_t^{l,F}\right) t^l - \tau_t^l t^l + \bar{\omega} u_t^l t^l + \bar{g}_t^{t,l} t^l \end{aligned} \tag{A.30}$$

Step 2: Use the government budget constraint, equation (A.29), and substitute out $\bar{\omega} t^h u_t^h + \bar{\omega} t^l u_t^l + t^h \bar{g}_t^{t,h} + t^l \bar{g}_t^{t,l} + g_t^c$ in equation (A.30):

$$\begin{aligned} &\left(1 - n_t^{h,F}\right) c_t^{h,H} t^h + e_t n_t^{h,F} c_t^{h,F} t^h + i_t + g_t^c + X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + e_t r_t^d d_t + b\left(z_t\right) n_t^{hl,H} t^h \\ &\quad + \left(1 - n_t^{l,F}\right) c_t^{l,H} t^l + e_t n_t^{l,F} c_t^{l,F} t^l + X^l\left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l = \left(w_t^{h,H} n_t^{h,H} + w_t^{hl,H} n_t^{hl,H} + e_t w^{h,F} n_t^{h,F}\right) t^h \\ &\qquad + r_t^k k_t + e_t d_{t+1} + \left(w_t^{l,H} n_t^{l,H} + e_t w^{l,F} n_t^{l,F}\right) t^l \end{aligned} \tag{A.31}$$

Step 3: Use the intermediate good firms profits, equation (A.23), and substitute out $w_t^{l,H} t^l n_t^{l,H} + w_t^{hl,H} t^h n_t^{hl,H} + w_t^{h,H} t^h n_t^{h,H} + r_t^k k_t$ in equation (A.31):

$$\begin{aligned}&\left(1-n_t^{h,F}\right) c_t^{h,H} t^h + \left(1-n_t^{l,F}\right) c_t^{l,H} t^l + e_t n_t^{h,F} \left(c_t^{h,F} - w^{h,F}\right) t^h + e_t n_t^{l,F} \left(c_t^{l,F} - w^{l,F}\right) t^l + i_t + g_t^c \\ &+ X^h \left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l + e_t \left(r_t^d d_t - d_{t+1}\right) + b\left(z_t\right) n_t^{hl,H} t^h = p_t^H y_{i,t} - \kappa^l v_t^l - \kappa^h v_t^h\end{aligned} \tag{A.32}$$

Step 4: Use the net foreign assets law of motion, equation (A.25), and substitute out $e_t\left(r_t^d d_t - d_{t+1}\right)$, and the intermediate good distribution, equation (A.26), and substitute out $p_t^H y_{i,t}^f$ in equation (A.32):

$$\begin{aligned}&\left(1-n_t^{h,F}\right) c_t^{h,H} t^h + \left(1-n_t^{l,F}\right) c_t^{l,H} t^l + e_t n_t^{h,F} \left(c_t^{h,F} - w^{h,F}\right) t^h \\ &+ e_t n_t^{l,F} \left(c_t^{l,F} - w^{l,F}\right) t^l + i_t + g_t^c + X^h \left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l \\ &+ p_t^H y^{F^*} - p_t^F y_t^F + b\left(z_t\right) n_t^{hl,H} t^h + \kappa^h v_t^h + \kappa^l v_t^l = p_t^H y_t^H + p_t^H y_t^{F^*}\end{aligned} \tag{A.33}$$

Step 5: Use the economy-wide final good profit function, equation (A.24), and substitute out $p_t^H + p_t^F$ in equation (A.33):

$$\begin{aligned}y_t = &\left(1-n_t^{h,F}\right) c_t^{h,H} t^h + \left(1-n_t^{l,F}\right) c_t^{l,H} t^l + e_t n_t^{h,F} \left(c_t^{h,F} - w^{h,F}\right) t^h + e_t n_t^{l,F} \left(c_t^{l,F} - w^{l,F}\right) t^l \\ &+ i_t + g_t^c + X^h \left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + X^l \left(\tilde{O}_t^l \tilde{u}_t^l\right) O_t^l u_t^l t^l + b\left(z_t\right) n_t^{hl,H} t^h + \kappa^h v_t^h + \kappa^l v_t^l\end{aligned} \tag{A.34}$$

C Decentralized Competitive Equilibrium

Given market prices $w_t^{h,H}$, $w_t^{hl,H}$, $w_t^{l,H}$, $w^{h,F}$, $w^{l,F}$, r_t^k, r_t^d, e_t, p_t^H, p_t^F, government policy τ_t^h, τ_t^l and economy-wide variables (A_t), each individual household of high-skilled households, $h = 1, 2, \ldots, N_t^h$, solves its problem as defined in Section 2.4.1, each individual household of low-skilled households, $l = 1, 2, \ldots, N_t^l$, solves its problem as defined in Section 2.4.2, each individual firm in the intermediate sector, $f = 1, 2, \ldots, N_t^f$, solves its problem as defined in Section 2.5.2, all markets clear and all constraints are satisfied. Thus the DCE, expressed in per capita terms, is given by equations (D.1)-(D.58):

Probabilities of a job seeker to be hired $\left\{\psi_{H,t}^{h,H}, \psi_{H,t}^{l,H}, \psi_{H,t}^{hl,H}\right\}$

$$\psi_{H,t}^{h,H} = \frac{m_t^{h,H}}{\left(1-O_t^h\right) s_t u_t^h t^h + \phi\left(z_t\right) n_t^{hl,H} t^h} \tag{D.1}$$

$$\psi_{H,t}^{l,H} = \frac{m_t^{l,H}}{\left(1-O_t^l\right) u_t^l t^l} \tag{D.2}$$

$$\psi_{H,t}^{hl,H} = \frac{m_t^{hl,H}}{\left(1-O_t^h\right)\left(1-s_t\right) u_t^h t^h} \tag{D.3}$$

Probabilities of a vacancy to be filled $\left\{\psi_{F,t}^{h,H}, \psi_{F,t}^{l,H}, \psi_{F,t}^{hl,H}\right\}$

$$\psi_{F,t}^{h,H} = \frac{m_t^{h,H}}{v_t^h} \tag{D.4}$$

$$\psi_{F,t}^{l,H} = \frac{m_t^{l,H}}{(1-x_t)\, v_t^l} \tag{D.5}$$

$$\psi_{F,t}^{hl,H} = \frac{m_t^{hl,H}}{x_t v_t^l} \tag{D.6}$$

Matches $\left\{m_t^{h,H}, m_t^{l,H}, m_t^{hl,H}\right\}$

$$m_t^{h,H} = \mu_1 \left(v_t^h\right)^{\mu_2} \left(\left(1-O_t^h\right) s_t u_t^h t^h + \phi\left(z_t\right) n_t^{hl,H} t^h\right)^{1-\mu_2} \tag{D.7}$$

$$m_t^{l,H} = \mu_1 \left((1-x_t)\, v_t^l\right)^{\mu_2} \left(\left(1-O_t^l\right) u_t^l t^l\right)^{1-\mu_2} \tag{D.8}$$

$$m_t^{hl,H} = \mu_1 \left(x_t v_t^l\right)^{\mu_2} \left(\left(1-O_t^h\right)(1-s_t)\, u_t^h t^h\right)^{1-\mu_2} \tag{D.9}$$

High-skilled h/h $\left\{c_t^{h,H}, c_t^{h,F}, i_t, k_{t+1}, d_{t+1}, n_{t+1}^{h,H}, n_{t+1}^{hl,H}, n_{t+1}^{h,F}, l_t^h, u_t^h, s_t, O_t^h, z_t, \lambda_{c_t^h}, \lambda_{n_t^{h,H}}, \lambda_{n_t^{hl,H}}, \lambda_{n_t^{h,F}}\right\}$

$$\begin{aligned} &\left(1-n_t^{h,F}\right) c_t^{h,H} t^h + e_t n_t^{h,F} c_t^{h,F} t^h + i_t + X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h u_t^h t^h + e_t r_t^d d_t + b\left(z_t\right) n_t^{hl,H} t^h \\ &= \left(w_t^{h,H} n_t^{h,H} + w_t^{hl,H} n_t^{hl,H} + e_t w^{h,F} n_t^{h,F}\right) t^h + r_t^k k_t - \tau_t^h t^h + e_t d_{t+1} + \bar{\omega} u_t^h t^h + \bar{g}_t^{t,h} t^h \end{aligned} \tag{D.10}$$

$$e_t \left(c_t^{h,F}\right)^{\eta} = \left(c_t^{h,H}\right)^{\eta} \tag{D.11}$$

$$i_t = k_{t+1} - (1-\delta)\, k_t + \frac{\Xi}{2}\left(\frac{k_{t+1}}{k_t} - 1\right)^2 k_t \tag{D.12}$$

$$\lambda_{c_t^h}\left(1 + \Xi\left(\frac{k_{t+1}}{k_t} - 1\right)\right) = \beta \mathbb{E}_t \lambda_{c_{t+1}^h}\left(1 + r_{t+1}^k - \delta + \frac{\Xi}{2}\left(\left(\frac{k_{t+2}}{k_{t+1}}\right)^2 - 1\right)\right) \tag{D.13}$$

$$\left(\frac{c_{t+1}^{h,H}}{c_t^{h,H}}\right)^{\eta} e_t = \beta \mathbb{E}_t e_{t+1} r_{t+1}^d \tag{D.14}$$

$$n_{t+1}^{h,H} = \left(1-\sigma^h\right) n_t^{h,H} + \frac{m_t^{h,H}}{t^h} \tag{D.15}$$

$$n_{t+1}^{hl,H} = \left(1 - \sigma^l - \phi\left(z_t\right) \psi_{H,t}^{h,H}\right) n_t^{hl,H} + \frac{m_t^{hl,H}}{t^h} \tag{D.16}$$

$$n_{t+1}^{h,F} = \left(1-\sigma^{h,F}\right) n_t^{h,F} + \psi^{h,F} O_t^h u_t^h \tag{D.17}$$

$$n_t^{h,H} + n_t^{hl,H} + u_t^h + l_t^h + n_t^{h,F} = 1 \tag{D.18}$$

$$\begin{aligned} \left(1-n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} &= -\lambda_{c_t^h}\left(X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) O_t^h - \bar{\omega}\right) + \lambda_{n_t^{h,H}} \psi_{H,t}^{h,H}\left(1-O_t^h\right) s_t \\ &+ \lambda_{n_t^{hl,H}} \psi_{H,t}^{hl,H}\left(1-O_t^h\right)(1-s_t) + \lambda_{n_t^{h,F}} \psi^{h,F} O_t^h \end{aligned} \tag{D.19}$$

$$\lambda_{n_t^{h,H}} \psi_{H,t}^{h,H} = \lambda_{n_t^{hl,H}} \psi_{H,t}^{hl,H} \tag{D.20}$$

$$\lambda_{n_t^{h,H}} \psi_{H,t}^{h,H} s_t + \lambda_{n_t^{hl,H}} \psi_{H,t}^{hl,H} (1 - s_t) = \lambda_{n_t^{h,F}} \psi^{h,F} - \lambda_{c_t^h} X^h \left(\tilde{O}_t^h \tilde{u}_t^h \right) \tag{D.21}$$

$$\lambda_{c_t^h} \frac{b'(z_t)}{\phi'(z_t)} = \psi_{H,t}^{h,H} \left(\lambda_{n_t^{h,H}} - \lambda_{n_t^{hl,H}} \right) \tag{D.22}$$

$$\lambda_{c_t^h} = \left(c_t^{h,H} \right)^{-\eta} \tag{D.23}$$

$$\lambda_{n_t^{h,H}} = \beta \mathbb{E}_t \left\{ - \left(1 - n_{t+1}^{h,F} \right) \Phi^h \left(l_{t+1}^h \right)^{-\phi^h} + \lambda_{c_{t+1}^h} w_{t+1}^{h,H} + \lambda_{n_{t+1}^{h,H}} \left(1 - \sigma^h \right) \right\} \tag{D.24}$$

$$\begin{aligned} \lambda_{n_t^{hl,H}} = \beta \mathbb{E}_t \Bigg\{ & - \left(1 - n_{t+1}^{h,F} \right) \Phi^h \left(l_{t+1}^h \right)^{-\phi^h} + \lambda_{c_{t+1}^h} \left(w_{t+1}^{hl,H} - b(z_{t+1}) \right) + \lambda_{n_{t+1}^{hl,H}} \left(1 - \sigma^l - \phi(z_{t+1}) \psi_{H,t+1}^{h,H} \right) \\ & + \lambda_{n_{t+1}^{h,H}} \psi_{H,t+1}^{h,H} \phi(z_{t+1}) \Bigg\} \end{aligned} \tag{D.25}$$

$$\begin{aligned} \lambda_{n_t^{h,F}} = \beta \mathbb{E}_t \Bigg\{ & \frac{\left(c_{t+1}^{h,F} \right)^{1-\eta}}{1-\eta} - \frac{\left(c_{t+1}^{h,H} \right)^{1-\eta}}{1-\eta} - \Phi^h \frac{\left(l_{t+1}^h \right)^{1-\phi^h}}{1-\phi^h} - \left(1 - n_{t+1}^{h,F} \right) \Phi^h \left(l_{t+1}^h \right)^{-\phi^h} \\ & - \lambda_{c_{t+1}^h} \left(e_t c_{t+1}^{h,F} - c_{t+1}^{h,H} - e_{t+1} w^{h,F} \right) + \lambda_{n_{t+1}^{h,F}} \left(1 - \sigma^{h,F} \right) \Bigg\} \end{aligned} \tag{D.26}$$

Low-skilled h/h $\left\{ c_t^{l,H}, c_t^{l,F}, n_{t+1}^{l,H}, n_{t+1}^{l,F}, l_t^l, u_t^l, O_t^l, \lambda_{c_t^l}, \lambda_{n_t^{l,H}}, \lambda_{n_t^{l,F}} \right\}$

$$\left(1 - n_t^{l,F} \right) c_t^{l,H} t^l + e_t n_t^{l,F} c_t^{l,F} t^l + X^l \left(\tilde{O}_t^l \tilde{u}_t^l \right) O_t^l u_t^l t^l = w_t^{l,H} n_t^{l,H} t^l + e_t w^{l,F} n_t^{l,F} t^l - \tau_t^l t^l + \bar{\omega} u_t^l t^l + \bar{g}_t^{t,l} t^l \tag{D.27}$$

$$e_t \left(c_t^{l,F} \right)^\eta = \left(c_t^{l,H} \right)^\eta \tag{D.28}$$

$$n_{t+1}^{l,H} = \left(1 - \sigma^l \right) n_t^{l,H} + \frac{m_t^{l,H}}{t^l} \tag{D.29}$$

$$n_{t+1}^{l,F} = \left(1 - \sigma^{l,F} \right) n_t^{l,F} + \psi^{l,F} O_t^l u_t^l \tag{D.30}$$

$$n_t^{l,H} + u_t^l + l_t^l + n_t^{l,F} = 1 \tag{D.31}$$

$$\left(1 - n_t^{l,F} \right) \Phi^l \left(l_t^l \right)^{-\phi^l} = -\lambda_{c_t^l} \left(X^l \left(\tilde{O}_t^l \tilde{u}_t^l \right) O_t^l - \bar{\omega} \right) + \lambda_{n_t^{l,H}} \psi_{H,t}^{l,H} \left(1 - O_t^l \right) + \lambda_{n_t^{l,F}} \psi^{l,F} O_t^l \tag{D.32}$$

$$\lambda_{n_t^{l,H}} \psi_{H,t}^{l,H} = \lambda_{n_t^{l,F}} \psi^{l,F} - \lambda_{c_t^l} X^l \left(\tilde{O}_t^l \tilde{u}_t^l \right) \tag{D.33}$$

$$\lambda_{c_t^l} = \left(c_t^{l,H}\right)^{-\eta} \tag{D.34}$$

$$\lambda_{n_t^{l,H}} = \beta \mathbb{E}_t \left\{ -\left(1 - n_{t+1}^{l,F}\right) \Phi^l \left(l_{t+1}^l\right)^{-\phi^l} + \lambda_{c_{t+1}^l} w_{t+1}^{l,H} + \lambda_{n_{t+1}^{l,H}} \left(1 - \sigma^l\right) \right\} \tag{D.35}$$

$$\begin{aligned} \lambda_{n_t^{l,F}} = \beta \mathbb{E}_t \Bigg\{ & \frac{\left(c_{t+1}^{l,F}\right)^{1-\eta}}{1-\eta} - \frac{\left(c_{t+1}^{l,H}\right)^{1-\eta}}{1-\eta} - \Phi^l \frac{\left(l_{t+1}^l\right)^{1-\phi^l}}{1-\phi^l} - \left(1 - n_{t+1}^{l,F}\right) \Phi^l \left(l_{t+1}^l\right)^{-\phi^l} \\ & - \lambda_{c_{t+1}^l} \left(e_t c_{t+1}^{l,F} - c_{t+1}^{l,H} - e_{t+1} w^{l,F}\right) + \lambda_{n_{t+1}^{l,F}} \left(1 - \sigma^{l,F}\right) \Bigg\} \end{aligned} \tag{D.36}$$

Wages $\left\{w_t^{h,H}, w_t^{l,H}, w_t^{hl,H}\right\}$

$$w_t^{h,H} = (1-\theta^h) \left(p_t^H y_{i,t}^h + \kappa^h \frac{(1-\sigma^h)}{\psi_{F,t}^{h,H}}\right) - \frac{\theta^h}{\lambda_{c_t^h}} \left(-\left(1 - n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} + \lambda_{n_t^{h,H}} \left(1 - \sigma^h\right)\right) \tag{D.37}$$

$$w_t^{l,H} = (1-\theta^l) \left(p_t^H y_{i,t}^l + \kappa^l \frac{(1-\sigma^l)}{\psi_{F,t}^{l,H}}\right) - \frac{\theta^l}{\lambda_{c_t^l}} \left(-\left(1 - n_t^{l,F}\right) \Phi^l \left(l_t^l\right)^{-\phi^l} + \lambda_{n_t^{l,H}} \left(1 - \sigma^l\right)\right) \tag{D.38}$$

$$\begin{aligned} w_t^{hl,H} = & (1-\theta^{hl}) \left(p_t^H y_{i,t}^{hl} + \kappa^l \frac{\left(1 - \sigma^l - \phi(z_t) \psi_{H,t}^{h,H}\right)}{\psi_{F,t}^{hl,H}}\right) \\ & - \frac{\theta^{hl}}{\lambda_{c_t^h}} \left(-\left(1 - n_t^{h,F}\right) \Phi^h \left(l_t^h\right)^{-\phi^h} - \lambda_{c_t^h} b(z_t) + \lambda_{n_t^{hl,H}} \left(1 - \sigma^l - \phi(z_t) \psi_{H,t}^{h,H}\right) + \lambda_{n_t^{h,H}} \psi_{H,t}^{h,H} \phi(z_t)\right) \end{aligned} \tag{D.39}$$

Intermediate-good firm focs $\left\{y_{i,t}, x_{i,t}, x_t, n_t^l, y_t^{F^*}, r_t^k, r_t^d, p_t^H, v_t^l, v_t^h, \Lambda_{t,t+1}\right\}$

$$y_{i,t} = A_t \left(\alpha \left(t^h n_t^l\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha) \left(x_{i,t}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{D.40}$$

$$x_{i,t} = \left(\zeta (k_t)^{\frac{\rho-1}{\rho}} + (1-\zeta) (t^h n_t^{h,H})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{D.41}$$

$$x_t = \frac{t^h \left(n_{t+1}^{hl,H} - \left(1 - \sigma^l - \phi(z_t) \psi_{H,t}^{h,H}\right) n_t^{hl,H}\right)}{v_t^l \psi_{F,t}^{hl,H}} \tag{D.42}$$

$$n_t^l = \frac{t^l}{t^h} n_t^{l,H} + q^h n_t^{hl,H} \tag{D.43}$$

$$\frac{\kappa^h}{\psi_{F,t}^{h,H}} = \mathbb{E}_t \Lambda_{t,t+1} \left\{ p_{t+1}^H y_{i,t+1}^h - w_{t+1}^{h,H} + \kappa^h \frac{(1-\sigma^h)}{\psi_{F,t+1}^{h,H}} \right\} \tag{D.44}$$

$$\frac{\kappa^l}{\psi_{F,t}^{l,H}} = \mathbb{E}_t \Lambda_{t,t+1} \left\{ p_{t+1}^H y_{i,t+1}^l - w_{t+1}^{l,H} + \kappa^l \frac{(1-\sigma^l)}{\psi_{F,t+1}^{l,H}} \right\} \tag{D.45}$$

$$\frac{\kappa^l}{\psi_{F,t}^{hl,H}} = \mathbb{E}_t\Lambda_{t,t+1}\left\{p_{t+1}^H y_{i,t+1}^{hl} - w_{t+1}^{hl,H} + \kappa^l \frac{\left(1-\sigma^l-\phi\left(z_{t+1}\right)\psi_{H,t+1}^{h,H}\right)}{\psi_{F,t+1}^{hl,H}}\right\} \tag{D.46}$$

$$\Lambda_{t,t+1} = \beta\left(\frac{c_{t+1}^{h,H}}{c_t^{h,H}}\right)^{-\eta} \tag{D.47}$$

$$y_{i,t} = y_t^H + y_t^{F^*} \tag{D.48}$$

$$r_t^k = p_t^H y_{i,t}^k \tag{D.49}$$

$$y_t^{F^*} = (1-\omega^*)\left(\frac{p_t^H}{e_t}\right)^{-\gamma^*} y_t^* \tag{D.50}$$

Economy-wide final good $\left\{y_t, y_t^H, y_t^F\right\}$

$$y_t = \left(\omega^{\frac{1}{\gamma}}\left(y_t^H\right)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}\left(y_t^F\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{D.51}$$

$$y_t^F = (1-\omega)\left(\frac{p_t^F}{P_t}\right)^{-\gamma} y_t \tag{D.52}$$

$$y_t^H = \frac{\omega}{1-\omega}\left(\frac{p_t^H}{p_t^F}\right)^{-\gamma} y_t^F \tag{D.53}$$

Government $\left\{g_t^c\right\}$

$$\bar{\omega}t^h u_t^h + \bar{\omega}t^l u_t^l + t^h\bar{g}_t^{t,h} + t^l\bar{g}_t^{t,l} + g_t^c = \tau_t \tag{D.54}$$

Market clearing condition $\left\{e_t\right\}$

$$p_t^H y_{i,t} = w_t^{l,H} t^l n_t^{l,H} + w_t^{hl,H} t^h n_t^{hl,H} + w_t^{h,H} t^h n_t^{h,H} + r_t^k k_t + \kappa^l v_t^l + \kappa^h v_t^h \tag{D.55}$$

Closing the SOE model $\left\{r^*, rp_t\right\}$

$$r_t^d = r_t^* + rp_t \tag{D.56}$$

$$rp_t = \psi^{rp}\left(exp\left(\frac{e_t d_{t+1}}{gdp_t} - \frac{ed}{gdp}\right) - 1\right) + \epsilon_t^{rp} \tag{D.57}$$

Additional definitions $\left\{nx_t, gdp_t, P\right\}$

$$e_t\left(r_t^d d_t - d_{t+1}\right) = p_t^H y_t^{F^*} - p_t^F y_t^F \tag{D.58}$$

$$gdp_t = y_t + nx_t \tag{D.59}$$

$$e_t = \frac{P_t^*}{P_t} \tag{D.60}$$

We thus have a system of 60 equations in the paths of 60 unknown endogenous variables: $\psi_{H,t}^{h,H}$, $\psi_{H,t}^{l,H}$, $\psi_{H,t}^{hl,H}$, $\psi_{F,t}^{h,H}$, $\psi_{F,t}^{l,H}$, $\psi_{F,t}^{hl,H}$, $m_t^{h,H}$, $m_t^{l,H}$, $m_t^{hl,H}$, $c_t^{h,H}$, $c_t^{h,F}$, i_t, k_{t+1}, d_{t+1}, $n_{t+1}^{h,H}$, $n_{t+1}^{hl,H}$, $n_{t+1}^{h,F}$, l_t^h, u_t^h, s_t, O_t^h, z_t, $\lambda_{c_t^h}$, $\lambda_{n_t^{h,H}}$, $\lambda_{n_t^{hl,H}}$, $\lambda_{n_t^{h,F}}$, $c_t^{l,H}$, $c_t^{l,F}$, $n_{t+1}^{l,H}$, $n_{t+1}^{l,F}$, l_t^l, u_t^l, O_t^l, $\lambda_{c_t^l}$, $\lambda_{n_t^{l,H}}$, $\lambda_{n_t^{l,F}}$, $w_t^{h,H}$, $w_t^{l,H}$, $w_t^{hl,H}$, $y_{i,t}$, $x_{i,t}$, x_t, n_t^l, $y_t^{F^*}$, r_t^k, p_t^H, v_t^l, v_t^h, $\Lambda_{t,t+1}$, r_t^d, y_t, y_t^H, y_t^F, g_t^c, e_t, r^*, rp_t, nx_t, gdp_t, P.

$$y_{i,t}^k = \zeta (1-\alpha) A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}} \left(\frac{x_{i,t}}{k_t}\right)^{\frac{1}{\rho}} \tag{D.61}$$

$$y_{i,t}^h = (1-\zeta)(1-\alpha) A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}}{x_{i,t}}\right)^{\frac{1}{\epsilon}} \left(\frac{x_{i,t}}{t^h n_t^{h,H}}\right)^{\frac{1}{\rho}} \tag{D.62}$$

$$y_{i,t}^l = \alpha A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}}{t^l n_t^{l,H} + q^h t^h n_t^{hl,H}}\right)^{\frac{1}{\epsilon}} \tag{D.63}$$

$$y_{i,t}^{hl} = q_h \alpha A_t^{\frac{\epsilon-1}{\epsilon}} \left(\frac{y_{i,t}}{t^l n_t^{l,H} + q^h t^h n_t^{hl,H}}\right)^{\frac{1}{\epsilon}} \tag{D.64}$$

We choose the following functional forms $\left\{b(z), \phi(z), X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right), X^l\left(\tilde{O}_t^l \tilde{u}_t^l\right)\right\}$

$$b(z_t) = b_1 (z_t)^{b_2} \tag{D.65}$$

$$\phi(z_t) = \phi_1 (z_t)^{\phi_2} \tag{D.66}$$

$$X^h\left(\tilde{O}_t^h \tilde{u}_t^h\right) = x_1^h \left(\tilde{O}_t^h \tilde{u}_t^h\right)^{x_2^h} \tag{D.67}$$

$$X^l\left(\tilde{O}_t^l \tilde{u}_t^l\right) = x_1^l \left(\tilde{O}_t^l \tilde{u}_t^l\right)^{x_2^l} \tag{D.68}$$

D Steady-state equilibrium

In the long-run, our economy reaches an equilibrium where no shocks exist and variables remain constant. To find the steady state equilibrium, we remove time subscripts and solve for the equilibrium. Thus, all variables satisfy that $x_{t+1} = x_t = x_{t-1} = x$. The steady-state equilibrium is given by equations (S.1) - (S.64):

Probabilities of a job seeker to be hired $\left\{\psi_H^{h,H}, \psi_H^{l,H}, \psi_H^{hl,H}\right\}$

$$\psi_H^{h,H} = \frac{m^{h,H}}{(1-O^h) s u^h t^h + \phi(z) n^{hl,H} t^h} \tag{S.1}$$

$$\psi_H^{l,H} = \frac{m^{l,H}}{(1-O^l) u^l t^l} \tag{S.2}$$

$$\psi_H^{hl,H} = \frac{m^{hl,H}}{\left(1-O^h\right)\left(1-s\right)u^h t^h} \tag{S.3}$$

Probabilities of a vacancy to be filled $\left\{\psi_F^{h,H}, \psi_F^{l,H}, \psi_F^{hl,H}\right\}$

$$\psi_F^{h,H} = \frac{m^{h,H}}{v^h} \tag{S.4}$$

$$\psi_F^{l,H} = \frac{m^{l,H}}{\left(1-x\right)v^l} \tag{S.5}$$

$$\psi_F^{hl,H} = \frac{m^{hl,H}}{xv^l} \tag{S.6}$$

Matchess $\left\{m^{h,H}, m^{l,H}, m^{hl,H}\right\}$

$$m^{h,H} = \mu_1\left(v^h\right)^{\mu_2}\left(\left(1-O^h\right)su^h t^h + \phi\left(z\right)n^{hl,H}t^h\right)^{1-\mu_2} \tag{S.7}$$

$$m^{l,H} = \mu_1\left(\left(1-x\right)v^l\right)^{\mu_2}\left(\left(1-O^l\right)u^l t^l\right)^{1-\mu_2} \tag{S.8}$$

$$m^{hl,H} = \mu_1\left(xv^l\right)^{\mu_2}\left(\left(1-O^h\right)\left(1-s\right)u^h t^h\right)^{1-\mu_2} \tag{S.9}$$

High-skilled h/h $\left\{c^{h,H}, c^{h,F}, i, k, d, n^{h,H}, n^{hl,H}, n^{h,F}, l^h, u^h, s, O^h, z, \lambda_{c^h}, \lambda_{n^{h,H}}, \lambda_{n^{hl,H}}, \lambda_{n^{h,F}}\right\}$

$$\begin{aligned}&\left(1-n^{h,F}\right)c^{h,H}t^h + en^{h,F}c^{h,F}t^h + i + X^h\left(\tilde{O}^h\tilde{u}^h\right)O^h u^h t^h + b\left(z\right)n^{hl,H}t^h + er^d d\\ &= \left(w^{h,H}n^{h,H} + w^{hl,H}n^{hl,H} + ew^{h,F}n^{h,F}\right)t^h + r^k k - \tau^h t^h + ed + \bar{\omega}u^h t^h + \bar{g}^{t,h}t^h\end{aligned} \tag{S.10}$$

$$e\left(c^{h,F}\right)^{\eta} = \left(c^{h,H}\right)^{\eta} \tag{S.11}$$

$$i = \delta k \tag{S.12}$$

$$1 = \beta\left(1 + r^k - \delta\right) \tag{S.13}$$

$$1 = \beta r^d \tag{S.14}$$

$$n^{h,H} = \frac{m^{h,H}}{\sigma^h t^h} \tag{S.15}$$

$$n^{hl,H} = \frac{m^{hl,H}}{\left(\sigma^l + \phi\left(z\right)\psi_H^{h,H}\right)t^h} \tag{S.16}$$

$$n^{h,F} = \frac{\psi^{h,F}O^h u^h}{\sigma^{h,F}} \tag{S.17}$$

$$n^{h,H}+n^{hl,H}+u^{h}+l^{h}+n^{h,F}=1 \tag{S.18}$$

$$\begin{aligned}\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}=&-\lambda_{c^{h}}\left(X^{h}\left(\tilde{O}^{h}\tilde{u}^{h}\right)O^{h}-\bar{\omega}\right)+\lambda_{n^{h,H}}\psi_{H}^{h,H}\left(1-O^{h}\right)s\\&+\lambda_{n^{hl,H}}\psi_{H}^{hl,H}\left(1-O^{h}\right)(1-s)+\lambda_{n^{h,F}}\psi^{h,F}O^{h}\end{aligned} \tag{S.19}$$

$$\lambda_{n^{h,H}}\psi_{H}^{h,H}=\lambda_{n^{hl,H}}\psi_{H}^{hl,H} \tag{S.20}$$

$$\lambda_{n^{h,H}}\psi_{H}^{h,H}s+\lambda_{n^{hl,H}}\psi_{H}^{hl,H}\left(1-s\right)=\lambda_{n^{h,F}}\psi^{h,F}-\lambda_{c^{h}}X^{h}\left(\tilde{O}^{h}\tilde{u}^{h}\right) \tag{S.21}$$

$$\lambda_{c^{h}}\frac{b'\left(z\right)}{\phi'\left(z\right)}=\psi_{H}^{h,H}\left(\lambda_{n^{h,H}}-\lambda_{n^{hl,H}}\right) \tag{S.22}$$

$$\lambda_{c^{h}}=\left(c^{h,H}\right)^{-\eta} \tag{S.23}$$

$$\lambda_{n^{h,H}}=\beta\left\{-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}+\lambda_{c^{h}}w^{h,H}+\lambda_{n^{h,H}}\left(1-\sigma^{h}\right)\right\} \tag{S.24}$$

$$\lambda_{n^{hl,H}}=\beta\left\{-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}+\lambda_{c^{h}}\left(w^{hl,H}-b(z)\right)+\lambda_{n^{hl,H}}\left(1-\sigma^{l}-\phi(z)\psi_{H}^{h,H}\right)+\lambda_{n^{h,H}}\psi_{H}^{h,H}\phi\left(z\right)\right\} \tag{S.25}$$

$$\begin{aligned}\lambda_{n^{h,F}}=\beta\Bigg\{&\frac{\left(c^{h,F}\right)^{1-\eta}}{1-\eta}-\frac{\left(c^{h,H}\right)^{1-\eta}}{1-\eta}-\Phi^{h}\frac{\left(l^{h}\right)^{1-\phi^{h}}}{1-\phi^{h}}-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}\\&-\lambda_{c^{h}}\left(ec^{h,F}-c^{h,H}-ew^{h,F}\right)+\lambda_{n^{h,F}}\left(1-\sigma^{h,F}\right)\Bigg\}\end{aligned} \tag{S.26}$$

Low-skilled h/h $\left\{c^{l,H},c^{l,F},n^{l,H},n^{l,F},l^{l},u^{l},O^{l},\lambda_{c^{l}},\lambda_{n^{l,H}},\lambda_{n^{l,F}}\right\}$

$$\begin{aligned}y=&\left(1-n^{h,F}\right)c^{h,H}t^{h}+\left(1-n^{l,F}\right)c^{l,H}t^{l}+en^{h,F}\left(c^{h,F}-w^{h,F}\right)t^{h}+en^{l,F}\left(c^{l,F}-w^{l,F}\right)t^{l}\\&+i+X^{h}\left(\tilde{O}^{h}\tilde{u}^{h}\right)O^{h}u^{h}t^{h}+X^{l}\left(\tilde{O}^{l}\tilde{u}^{l}\right)O^{l}u^{l}t^{l}+b\left(z\right)n^{hl,H}t^{h}+\kappa^{h}v^{h}+\kappa^{l}v^{l}\end{aligned} \tag{S.27}$$

$$e\left(c^{l,F}\right)^{\eta}=\left(c^{l,H}\right)^{\eta} \tag{S.28}$$

$$n^{l,H}=\frac{m^{l,H}}{\sigma^{l}t^{l}} \tag{S.29}$$

$$n^{l,F}=\frac{\psi^{l,F}O^{l}u^{l}}{\sigma^{l,F}} \tag{S.30}$$

$$n^{l,H}+u^{l}+l^{l}+n^{l,F}=1 \tag{S.31}$$

$$\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}=-\lambda_{c^{l}}\left(X^{l}\left(\tilde{O}^{l}\tilde{u}^{l}\right)O^{l}-\bar{\omega}\right)+\lambda_{n^{l,H}}\psi_{H}^{l,H}\left(1-O^{l}\right)+\lambda_{n^{l,F}}\psi^{l,F}O^{l} \tag{S.32}$$

$$\lambda_{n^{l,H}}\psi_{H}^{l,H}=\lambda_{n^{l,F}}\psi^{l,F}-\lambda_{c^{l}}X^{l}\left(\tilde{O}^{l}\tilde{u}^{l}\right) \tag{S.33}$$

$$\lambda_{c^{l}}=\left(c^{l,H}\right)^{-\eta} \tag{S.34}$$

$$\lambda_{n^{l,H}}=\beta\left\{-\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}+\lambda_{c^{l}}w^{l,H}+\lambda_{n^{l,H}}\left(1-\sigma^{l}\right)\right\} \tag{S.35}$$

$$\begin{aligned}\lambda_{n^{l,F}}=\beta\Bigg\{&\frac{\left(c^{l,F}\right)^{1-\eta}}{1-\eta}-\frac{\left(c^{l,H}\right)^{1-\eta}}{1-\eta}-\Phi^{l}\frac{\left(l^{l}\right)^{1-\phi^{l}}}{1-\phi^{l}}-\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}-\lambda_{c^{l}}\left(ec^{l,F}-c^{l,H}-ew^{l,F}\right)\\&+\lambda_{n^{l,F}}\left(1-\sigma^{l,F}\right)\Bigg\}\end{aligned} \tag{S.36}$$

Wages $\left\{w^{h,H},w^{l,H},w^{hl,H}\right\}$

$$w^{h,H}=\left(1-\theta^{h}\right)\left(p^{H}y_{i}^{h}+\kappa^{h}\frac{\left(1-\sigma^{h}\right)}{\psi_{F}^{h,H}}\right)-\frac{\theta^{h}}{\lambda_{c^{h}}}\left(-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}+\lambda_{n^{h,H}}\left(1-\sigma^{h}\right)\right) \tag{S.37}$$

$$w^{l,H}=\left(1-\theta^{l}\right)\left(p^{H}y_{i}^{l}+\kappa^{l}\frac{\left(1-\sigma^{l}\right)}{\psi_{F}^{l,H}}\right)-\frac{\theta^{l}}{\lambda_{c^{l}}}\left(-\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}+\lambda_{n^{l,H}}\left(1-\sigma^{l}\right)\right) \tag{S.38}$$

$$\begin{aligned}w^{hl,H}=&\left(1-\theta^{hl}\right)\left(p^{H}y_{i}^{hl}+\kappa^{l}\frac{\left(1-\sigma^{l}-\phi\left(z\right)\psi_{H}^{h,H}\right)}{\psi_{F}^{hl,H}}\right)\\&-\frac{\theta^{hl}}{\lambda_{c^{h}}}\left(-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}-\lambda_{c^{h}}b\left(z\right)+\lambda_{n^{hl,H}}\left(1-\sigma^{l}-\phi\left(z\right)\psi_{H}^{h,H}\right)+\lambda_{n^{h,H}}\psi_{H}^{h,H}\phi\left(z\right)\right)\end{aligned} \tag{S.39}$$

Intermediate good firm $\left\{y_{i},x_{i},x,n^{l},y^{F^{*}},r^{k},r^{d},p^{H},v^{l},v^{h},\Lambda\right\}$

$$y_{i}=A\left(\alpha\left(t^{h}n^{l}\right)^{\frac{\epsilon-1}{\epsilon}}+\left(1-\alpha\right)\left(x_{i}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{S.40}$$

$$x_{i}=\left(\zeta(k)^{\frac{\rho-1}{\rho}}+(1-\zeta)(t^{h}n^{h,H})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{S.41}$$

$$x=\frac{t^{h}\left(\sigma^{l}+\phi\left(z\right)\psi_{H}^{h,H}\right)n^{hl,H}}{v^{l}\psi_{F}^{hl,H}} \tag{S.42}$$

$$n^{l}=\frac{t^{l}}{t^{h}}n^{l,H}+q^{h}n^{hl,H} \tag{S.43}$$

$$\frac{\kappa^{h}}{\psi_{F}^{h,H}}=\beta\left\{p^{H}y_{i}^{h}-w^{h,H}+\kappa^{h}\frac{\left(1-\sigma^{h}\right)}{\psi_{F}^{h,H}}\right\} \tag{S.44}$$

$$\frac{\kappa^l}{\psi_F^{l,H}} = \beta\left\{p^H y_i^l - w^{l,H} + \kappa^l \frac{\left(1-\sigma^l\right)}{\psi_F^{l,H}}\right\} \tag{S.45}$$

$$\frac{\kappa^l}{\psi_F^{hl,H}} = \beta\left\{p^H y_i^{hl} - w^{hl,H} + \kappa^l \frac{\left(1-\sigma^l-\phi\left(z\right)\psi_H^{h,H}\right)}{\psi_F^{hl,H}}\right\} \tag{S.46}$$

$$\Lambda = \beta \tag{S.47}$$

$$y_i = y^H + y^{F^*} \tag{S.48}$$

$$r^k = p^H y_i^k \tag{S.49}$$

$$y^{F^*} = \left(1-\omega^*\right)\left(\frac{p^H}{e}\right)^{-\gamma^*} y^* \tag{S.50}$$

Economy-wide final good $\left\{y, y^H, y^F\right\}$

$$y = \left(\omega^{\frac{1}{\gamma}}\left(y^H\right)^{\frac{\gamma-1}{\gamma}} + \left(1-\omega\right)^{\frac{1}{\gamma}}\left(y^F\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{S.51}$$

$$y^F = \left(1-\omega\right)\left(\frac{p^F}{P}\right)^{-\gamma} y \tag{S.52}$$

$$y^H = \frac{\omega}{1-\omega}\left(\frac{p^H}{p^F}\right)^{-\gamma} y^F \tag{S.53}$$

Government $\left\{\tau\right\}$

$$\bar{\omega} t^h u^h + \bar{\omega} t^l u^l + t^h \bar{g}^{t,h} + t^l \bar{g}^{t,l} = \tau \tag{S.54}$$

Market clearing condition $\left\{e\right\}$

$$p^H y_i = w^{l,H} t^l n^{l,H} + w^{hl,H} t^h n^{hl,H} + w^{h,H} t^h n^{h,H} + r^k k + \kappa^l v^l + \kappa^h v^h \tag{S.55}$$

Closing the SOE model $\left\{r^d, rp\right\}$

$$r^d = r^* + rp \tag{S.56}$$

$$rp = \psi^{rp} \tag{S.57}$$

Additional definitions $\left\{nx, gdp\right\}$

$$ed(r^d - 1) = p^H y^{F^*} - p^F y^F \tag{S.58}$$

$$gdp = y + nx \tag{S.59}$$

$$e = \frac{P^*}{P} \tag{S.60}$$

$\{b_1, \phi_2\}$

$$b(z) = b_1 (z)^{b_2} \tag{S.61}$$

$$\phi(z) = \phi_1 (z)^{\phi_2} \tag{S.62}$$

$\{x_2^h, x_2^l\}$

$$X^h \left(\tilde{O}^h \tilde{u}^h\right) = x_1^h \left(O^h u^h\right)^{x_2^h} \tag{S.63}$$

$$X^l \left(\tilde{O}^l \tilde{u}^l\right) = x_1^l \left(O^l u^l\right)^{x_2^l} \tag{S.64}$$

E Calibration strategy

We set employment and unemployment rates per household type using data from the Labor Force Survey (Hellenic Statistical authority). Then, l^l and l^h are obtained residually from:

$$n^{l,H} + u^l + l^l + n^{l,F} = 1 \tag{S.31}$$

$$n^{h,H} + n^{hl,H} + u^h + l^h + n^{h,F} = 1 \tag{S.18}$$

Setting $\psi^{h,F}/\psi_H^{h,H} = 1.6$ and $\psi_H^{h,H} = 0.10$, we find $\psi^{h,F} = 0.160$. We compute $m^{h,H} = 0.0115$ and $m^{l,H} = 0.0270$ from:

$$n^{h,H} = \frac{m^{h,H}}{\sigma^h t^h} \tag{S.15}$$

$$n^{l,H} = \frac{m^{l,H}}{\sigma^l t^l} \tag{S.29}$$

Setting $\psi^{l,F} = 0.5$, we find $O^l = 0.0033$ from:

$$n^{l,F} = \frac{\psi^{l,F} O^l u^l}{\sigma^{l,F}} \tag{S.30}$$

We find $\psi_H^{l,H} = 0.3277$ from:

$$\psi_H^{l,H} = \frac{m^{l,H}}{(1 - O^l)\, u^l t^l} \tag{S.2}$$

We set $\psi^{h,F} = 0.70$, as in Bandeira et al. (2022), and find $O^h = 0.0256$ from:

$$n^{h,F} = \frac{\psi^{h,F} O^h u^h}{\sigma^{h,F}} \tag{S.17}$$

We solve a system of two equations in two unknowns and obtain $s = 0.1647$ and $\phi(z) = 1.8988$ from:

$$\psi_H^{h,H} = \frac{m^{h,H}}{(1 - O^h) s u^h t^h + \phi(z) n^{hl,H} t^h} \tag{S.1}$$

$$\psi_H^{hl,H} = \frac{m^{hl,H}}{(1 - O^h)(1 - s) u^h t^h} \tag{S.3}$$

We calculate $m^{hl,H} = 0.0159$ and $v^h = 0.0165$ from:

$$n^{hl,H} = \frac{m^{hl,H}}{\left(\sigma^l + \phi(z) \psi_H^{h,H}\right) t^h} \tag{S.16}$$

$$\psi_F^{h,H} = \frac{m^{h,H}}{v^h} \tag{S.4}$$

To find $\mu_1 = 0.4129$ and $\mu_2 = 0.7288$, we solve the following system of equations:

$$m^{h,H} = \mu_1 \left(v^h\right)^{\mu_2} \left((1 - O^h) s u^h t^h + \phi(z) n^{hl,H} t^h\right)^{1-\mu_2} \tag{S.7}$$

$$m^{l,H} = \mu_1 \left(m^{l,H}/\psi_F^{l,H}\right)^{\mu_2} \left((1 - O^l) u^l t^l\right)^{1-\mu_2} \tag{S.8}$$

where $(1 - x) v^l = 0.0601$ and $xv^l = 0.0515$ are obtained from:

$$\psi_F^{l,H} = \frac{m^{l,H}}{(1 - x) v^l} \tag{S.5}$$

$$m^{hl,H} = \mu_1 \left(xv^l\right)^{\mu_2} \left((1 - O^h)(1 - s) u^h t^h\right)^{1-\mu_2} \tag{S.9}$$

Then, $v^l = 0.1116$ and $x = 0.4612$. Finally, $\psi_F^{hl,H} = 0.3090$ is obtained from:

$$\psi_F^{hl,H} = \frac{m^{hl,H}}{xv^l} \tag{S.6}$$

We control for the elasticity of substitution between physical capital and skilled labor ($\rho = 0.7$), for the elasticity substitution between unskilled labor and capital-skilled labor ($\epsilon = 1.46$), sthe hare of unskilled labor ($\alpha = 0.47$), the share of government transfers ($s^{g,t} = 0.15$) and total vacancy costs, $\kappa^h v^h + \kappa^l v^l$, to be 3% of GDP. Furthermore, by setting the price levels P, p^{F^*} equal to 1, and in order to match k/y, y^F/y, y^{F^*}/y, d/y, $w^{h,H}/w^{l,H}$, $w^{hl,H}/w^{l,H}$, c^p/y from the data, the following equations form a system of 43 equations in 43 unknowns: β, γ, δ, ζ, ω, Φ^h, Φ^l, $X^h\left(\tilde{O}^h\tilde{u}^h\right)$, $X^l\left(\tilde{O}^l\tilde{u}^l\right)$, κ^l, $b(z)$, r^d, r^k, k, d, $c^{h,H}$, $c^{h,F}$, $c^{l,H}$, $c^{l,F}$, y, y^H, y^F, y_i, x_i, y^H/y, y^{F^*}, y^*, $w^{h,H}$, $w^{hl,H}$, $w^{l,H}$, p^H, p^F, e, q^h, τ, $\bar{\omega}$, λ_{c^h}, λ_{n^h}, $\lambda_{n^{hl}}$, $\lambda_{n^{h,F}}$, λ_{c^l}, λ_{n^l}, $\lambda_{n^{l,F}}$.

$$\frac{y^H}{y} = \omega \left(\frac{p^H}{P}\right)^{-\gamma} \tag{S.53}$$

$$\frac{y^F}{y} = (1-\omega)\left(\frac{p^F}{P}\right)^{-\gamma} \tag{S.52}$$

$$1 = \left(\omega^{\frac{1}{\gamma}}\left(y^H/y\right)^{\frac{\gamma-1}{\gamma}} + (1-\omega)^{\frac{1}{\gamma}}\left(y^F/y\right)^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}} \tag{S.51}$$

$$1 = (\omega p_H^{1-\gamma} + (1-\omega)p_F^{1-\gamma})^{\frac{1}{1-\gamma}} \tag{SS.1}$$

$$\frac{y_i}{y} = \frac{y^H}{y} + \frac{y^{F^*}}{y} \tag{S.48}$$

$$y^H = \left(y^H/y\right)y \tag{SS.2}$$

$$y^F = \left(y^F/y\right)y \tag{SS.3}$$

$$p^H y_i = w^{l,H}t^l n^{l,H} + w^{hl,H}t^h n^{hl,H} + w^{h,H}t^h n^{h,H} + r^k k + \kappa^l v^l + \kappa^h v^h \tag{S.55}$$

$$k = \left(\frac{k}{y}\right)y \tag{SS.4}$$

$$x_i = \left(\zeta(k)^{\frac{\rho-1}{\rho}} + (1-\zeta)(t^h n^{h,H})^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{S.41}$$

$$y_i = A\left(\alpha\left(t^h n^l\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\alpha)\left(x_i\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{S.40}$$

$$\frac{\kappa^h}{\psi_F^{h,H}} = \beta\left\{p^H(1-\zeta)(1-\alpha)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{t^h n^{h,H}}\right)^{\frac{1}{\rho}} - w^{h,H} + \kappa^h\frac{(1-\sigma^h)}{\psi_F^{h,H}}\right\} \tag{S.44}$$

$$\frac{w^{hl,H}}{w^{l,H}} = \frac{\beta p^H \alpha q^h A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{n^{l,H}t^l + q^h n^{hl,H}t^h}\right)^{\frac{1}{\epsilon}} + \frac{\kappa^l}{\psi_F^{hl,H}}\left(\beta\left(1-\sigma^l-\phi(z)\psi_H^{h,H}\right)-1\right)}{\beta p^H \alpha A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{n^{l,H}t^l + q^h n^{hl,H}t^h}\right)^{\frac{1}{\epsilon}} + \frac{\kappa^l}{\psi_F^{l,H}}\left(\beta\left(1-\sigma^l\right)-1\right)} \tag{S.45}$$

$$w^{l,H} = \frac{w^{h,H}}{(w^{h,H}/w^{l,H})} \tag{SS.5}$$

$$w^{hl,H} = w^{l,H}\left(\frac{w^{hl,H}}{w^{l,H}}\right) \tag{SS.6}$$

$$r^d + \frac{i/y}{k/y} - 1 = p^H\zeta(1-\alpha)A^{\frac{\epsilon-1}{\epsilon}}\left(\frac{y_i}{x_i}\right)^{\frac{1}{\epsilon}}\left(\frac{x_i}{k}\right)^{\frac{1}{\rho}} \tag{S.49}$$

$$\frac{\kappa^l v^l + \kappa^h v^h}{y} = 0.01 \tag{SS.7}$$

$$1 = \frac{c^p}{y} + \frac{i}{y} + \frac{g^c}{y} + \frac{\kappa^l v^l + \kappa^h v^h}{y} + \frac{X^h\left(\bar{O}^h \bar{u}^h\right) O^h u^h t^h}{y} + \frac{X^l\left(\bar{O}^l \bar{u}^l\right) O^l u^l t^l}{y} + \frac{b\left(z\right) n^{hl,H} t^h}{y} \tag{S.27}$$

$$p^F = e p^{F^*} \tag{SS.8}$$

$$\frac{y^{F^*}}{y} = \left(1-\omega\right)\left(\frac{p^H}{e}\right)^{-\gamma} \frac{y^*}{y} \tag{SS.9}$$

$$d = \left(\frac{d}{y}\right) y \tag{SS.10}$$

$$p^H\left(y_i - y^H\right) - p^F y^F = e\left(\frac{d}{y}\right) y\left(r^d - 1\right) \tag{S.58}$$

$$y^{F^*} = \left(\frac{y^{F^*}}{y}\right) y \tag{SS.11}$$

$$\lambda_{c^h} = \left(c^{h,H}\right)^{-\eta} \tag{S.23}$$

$$\left(1 - n^{h,F}\right)\Phi^h\left(l^h\right)^{-\phi^h} = \lambda_{c^h}\bar{\omega} + \lambda_{n^{h,H}}\psi_H^{h,H} \tag{S.19}$$

$$e\left(c^{h,F}\right)^{\eta} = \left(c^{h,H}\right)^{\eta} \tag{S.11}$$

$$\lambda_{n^{h,H}}\left(1 - \beta\left(1 - \sigma^h - \psi_H^{h,H}\right)\right) = \beta\lambda_{c^h}\left(w^{h,H} - \bar{\omega}\right) \tag{S.24}$$

$$\begin{aligned}\lambda_{n^{h,F}}\left(1 - \beta\left(1 - \sigma^{h,F}\right)\right) = \beta\left\{\frac{\left(c^{h,F}\right)^{1-\eta}}{1-\eta} - \frac{\left(c^{h,H}\right)^{1-\eta}}{1-\eta} - \Phi^h\frac{\left(l^h\right)^{1-\phi^h}}{1-\phi^h} - \left(1 - n^{h,F}\right)\Phi^h\left(l^h\right)^{-\phi^h}\right\} \\ - \beta\lambda_{c^h}\left(e c^{h,F} - c^{h,H} - e\left(\frac{w^{h,F}}{w^{h,H}}\right) w^{h,H}\right)\end{aligned} \tag{S.26}$$

$$\lambda_{n^{h,F}}\psi^{h,F} = \lambda_{c^h} X^h\left(\tilde{O}^h\tilde{u}^h\right) + \lambda_{n^{h,H}}\psi_H^{h,H} \tag{S.21}$$

$$\lambda_{n^{h,H}}\psi_H^{h,H} = \lambda_{n^{hl,H}}\psi_H^{hl,H} \tag{S.20}$$

$$\lambda_{n^{hl,H}}\left(1 - \beta\left(1 - \sigma^l - \phi\left(z\right)\phi_H^{h,H}\right)\right) = \beta\left\{-\left(1 - n^{h,F}\right)\Phi^h\left(l^h\right)^{-\phi^h} + \lambda_{c^h}\left(w^{hl,H} - b(z)\right) + \lambda_{n^{h,H}}\psi_H^{h,H}\phi\left(z\right)\right\} \tag{S.25}$$

$$
\begin{aligned}
&\left(1-n^{h,F}\right)c^{h,H}t^{h}+en^{h,F}c^{h,F}t^{h}+\left(\frac{i}{y}\right)y+X^{h}\left(\tilde{O}^{h}\tilde{u}^{h}\right)O^{h}u^{h}t^{h}+b\left(z\right)n^{hl,H}t^{h}+er^{d}\left(\frac{d}{y}\right)y\\
&=\left(w^{h,H}n^{h,H}+w^{hl,H}n^{hl,H}+e\left(\frac{w^{h,F}}{w^{h,H}}\right)w^{h,H}n^{h,F}\right)t^{h}+r^{k}k-\tau t^{h}+e\left(\frac{d}{y}\right)y+\bar{\omega}u^{h}t^{h}+s^{g,t}yt^{h}
\end{aligned}
\tag{S.10}
$$

$$
\begin{aligned}
\frac{c^{p}}{y}=\frac{\left(1-n^{h,F}\right)c^{h,H}t^{h}}{y}&+\frac{\left(1-n^{l,F}\right)c^{l,H}t^{l}}{y}\\
&+\frac{en^{h,F}\left(c^{h,F}-\left(\frac{w^{h,F}}{w^{h,H}}\right)w^{h,H}\right)t^{h}}{y}+\frac{en^{l,F}\left(c^{l,F}-\left(\frac{w^{l,F}}{w^{l,H}}\right)w^{l,H}\right)t^{l}}{y}
\end{aligned}
\tag{SS.11}
$$

$$
e\left(c^{l,F}\right)^{\eta}=\left(c^{l,H}\right)^{\eta}
\tag{S.28}
$$

$$
\bar{\omega}t^{h}u^{h}+\bar{\omega}t^{l}u^{l}+t^{h}\bar{g}^{t,h}+t^{l}\bar{g}^{t,l}+g^{c}=\tau
\tag{S.54}
$$

$$
\lambda_{c^{l}}=\left(c^{l,H}\right)^{-\eta}
\tag{S.34}
$$

$$
\lambda_{n^{l,H}}\left(1-\beta\left(1-\sigma^{l}-\psi_{H}^{l,H}\right)\right)=\beta\lambda_{c^{l}}\left(w^{l,H}-\bar{\omega}\right)
\tag{S.35}
$$

$$
\lambda_{n^{l,F}}\psi^{l,F}=\lambda_{c^{l}}X^{l}\left(\tilde{O}^{l}\tilde{u}^{l}\right)+\lambda_{n^{l,H}}\psi_{H}^{l,H}
\tag{S.33}
$$

$$
\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}=\lambda_{c^{l}}\bar{\omega}+\lambda_{n^{l,H}}\psi_{H}^{l,H}
\tag{S.32}
$$

$$
\begin{aligned}
\lambda_{n^{l,F}}\left(1-\beta\left(1-\sigma^{l,F}\right)\right)=\beta&\left\{\frac{\left(c^{l,F}\right)^{1-\eta}}{1-\eta}-\frac{\left(c^{l,H}\right)^{1-\eta}}{1-\eta}-\Phi^{l}\frac{\left(l^{l}\right)^{1-\phi^{l}}}{1-\phi^{l}}-\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}}\right\}\\
&-\beta\lambda_{c^{l}}\left(ec^{l,F}-c^{l,H}-e\left(\frac{w^{l,F}}{w^{l,H}}\right)w^{l,H}\right)
\end{aligned}
\tag{S.36}
$$

$$
1=\beta r^{d}
\tag{S.14}
$$

$$
\frac{i}{y}=\delta\frac{k}{y}
\tag{S.12}
$$

$$
r^{d}=1+r^{k}-\delta
\tag{S.13}
$$

Then, we obtain the bargaining power of firms, θ^{h}, θ^{l} and θ^{hl}, from equations:

$$
w^{h,H}=\left(1-\theta^{h}\right)\left(p^{H}y_{i}^{h}+\kappa^{h}\frac{\left(1-\sigma^{h}\right)}{\psi_{F}^{h,H}}\right)-\frac{\theta^{h}}{\lambda_{c^{h}}}\left(-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}}+\lambda_{n^{h,H}}\left(1-\sigma^{h}\right)\right)
\tag{S.37}
$$

$$w^{l,H} = \left(1-\theta^{l}\right)\left(p^{H}y_{i}^{l} + \kappa^{l}\frac{\left(1-\sigma^{l}\right)}{\psi_{F}^{l,H}}\right) - \frac{\theta^{l}}{\lambda_{c^{l}}}\left(-\left(1-n^{l,F}\right)\Phi^{l}\left(l^{l}\right)^{-\phi^{l}} + \lambda_{n^{l,H}}\left(1-\sigma^{l}\right)\right) \tag{S.38}$$

$$\begin{aligned} w^{hl,H} = &\left(1-\theta^{hl}\right)\left(p^{H}y_{i}^{hl} + \kappa^{l}\frac{\left(1-\sigma^{l}-\phi\left(z\right)\psi_{H}^{h,H}\right)}{\psi_{F}^{hl,H}}\right) \\ &- \frac{\theta^{hl}}{\lambda_{c^{h}}}\left(-\left(1-n^{h,F}\right)\Phi^{h}\left(l^{h}\right)^{-\phi^{h}} - \lambda_{c^{h}}b\left(z\right) + \lambda_{n^{hl,H}}\left(1-\sigma^{l}-\phi\left(z\right)\psi_{H}^{h,H}\right) + \lambda_{n^{h,H}}\psi_{H}^{h,H}\phi\left(z\right)\right) \end{aligned} \tag{S.39}$$

Regarding the parameters of the on-the-job search cost, b_1, b_2, the efficacy of this search ϕ_1 ϕ_2, and the search effort to end mismatch z, we use the following procedure: first, we normalize $\phi_1 = 1$ and we set the cost of search, b_2, to be quadratic. Then, by using the following equations, we solve for ϕ_2, b_1, z:

$$b(z) = b_1(z)^{b_2} \tag{S.61}$$

$$\phi(z) = \phi_1(z)^{\phi_2} \tag{S.62}$$

$$\lambda_{c^h}\frac{b'(z)}{\phi'(z)} = \psi_H^{h,h}(\lambda_{n^{h,H}} - \lambda_{n^{hl,H}}) \tag{S.22}$$

Finally, we set x_1^h, x_1^l by jointly targeting (a) the share of emigration flows in the working age population around 2010 (0.7%) and (b) an average skilled to unskilled emigrants ratio of 2/3 (see also Bandeira et al. (2022)). This is in line with survey evidence from Labrianidis et al. (2016) who report that more than 65% of Greek emigrants post 2010 were highly educated graduates (as measured by ISCED levels of 5 and above). Hence, given that in equilibrium $\tilde{O}^h\tilde{u}^h = O^h u^h$ and $\tilde{O}^l\tilde{u}^l = O^l u^l$, we obtain x_2^h, x_2^l from the following equations:

$$X^h\left(O^h u^h\right) = x_1^h\left(O^h u^h\right)^{x_2^h} \tag{S.63}$$

$$X^l\left(O^l u^l\right) = x_1^l\left(O^l u^l\right)^{x_2^l} \tag{S.64}$$