- Do we use CPI inflation, GDP deflator or PCE inflation?

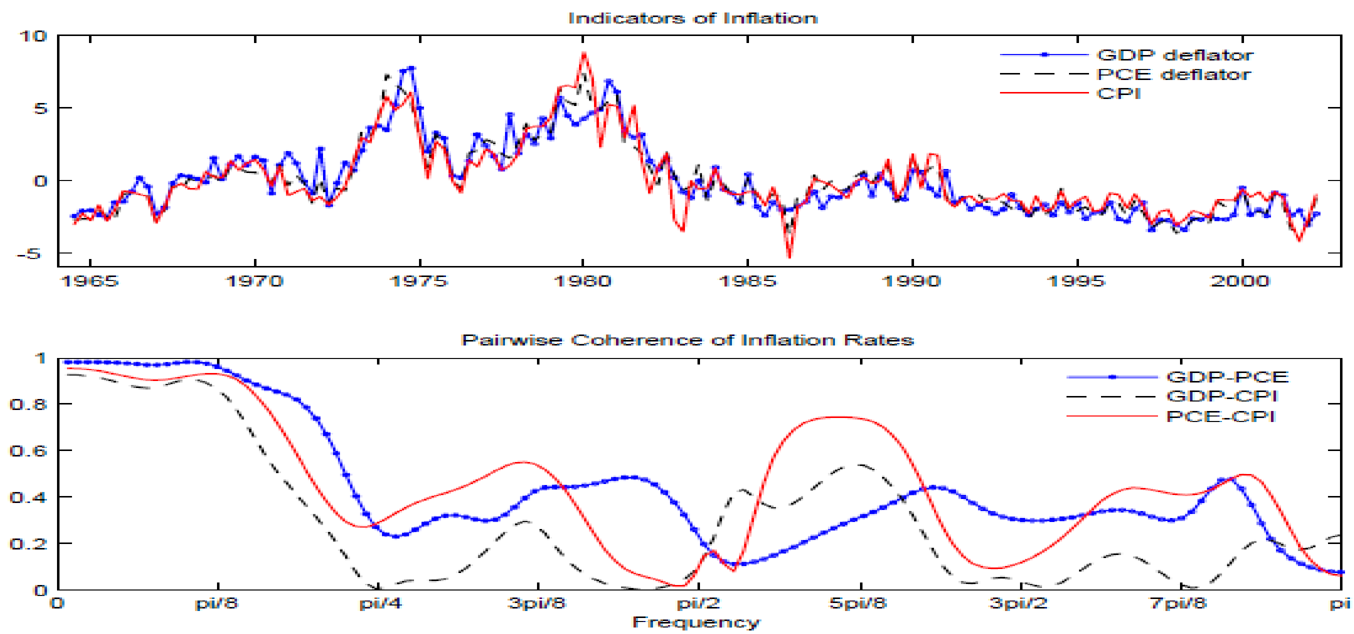

Figure 2: Indicators of quarterly inflation rates (de-meaned) and pairwise coherence.

• Idea: different measures contain (noisy) information about the true series. Not perfectly correlated among each other.

Case 1: Measurement error is present.

Observable x_{0t}. Model based quantity: $x_{0t}^{m}(\theta) = S_0[y_{1t}, y_{2t}]$, S_0 is a selection matrix.

$$x_{0t} = x_t^m(\theta) + u_{0t} \tag{65}$$

where u_{0t} is iid measurement error.

- Difference with the setup of previous section is that measurement error is now economically motivated here.

- Cases 2-4 use ideas underlying **factor models**

- Case 2 (Boivin and Giannoni, 2005) let x_{1t} be a $k \times 1$ vector of observables and let $x_{1t}^m(\theta) = S_1[y_{1t}, y_{2t}]$, where S_1 is another selection matrix of dimension $N \times 1$, dim (N) <dim(k). Then measurement equation is:

$$x_{1t} = \Lambda_1 x_t^m(\theta) + u_{1t} \tag{66}$$

where the first row of Λ_1 is normalized to 1, and u_{1t} is iid measurement error.

• Use x_{1t} to jointly estimate $\theta, \Lambda_1, \sigma_u^2$. and recover the states y_{1t} if it is of interest.

• Interpretation of $\Lambda_{1j}, j = 2, ...N$: information content of indicator j for x_t^m relative to indicator 1.

- What is the advantage of this procedure? If only one component of x_{1t} is used, estimates of θ will probably be noisy.

- Using a vector and assuming that the elements of u_{1t} are idiosyncratic:

i) can reduce the noise in the estimate of the states y_{1t} (the estimated variance of y_{1t} will be asymptotically of the order $1/k$ $\times$ the variance obtained when only one indicator is used, see Stock and Watson, 2002).

ii) estimates of θ more precise, see Justiniano et al. (2012).

- What is the difference with factor models? Here the DSGEs structure is imposed in the specification of the law of motion of the states (states have economic content). In factor models, the states have an unrestricted time series specification, say a random walk, and are uninterpretable.

- How do we identify the dynamics induced by the structural shocks and the measurement errors? Since the measurement error is identified from the cross sectional properties of the observables x_{1t}, it is possible to have both the structural disturbances and measurement errors serially correlated.

Many cases fit in case 3):

- Sometimes we may have proxy measures for the unobservable states. (commodity prices are often used as proxies for future inflation shocks, stock market shocks are used as proxies for future technology shocks, see Beaudry and Portier (2006).

- Sometimes we have survey data to proxy for unobserved states (e.g. business cycles).

- Sometimes we have flash information (preliminary estimates).

- Assume that these indicators give you information about the states y_{1t}. Let x_{2t} a $q \times 1$ vector of variables and let $x^m_{2t}(\theta) = S_2[y_{1t}, y_{2t}]$, S_2 is another selection matrix. Measurement equation:

$$x_{2t} = \Lambda_2 x^m_{2t}(\theta) + u_{2t} \tag{67}$$

Λ_2 is unrestricted, except for the first row, which is normalized to 1.

- Combining all sources of available information:

$$X_t = \Lambda S y_t(\theta) + u_t \tag{68}$$

where $X_t = [x_{0t}, x_{1t}, x_{2t}]'$, $u_t = [u_{0t}, u_{1t}, u_{2t}]'$ and $\Lambda = [I, \Lambda_1, \Lambda_2]'$, $y_t = [y_{1t}, y_{2t}]$ and $S = diag[S_0, S_1, S_2]$.

- The fact that we are using the DSGE structure ($x_t^m(\theta)$ depends on θ) imposes restrictions on the data.

- We interpret data information through the lenses of the DSGE model, even though the model does not feature the variables used in estimation.

Case 4): use **transformations** of the data which are, hopefully, less noisy. For example, output and hours may be poorly estimated, but labor productivity may be better estimated.

Observables x_{3t}. Model based quantities $x_{3t}^m(\theta) = S_3[y_{1t}, y_{2t}]$, S_3 is a selection matrix. Then

$$x_{3t} = Mx_{3t}^m(\theta) + u_{3t} \tag{69}$$

where u_{3t} is iid measurement error, and M is matrix of zero and ones.

Example 6.1 *Consider a three equation New-Keynesian model:*

$$o_t = E_t(o_{t+1}) - \frac{1}{\phi}(i_t - E_t\pi_{t+1}) + e_{1t} \quad (70)$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa o_t + e_{2t} \quad (71)$$

$$i_t = \psi_r i_{t-1} + (1-\psi_r)(\psi_\pi \pi_t + \psi_x o_t) + e_{3t} \quad (72)$$

where β is the discount factor, ϕ the relative risk aversion coefficient, κ the slope of Phillips curve, $(\psi_r, \psi_\pi, \psi_x)$ policy parameters. Here o_t is the output gap, π_t the inflation rate and i_t the nominal interest rate. Assume

$$e_{1t} = \rho_1 e_{1t-1} + v_{1t} \quad (73)$$

$$e_{2t} = \rho_2 e_{2t-1} + v_{2t} \quad (74)$$

$$e_{3t} = v_{3t} \quad (75)$$

where $\rho_1, \rho_2 < 1$, $v_{jt} \sim (0, \sigma_j^2), j = 1, 2, 3$.

- How do we link the output gap, the inflation rate and the nominal interest rate to empirical counterparts? Which the nominal interest rate should we use? How do we measure the gap?

- Model solution (state equations of the state space system)

$$y_t = R(\theta)y_{t-1} + S(\theta)v_t \tag{76}$$

where y_t *is a* 8×1 *vector including* $(o_t, \pi_t, i_t, e_{1t}, e_{2t}, e_{3t})$, *the expectations of* x_t *and* π_t *and* $\theta = (\beta, \phi, \kappa, \psi_r, \psi_y, \psi_\pi, \rho_1, \rho_2, \sigma_1, \sigma_2, \sigma_3)$.

Let $o_t^j, j = 1, \ldots N_x$ *be indicators for* o_t, *let* $\pi_t^j, j = 1, \ldots N_\pi$ *indicators for* π_t, *and* $i_t^j, j = 1, \ldots N_i$ *indicators for* i_t. *Let*

$X_t = [o_t^1, \ldots, o_t^{N_x}, \pi_t^1, \ldots, \pi_t^{N_\pi}, i_t^1, \ldots i_t^{N_i}]'$ *be a* $N_x + N_\pi + N_i \times 1$ *vector.*

- *The measurement equation is*

$$X_t = \Lambda y_t + u_t \tag{77}$$

where Λ *is* $N_x + N_\pi + N_i \times 3$ *matrix with at most one nonzero element in each row and* u_t *is iid.*

- *(76)-(77) is an* **extended state space**. *Kalman filter routine gives us estimates of* θ, Λ *and* y_t, *which are consistent with the data* x_t.