Utility: CRRA

$$u(c_t, l_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \frac{A(1-n_t-e_t)^{1-\gamma}}{1-\gamma}$$

Constraints

- Human capital

$$h_{t+1} = (1-\delta_h)h_t + \Omega h_t x_t^{\phi_x} e_t^{\phi_e} \quad \text{—— (1)}$$

second alternative

$$h_{t+1} = (1-\delta_h)h_t + \Omega h_t e_t^{\phi_e} + x_t$$

$$x_t = h_{t+1} - (1-\delta_h)h_t - \Omega h_t e_t^{\phi_e}$$

- time endowment constraint

$$n_t + e_t + l_t = 1 \quad \text{—— (2)}$$

- non-negative constraint

$$n_t, e_t, l_t \geq 0 \quad \text{—— (3)}$$

- Cash-in-advance constraint

Case 1: human capital expenditure as cash good

$$c_t + x_t \leq \frac{g_t - 1 + \hat{m}_{t-1}}{g_t \hat{p}_t} \quad \text{—— (4)}$$

$$\hat{p}_t = \frac{p_t}{M_t}$$

$$\hat{m}_t = \frac{m_t}{M_t}$$

Case 2: human capital expenditure as credit good

$$c_t \leq \frac{g_{t-1} + \hat{m}_{t-1}}{g_t \hat{p}_t} \quad \text{—— (5)}$$

- Physical capital Accumulation

$$K_{t+1} = (1-\delta_k)K_t + y_t \quad \text{—— (6)} \qquad y_t = K_{t+1} - (1-\delta_k)K_t$$

- Budget constraint

$$c_t + x_t + y_t + \frac{\hat{m}_t}{\hat{p}_t} \leq w_t . h_t . n_t + r_t . K_t + \frac{\hat{m}_{t-1} + g_t - 1}{g_t \hat{p}_t} \quad \text{—— (7)}$$

roduction function

$$Y_t = e^{z_t} K_t^{\theta} (H_t N_t)^{1-\theta} \quad — (8)$$

o Competitive firm maximize profit

$$\max_{\Pi} \Pi = Y_t - w_t H_t N_t - r_t K_t \quad — (9)$$

where $H_t N_t$ is the effective labor

in equilibrium $h_t = H_t$ ~~and~~, $n_t = N_t$ and $k_t = K_t$

firms doesn't choose H_t as the human capital accumulation is decided by household

maximize profit with respect to N_t and K_t

$$[N_t]: (1-\theta) e^{z_t} \left(\frac{K_t}{H_t N_t}\right)^{\theta} . H_t - w_t H_t = 0$$

$$w_t = (1-\theta) e^{z_t} \left(\frac{K_t}{H_t N_t}\right)^{\theta} \quad — (10)$$

$$[K_t]: \theta e^{z_t} K_t^{\theta-1} (H_t N_t)^{1-\theta} - r_t = 0$$

$$r_t = \theta e^{z_t} \left(\frac{H_t N_t}{K_t}\right)^{1-\theta} \quad — (11)$$

Market clearing condition

$$C_t + X_t + Y_t = e^{z_t} K_t^{\theta} (H_t N_t)^{1-\theta} \quad — (12)$$

0 ~~Cash~~ Credit good

$[C_t]: C_t^{-\sigma} - \lambda_1^t - \lambda_2^t = 0$ —— ①

$[e_t]: -A(1-n_t-e_t)^{-\gamma} + \lambda_1^t(\Omega h_t \phi e_t^{\phi-1}) = 0$ —— ②

$[n_t]: -A(1-n_t-e_t)^{-\gamma} + \lambda_1^t(\omega_t h_t) = 0$ —— ③

$[\lambda_{t+1}]: \beta V(t+1) - \lambda_1^t = 0$ —— ④

$[\hat{m}_t]: \beta V(t+1) - \lambda_1^t\left(\frac{1}{\hat{p}_t}\right) = 0$ —— ⑤

$[k_{t+1}]: \beta V(t+1) - \lambda_1^t = 0$ —— ⑥

$(h)\ V(t) = \lambda_1^t(\omega_t n_t + (1-\delta_h) + \Omega e_t^{\phi})$

$V(t+1) = \lambda_1^{t+1}(\omega_{t+1} n_{t+1} + (1-\delta_h) + \Omega e_{t+1}^{\phi})$

$[h_{t+1}]: \beta[\lambda_1^{t+1}(\omega_{t+1} n_{t+1} + (1-\delta_h) + \Omega e_{t+1}^{\phi})] = \lambda_1^t$ —— ⑦

$(\hat{m})\ V(t) = \lambda_1^t\left(\frac{1}{g_t \hat{p}_t}\right) + \lambda_2^t\left(\frac{1}{g_t \hat{p}_t}\right) = (\lambda_1^t + \lambda_2^t)\left(\frac{1}{g_t \hat{p}_t}\right)$

$V(t+1) = (\lambda_1^{t+1} + \lambda_2^{t+1})\left(\frac{1}{g_{t+1}\hat{p}_{t+1}}\right)$

$[\hat{m}_t]: \beta\left[(\lambda_1^{t+1} + \lambda_2^{t+1})\left(\frac{1}{g_{t+1}\hat{p}_{t+1}}\right)\right] = \lambda_1^t\left[\frac{1}{\hat{p}_t}\right]$ —— ⑧

$(k)\ V(t) = \lambda_1^t((1-\delta_k) + r_t)$

$V(t+1) = \lambda_1^{t+1}((1-\delta_k) + r_{t+1})$

$[k_{t+1}]: \beta[\lambda_1^{t+1}((1-\delta_k) + r_{t+1})] = \lambda_1^t$ —— ⑨

from eq (1)

$$C_t^{-\sigma} = \lambda_1^t + \lambda_2^t$$

from eq (2)

$$\lambda_1^t = \frac{A(1-h_t-e_t)^{-r}}{\Omega h_t \phi e_t^{\phi-1}} \quad \text{—— (10)}$$

from eq (3)

$$\lambda_1^t = \frac{A(1-h_t-e_t)^{-r}}{w_t h_t} \quad \text{—— (11)}$$

from eq (10) & (11)

$$\frac{A(1-h_t-e_t)^{-r}}{\Omega h_t \phi e_t^{\phi-1}} = \frac{A(1-h_t-e_t)^{-r}}{w_t h_t}$$

$$w_t = \Omega \phi e_t^{\phi-1} \quad *$$

eq (11) into eq (7)

$$\beta\left[\frac{A(1-h_{t+1}-e_{t+1})^{-r}}{w_{t+1} h_{t+1}} \cdot \left(w_{t+1} h_{t+1} + (1-\delta_h) + \Omega e_{t+1}^{\phi}\right)\right] = \frac{A(1-h_t-e_t)^{-r}}{w_t h_t} \quad *$$

eq (11) & (1) into eq (8)

$$\beta C_{t+1}^{-\sigma} \left(\frac{1}{g_{t+1} \hat{p}_{t+1}}\right) = \frac{A(1-h_t-e_t)^{-r}}{w_t h_t \hat{p}_t} \quad *$$

eq (11) into eq (9)

$$\beta\left(\frac{A(1-h_{t+1}-e_{t+1})^{-r}}{w_{t+1} h_{t+1}}\right)\left((1-\delta_k)+r_{t+1}\right) = \frac{A(1-h_t-e_t)^{-r}}{w_t h_t} \quad *$$