

$$(\partial B_{t+1}) \quad \mathbb{E}_t \left[ (1 - \Gamma(\bar{\omega}_{t+1}, \sigma_t)) \frac{R_{t+1}^k}{R_{t+1}} + \eta_{t+1} \left[ \frac{R_{t+1}^k}{R_{t+1}} (\Gamma(\bar{\omega}_{t+1}, \sigma_t)) - \mu G(\bar{\omega}_{t+1}, \sigma_t) - 1 \right] \right] = 0 \quad (1)$$

$$(\partial \bar{\omega}_{t+1}) \quad \mathbb{E}_t \left[ \eta_{t+1} - \frac{\Gamma'(\bar{\omega}_{t+1}, \sigma_t)}{\Gamma'(\bar{\omega}_{t+1}, \sigma_t) - \mu G'(\bar{\omega}_{t+1}, \sigma_t)} \right] = 0 \quad (2)$$

By combining the two FOCs I get

$$\left( 1 - \Gamma(\bar{\omega}_{t+1}, \sigma_t) + \frac{\Gamma'(\bar{\omega}_{t+1}, \sigma_t)}{\Gamma'(\bar{\omega}_{t+1}, \sigma_t) - \mu G'(\bar{\omega}_{t+1}, \sigma_t)} \right) \frac{1 + R_{t+1}^k}{1 + R_{t+1}} - \frac{\Gamma'(\bar{\omega}_{t+1}, \sigma_t)}{\Gamma'(\bar{\omega}_{t+1}, \sigma_t) - \mu G'(\bar{\omega}_{t+1}, \sigma_t)} = 0 \quad (3)$$

To linearize GAMMA function

$$\Gamma_t = \bar{\omega}_{t+1} [1 - F_t] + G_t \quad (4)$$

which results in the following log linear form

$$\Gamma(1 + \hat{\Gamma}_t) = \bar{\omega}(1 - F) \left[ 1 + \widehat{\omega}_{t+1} + \widehat{1 - F_t} \right] + G(1 + \hat{G}_t) \quad (5)$$

Then

$$\widehat{1 - F_t} = \frac{F}{F - 1} \hat{F}_t \quad (6)$$

and then I proceed by applying taylor expansion on F and G which are two-variables functions The log linear FOC reads

$$\mathbb{E}_t \left\{ \frac{1 + R^k}{1 + R} (1 - \Gamma) \left[ 1 + \frac{R^k}{1 + R^k} \hat{R}_{t+1}^k + \frac{\Gamma}{\Gamma - 1} \hat{\Gamma}_t - \frac{R}{1 + R} \hat{R}_{t+1} \right] + \right. \quad (7)$$

$$\left. \frac{\Gamma'}{\Gamma' - \mu G'} \left[ 1 + \hat{\Gamma}_t - \frac{\Gamma' \hat{\Gamma}_t - \mu G' \hat{G}_t}{\Gamma' - \mu G'} \right] \times \right. \quad (8)$$

$$\left. \left( \frac{1 + R^k}{1 + R} (\Gamma - \mu G) \left[ 1 + \frac{R^k}{1 + R^k} \hat{R}_{t+1}^k + \frac{\Gamma \hat{\Gamma}_t - \mu G \hat{G}_t}{\Gamma - \mu G} - \frac{R}{1 + R} \hat{R}_{t+1} \right] - 1 \right) \right\} \quad (9)$$

To log linearize G'

$$\hat{G}_t' = \frac{G''_{\omega\omega}}{G'_\omega} \widehat{\omega}_{t+1} \bar{\omega} + \frac{G''_{\omega\sigma}}{G'_\omega} \hat{\sigma}_t \sigma \quad (10)$$

Log linear  $\Gamma$  is

$$\Gamma(1 + \hat{\Gamma}_t) = \bar{\omega}(1 - F) \left[ 1 + \widehat{\omega}_{t+1} + \widehat{1 - F_t} \right] + G(1 + \hat{G}_t) \quad (11)$$

with

$$\widehat{1 - F_t} = \frac{F}{F - 1} \hat{F}_t \quad (12)$$

How to express  $\hat{F}_t$