$$\frac{C_{t+1}}{C_t} = \frac{\beta\left(1+i_t^d\right)}{1+\pi_{t+1}} \tag{1}$$

$$\frac{\psi}{(1-\psi)}\frac{C_t}{1-H_t} = w_t \tag{2}$$

$$C_t = \frac{b_{t-1}}{1+\pi_t} + m_t + w_t H_t \tag{3}$$

$$d_t^h = \frac{d_{t-1}^h}{1+\pi_t}\left(1+i_{t-1}^d\right) - m_t \tag{4}$$

$$b_t = \pi_t^f + \pi_t^b + \pi_t^c + x_t \tag{5}$$

$$Y_{j.t} = A_t \omega_{j.t}^Y K_{j.t-1}{}^{\theta} H_{j.t}{}^{\xi} S_{j.t-1}{}^{1-\theta-\xi} \tag{6}$$

$$debt_t^f = \left(1+i_{t-1}^l\right)\frac{debt_{t-1}^f}{1+\pi_t} - \Phi_d\left(1+i_{t-1}^l\right)\frac{debt_{t-1}^f}{1+\pi_t}(1-\Phi_t) - \left(1+i_{t-1}^l\right)\frac{debt_{t-1}^f}{1+\pi_t}\Phi_t^K + l_t \tag{7}$$

$$\pi_t^f = Y_t^{ND} - (1-\Phi_t)[S_t(1+\chi) + w_t H_t] - (1-\Phi_t)\left(1+i_{t-1}^l\right)\frac{\Phi_d debt_{t-1}^f}{\mathbf{1}+\pi_t} \tag{8}$$

$$\Phi_t = \frac{\tilde{\boldsymbol{\omega}}_t^{\boldsymbol{Y}} - (1-q^Y)}{2q^Y} \tag{9}$$

$$\tilde{\boldsymbol{\omega}}_t^{\boldsymbol{Y}} = \frac{\left(1+i_{t-1}^l\right)\frac{\Phi_d debt_{t-1}^f}{1+\pi_t} + S_t + w_t H_t}{Y_t} \tag{10}$$

$$\Phi_t^K = \frac{\tilde{\boldsymbol{\omega}}_t^{\boldsymbol{K}} - (1-q^K)}{2q^K} \tag{11}$$

$$\widetilde{\omega}_t^K = \frac{\left(1+i_{t-1}^l\right)\frac{debt_{t-1}^f}{1+\pi_t} - Y_t + S_t + w_t H_t}{p_t^K(1-\delta)K_{t-1}} \tag{12}$$

$$Y_t^{ND} = \boldsymbol{Y_t}[(1+q\Phi_t)(1-\Phi_t)] \tag{13}$$

FOC H

$$\xi Y_t^{ND} - (1-\Phi_t)w_t H_t - \mu_t(1-\rho)\xi Y_t^{ND} + \mu_t(1-\rho)(1-\Phi_t)w_t H_t - \mu_t \xi Y_t + \mu_t w_t H_t + \mu_t \xi Y_t^{ND} - \mu_t w_t H_t(1-\Phi_t) + \mu_t \xi Y_t \Phi_t^K - \mu_t w_t H_t \Phi_t^K = 0 \tag{14}$$

FOC DEBT

$$-\left(\mathbf{1}+\boldsymbol{i_t^l}\right)\Phi_d(1-\Phi_{t+1}) + \mu_{t+1}(1-\chi)\left(\mathbf{1}+\boldsymbol{i_t^l}\right)\Phi_d(1-\Phi_{t+1}) - \mu_{t+1}\left(\mathbf{1}+\boldsymbol{i_t^l}\right) + \mu_{t+1}\boldsymbol{\Phi_d}\left(\mathbf{1}+\boldsymbol{i_t^l}\right)(1-\Phi_{t+1}) + \mu_{t+1}\left(\mathbf{1}+\boldsymbol{i_t^l}\right)\Phi_{t+1}^K = 0 \tag{15}$$

FOC K

$$\frac{\beta}{c_{t+1}}\theta Y_{t+1}^{ND} - \frac{\beta}{c_{t+1}}\mu_{t+1}(1-\rho)\theta Y_{t+1}^{ND} - \mu_{t+1}\frac{\beta}{c_{t+1}}\theta Y_{t+1} + \frac{\beta}{c_{t+1}}\mu_{t+1}\theta Y_{t+1}^{ND} + \mu_{t+1}\frac{\beta}{c_{t+1}}\theta Y_{t+1}\Phi_{t+1}^K - \frac{1}{c_t}\mu_t\frac{\vartheta \boldsymbol{p_t^K} K_t}{\left(\mathbf{1}+\boldsymbol{i_t^l}\right)} + \frac{\beta}{c_{t+1}}\mu_{t+1}\frac{\vartheta \boldsymbol{p_{t+1}^K} K_t}{\left(\mathbf{1}+\boldsymbol{i_{t+1}^l}\right)}(1-\delta)(1-[(q^K\Phi_{t+1}^K + 1 - q^K)\Phi_{t+1}^K]) + \frac{1}{c_t}\mu_t \boldsymbol{p_t^K} K_t = 0 \tag{16}$$

$$\frac{\beta}{c_{t+1}}(1-\theta-\xi)Y_{t+1}^{ND} - \frac{S_t}{c_t}(1-\Phi_t)(1+\chi) - \frac{\beta}{c_{t+1}}\mu_{t+1}(1-\rho)(1-\theta-\xi)Y_{t+1}^{ND} + \frac{S_t}{c_t}\mu_t(1-\rho)(1-\Phi_t)(1+\chi) - \mu_{t+1}\frac{\beta}{c_{t+1}}(1-\theta-\xi)Y_{t+1} + \frac{S_t}{c_t}\mu_t(1+\chi) + \mu_{t+1}\frac{\beta}{c_{t+1}}(1-\theta-\xi)Y_{t+1}^{ND} - \frac{S_t}{c_t}\mu_t(1-\Phi_t)(1+\chi) + \mu_{t+1}\frac{\beta}{c_{t+1}}\Phi_{t+1}^K(1-\theta-\xi)Y_{t+1} - \frac{S_t}{c_t}\Phi_t^K\mu_t(1+\chi) = 0 \tag{17}$$

$$l_t\left(1+i_t^l\right)=\vartheta p_t^K[K_t-(1-\delta)K_{t-1})(1-(q^K\Phi_t^K+1-q^K)\Phi_t^K)] \tag{18}$$

$$p_t^K K_t = n_t^f + debt_t^f \tag{19}$$

$$\begin{aligned} n_t^f &= (1-\Phi_t)(1-\chi)\pi_t^f + Y_t - S_t(1+\chi) - w_t H_t - Y_t^{ND} + (1-\Phi_t)S_t(1+\chi) + (1-\Phi_t)w_t H_t \\ &\quad - \Phi_t^K Y_t + \Phi_t^K S_t(1+\chi) + \Phi_t^K w_t H_t \end{aligned} \tag{20}$$

$$\boldsymbol{\pi_t^c} = p_t^K(K_t-(1-\delta)K_{t-1}) - I_t \tag{21}$$

$$K_t = (1-\delta)K_{t-1} + I_t - \frac{\kappa}{2}(\frac{I_t}{I_{t-1}}-1)^2 I_t \tag{22}$$

$$p_t^K = 1 + p_t^K\frac{I_t}{I_{t-1}}\kappa\left(\frac{I_t}{I_{t-1}}-1\right) - \beta\frac{\lambda_{t+1}}{\lambda_t}p_{t+1}^K\left(\frac{I_{t+1}}{I_t}\right)^2\kappa\left(\frac{I_{t+1}}{I_t}-1\right) \tag{23}$$

$$d_t^h + b_t + bd_t = r_t + z_t + debt_t^f \tag{24}$$

$$r_t = \frac{r_{t-1}}{1+\pi_t} - \frac{bd_{t-1}}{1+\pi_t}\left(1+i_{t-1}^{bd}\right) - i_{t-1}^{bd}\frac{z_{t-1}}{(1+\pi_t)} + bd_t + x_t \tag{25}$$

$$bd_t = (rr_D(b_t+d_t^h) - \left[\frac{r_{t-1}}{(1+\pi_t)} - \frac{bd_{t-1}}{(1+\pi_t)}\left(1+i_{t-1}^{bd}\right) - i_{t-1}^{bd}\frac{z_{t-1}}{(1+\pi_t)} + x_t\right] \tag{26}$$

$$\pi_t^b = \frac{i_{t-1}^l debt_{t-1}^f}{1+\pi_t} - \frac{i_{t-1}^d d_{t-1}}{1+\pi_t} - \frac{i_{t-1}^{bd} bd_{t-1}}{1+\pi_t} - \frac{z_{t-1}\left(1+i_{t-1}^{bd}\right)}{1+\pi_t} \tag{27}$$

$$\begin{aligned} z_t &= \left(1+i_{t-1}^l\right)\frac{debt_{t-1}^f}{1+\pi_t}\Phi_t^K - (1-\delta)q_t^K K_{t-1}(q^K\Phi_t^K+1-q^K)\Phi_t^K \\ &\quad - [\Phi_t^K Y_t - \Phi_t^K S_t(1+\chi) - \Phi_t^K w_t H_t] \end{aligned} \tag{28}$$

$$r_t = \frac{(1+\tau_t)}{(1+\pi_t)}r_{t-1} \tag{29}$$

$$\chi S_t = \pi_t^s \quad (30)$$

Market Clearing:

$$C_t + I_t + S_t = Y_t \quad (31)$$

Shocks:

$$\hat{A}_{\boldsymbol{t}} = \rho_a \hat{A}_{\boldsymbol{t-1}} + \varepsilon_{a.t} \quad (32)$$

Additional equations:

$$\hat{\tau}_{\boldsymbol{t}} = \rho_\tau \hat{\tau}_{\boldsymbol{t-1}} + \varepsilon_{\tau.t} \quad (33)$$

$$\hat{\imath}_t^{bd} = \rho_{bd} \hat{\imath}_{t-1}^{bd} + \varepsilon_{bd.t} \quad (34)$$