## 1 System of equations

Main features

- 2 sectors: tradable $(\mathrm{T})+$ non-tradable( N$)$
- 2 workers: 1,2
- Workers consume, supply labor and save in foreign bonds
- Perfect labor mobility for each worker type between T-NT sectors
- shock: price of the endowment: "p Co"
- Fraction "1- $\mu$ " of the endowment goes for worker "1", fraction $\mu$ goes for worker "2"

Euler equations:

$$
\begin{align*}
& {\left[c_{1, t}-\frac{1}{\omega}\left(n_{1, t}\right)^{\omega}\right]^{-\sigma}=\beta \mathbb{E}_{t}\left\{\left[c_{1, t+1}-\frac{1}{\omega}\left(n_{1, t+1}\right)^{\omega}\right]^{-\sigma} \frac{p_{t}}{p_{t+1}}\left(1+R_{t}\right)\right\}}  \tag{Eq.1}\\
& {\left[c_{2, t}-\frac{1}{\omega}\left(n_{2, t}\right)^{\omega}\right]^{-\sigma}=\beta \mathbb{E}_{t}\left\{\left[c_{2, t+1}-\frac{1}{\omega}\left(n_{2, t+1}\right)^{\omega}\right]^{-\sigma} \frac{p_{t}}{p_{t+1}}\left(1+R_{t}\right)\right\}} \tag{Eq.2}
\end{align*}
$$

Budget constraints:

$$
\begin{align*}
& c_{1, t}+\left(1+R_{t-1}\right) d_{1, t-1}=\alpha_{T} \frac{Y_{t}^{T}}{N_{1, t}^{T}} n_{1, t}+d_{1, t}+(1-\mu) p_{t}^{C o} y^{C o}  \tag{Eq.3}\\
& c_{2, t}+\left(1+R_{t-1}\right) d_{2, t-1}=\left(1-\alpha_{T}\right) \frac{Y_{t}^{T}}{N_{2, t}^{T}} n_{2, t}+d_{2, t}+\mu p_{t}^{C o} y^{C o} \tag{Eq.4}
\end{align*}
$$

Consumption aggregation:

$$
\begin{align*}
& c_{1, t}^{N}=\varphi\left(\frac{p_{N, t}}{p_{t}}\right)^{-\chi} c_{1, t}  \tag{Eq.5}\\
& c_{1, t}^{N}=\varphi\left(\frac{p_{N, t}}{p_{t}}\right)^{-\chi} c_{2, t}  \tag{Eq.6}\\
& c_{1, t}^{T}=(1-\varphi)\left(\frac{1}{p_{t}}\right)^{-\chi} c_{1, t}  \tag{Eq.7}\\
& c_{2, t}^{T}=(1-\varphi)\left(\frac{1}{p_{t}}\right)^{-\chi} c_{2, t}  \tag{Eq.8}\\
& C_{t}=c_{1, t}+c_{2, t}  \tag{Eq.9}\\
& C_{t}^{N}=c_{1, t}^{N}+c_{2, t}^{N}  \tag{Eq.10}\\
& C_{t}^{T}=c_{1, t}^{T}+c_{2, t}^{T}  \tag{Eq.11}\\
& p_{t}=\left[\varphi\left(p_{N, t}\right)^{1-\chi}+(1-\varphi)\right]^{\frac{1}{1-\chi}} \tag{Eq.12}
\end{align*}
$$

Production functions and optimality conditions:

$$
\begin{align*}
& Y_{t}^{T}=\left(N_{1, t}^{T}\right)^{\alpha_{T}}\left(N_{2, t}^{T}\right)^{1-\alpha_{T}}  \tag{Eq.13}\\
& Y_{t}^{N}=\left(N_{1, t}^{N}\right)^{\alpha_{N}}\left(N_{2, t}^{N}\right)^{1-\alpha_{N}}  \tag{Eq.14}\\
& \left(n_{1, t}\right)^{\omega-1}=\alpha_{T} \frac{Y_{t}^{T}}{N_{1, t}^{T}} \frac{1}{p_{t}}  \tag{Eq.15}\\
& \left(n_{1, t}\right)^{\omega-1}=\alpha_{N} \frac{Y_{t}^{N}}{N_{1, t}^{N}} \frac{p_{N, t}}{p_{t}}  \tag{Eq.16}\\
& \left(n_{2, t}\right)^{\omega-1}=\left(1-\alpha_{T}\right) \frac{Y_{t}^{T}}{N_{2, t}^{T}} \frac{1}{p_{t}}  \tag{Eq.17}\\
& \left(n_{2, t}\right)^{\omega-1}=\left(1-\alpha_{N}\right) \frac{Y_{t}^{N}}{N_{2, t}^{N}} \frac{p_{N, t}}{p_{t}} \tag{Eq.18}
\end{align*}
$$

Interest rate

$$
\begin{equation*}
R_{t}=r+\psi\left(e^{d_{1, t}+d_{2, t}-d_{1}-d_{2}}-1\right) \tag{Eq.19}
\end{equation*}
$$

Market clearing

$$
\begin{equation*}
Y_{t}^{N}=C_{t}^{N} \tag{Eq.20}
\end{equation*}
$$

Labor market clearing:

$$
\begin{align*}
n_{1, t} & =N_{1, t}^{T}+N_{1, t}^{N}  \tag{Eq.21}\\
n_{2, t} & =N_{2, t}^{T}+N_{2, t}^{N} \tag{Eq.22}
\end{align*}
$$

## Exogenous variables

$$
\begin{equation*}
\ln \left(p_{t}^{C o}\right)=\left(1-\rho_{c o}\right) \ln \left(p_{t-1}^{C o}\right)+\rho_{c o} \ln \left(p_{t}^{C o}\right)+\epsilon_{p^{c o}} \tag{Eq.23}
\end{equation*}
$$

