Energy Price Shocks and the Macroeconomy: The Role of Consumer Durables *

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Abstract

Hamilton (2005) conjectures that oil shocks can significantly affect the economy by disrupting spending on goods other than energy. Thus, we extend Kim and Loungani's (1992) framework to include a distinction between investment in consumer durables and capital goods, as well as energy use by the households, to evaluate the importance of energy price shocks for output fluctuations. The model economy is calibrated to match total energy use and durable goods consumption as observed in the U.S. data. Simulation results indicate that this economy, despite higher total energy use, has a smaller proportion of output fluctuations attributable to energy price shocks than the one without durable goods. This results from the fact that the representative household in our model has the flexibility to rebalance its portfolio of durable and fixed capital. Specifically, the energy price hike is absorbed by reducing durable goods investment more than the investment in capital goods, thereby cushioning the hit to future production at the expense of current durables consumption. Consequently, productivity shocks continue to be the driving force behind output fluctuations.

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1 Introduction

As Hamilton and Herrera (2004) and Hamilton (2005) point out, nine out of ten of the U.S. recessions since World War II and every recession since 1973 were preceded by a spike in oil prices. However, when one calculates the dollar share of energy expenditure in the economy and uses the elasticity of output with respect to a given change in energy use, it can only explain a small fraction of the drop in GDP during a typical recession (see Hamilton (2005)).[1] This is also evident in Kim and Loungani's (1992) Dynamic Stochastic General Equilibrium (DSGE) model that has a role for energy use exclusively on the production side. Their model simulation showed that energy price shocks can only generate a small fraction of the output fluctuations observed in the U.S. data.[2] A strong conclusion from their research is that output volatility is mainly driven by shocks to total factor productivity (TFP), and - going one step further - all previous recessions would have occurred even without energy price shocks.

Hamilton (2005), however, proposes an alternative transmission mechanism whereby oil shocks can significantly affect the economy by disrupting spending on goods other than energy. Specifically, oil shocks can make consumers postpone their purchase of durable goods apart from reducing demand for investment. Empirical evidence exists to this effect. For example, Edelstein and Kilian (2007) find that the drop in auto expenditures is seven times as much as expenditures on nondurables and services in response to an energy price increase.[3] Hence, we extended Kim and Loungani (1992) by explicitly modeling household consumption of durable goods and energy use. Thus, a DSGE model with higher total energy use (firms plus households), and durable goods demand with a large energy price elasticity, can potentially increase the share of output fluctuations attributable to energy price shocks.

Introducing durable goods and household energy consumption actually *decreases* the relevance of energy price shocks for output volatility, despite higher total energy use. This surprising outcome happens because households now have two margins of adjustment for their investment decision – durable consumption goods and fixed capital – in response to exogenous shocks. This ability to rebalance their portfolio is missing in a typical DSGE model, with or without energy use, when responding to a shock (TFP or oil).

In our calibrated model economy, we show that an energy price increase has a larger negative effect on durables than on fixed capital. Even though both capital stocks decrease in response to higher

[1] According to the Bureau of Economic Analysis and the Energy Information Administration, between 1970 and 2005, residential energy consumption was on average 4.11 percent of GDP with commercial and industrial being 4.67 percent of GDP.

[2] Rotemberg and Woodford (1999) study output impulse response functions and show that under imperfect competition the effect of an oil price shock is stronger than under perfect competition. Finn (2000) shows that one can increase the economy's response to an oil price shock even under perfect competition when one models energy use as a function of capacity utilization. However, both papers are silent on the business cycle properties of the model in response to energy shocks. Specifically, they do not report the share of output fluctuations explained by energy price shocks and the other business cycle facts such as volatility of key economic variables such as investment, consumption and output.

[3] See Hamilton (2003) for a detailed list of other empirical studies that have investigated the various channels through which oil price hikes affect economic activity. For example, Lee and Ni (2002) found that oil price shocks reduced production in oil-intensive industries reduced demand for durable goods such as autos. Mehra and Peterson (2005) found that oil price increases negatively impacted consumer spending by affecting demand for durable goods.

energy prices, the fixed capital drops by less than the stock of durables after households rebalance their portfolio. Furthermore, the drop in fixed capital is less than that in a Kim and Loungani type economy, which explains why energy accounts for less output fluctuation in our model. Consequently, TFP shocks alone account for the majority of output fluctuations.

In a basic DSGE model without energy use and a single consumption good as in Cooley and Prescott (1995), volatility of consumption is far lower than the one observed in the data. An interesting byproduct of our modeling structure is that having durable consumption goods construct and energy price shocks *together* raise model consumption volatility closer to the value observed in the data.[4] This is again due to the rebalancing effect, which results in the household reducing the hit to future production by reducing spending on durable consumption goods in response to energy shocks.

2 Model

The representative household gets utility from consuming three types of consumption goods: consumption of nondurables and services excluding energy (N), the flow of services from the stock of durables goods (D) and energy use (E_h). The household uses the following aggregator function to combine these three types of consumption into C^A:

$$C_t^A = N_t^\gamma \left(\theta D_{t-1}^\rho + (1-\theta) E_{h,t}^\rho\right)^{\frac{1-\gamma}{\rho}} \tag{1}$$

where $\theta \in (0,1)$ and $\rho \leq 1$. With this aggregation function the elasticity of substitution between energy and durable goods is $\frac{1}{1-\rho}$. We will choose $\rho < 0$, which implies that the durable goods and energy are complements. The elasticity of substitution between non-durable consumption and the composite of durables and energy goods is one in our model.[5] This feature is motivated by Ogaki and Reinhart (1998) who found that in the U.S. data the elasticity of substitution between durables and nondurable goods was close to one.[6] Notice that the stock of durables from last period enters today's utility function. That way the timing of durable goods investment is analogous to fixed investment where yesterday's capital stock K_{t-1} enters today's production function.[7]

We write the period t utility function as following:

$$u\left(C_t^A, H_t\right) = \varphi \log C_t^A + (1-\varphi)\log\left(1-H_t\right) \tag{2}$$

[4] The two shock construct has also been used by other researchers such as Braun (1994), McGratten (1994) and Chang (1995) who introduce fiscal shocks (such as changes in tax rates and government spending) to improve the ability of the DSGE model to mimic the data characteristics such as volatility of consumption, hours worked and productivity.

[5] This is similar to the aggregator function used by Fernandez-Villaverde and Krueger (2001) who use a Cobb-Douglas aggregator between non-durable and durable consumption. We have extended it to include the third type of consumption good, which is energy.

[6] Similarly, Rupert et. al. (1995) found that the elasticity of substitution between market goods and home production was not significantly different from one.

[7] Notice that the variable C_t^A does not correspond to consumption observed in the National Income and Product Accounts (NIPA) data. Total real consumption based on NIPA definition is defined as $C_t = I_{d,t} + N_t + E_{h,t}$. This distinction is relevant when we simulate the economy. When we compute second moments and plot impulse responses for consumption we are always referring to this NIPA based C_t of consumption rather than the aggregator-based C_t^A.

where $\varphi \in (0,1)$ and H denotes hours worked. This log-utility specification is the same as in Kim and Loungani (1992).

The timing convention is as follows: Households set the durable goods stock D_{t-1} in period $t-1$ and this stock will produce the flow of durable good services in period t. In other words, the durable goods stock D_{t-1} is a state variable at time t. Durable goods depreciate at rate δ_d per period. Moreover, there are convex adjustment costs for adjusting the stock of durable goods.[8] Thus the durable goods investment $I_{d,t}$ necessary to alter the durable goods stock from D_{t-1} to D_t is:

$$I_{d,t} = D_t - (1-\delta_d) D_{t-1} + \frac{\omega_{1d}}{1+\omega_{2d}} \left(\frac{D_t - D_{t-1}}{D_{t-1}} \right)^{1+\omega_{2d}} \tag{3}$$

where $\omega_{1d} \geq 0, \omega_{2d} > 0$. Notice that in steady state adjustment costs will be zero.

Following Kim and Loungani (1992), firms produce output by combining three inputs: Labor H, capital K and energy E_f according to the following production function:

$$Y_t = Z_{y,t} \left(\eta K_{t-1}^{\psi} + (1-\eta) E_{f,t}^{\psi} \right)^{\frac{\alpha}{\psi}} H_t^{1-\alpha} \tag{4}$$

where the term Z_y is a TFP shock that follows a stochastic process and $\psi \leq 1$.

Just as for durable goods, there is an adjustment cost for altering the capital stock from K_{t-1} to K_t, which implies that capital investment $I_{k,t}$ is

$$I_{k,t} = K_t - (1-\delta_k) K_{t-1} + \frac{\omega_{1k}}{1+\omega_{2k}} \left(\frac{K_t - K_{t-1}}{K_{t-1}} \right)^{1+\omega_{2k}} \tag{5}$$

where $\omega_{1k} \geq 0, \omega_{2k} > 0$.

We assume that all of the energy inputs need to be imported as in Kim and Loungani (1992). The social planner's problem is then:

$$\max E \sum_{t=0}^{\infty} \beta^t u \left(N_t^{\gamma} \left(\theta D_{t-1}^{\rho} + (1-\theta) E_{h,t}^{\rho} \right)^{\frac{1-\gamma}{\rho}}, H_t \right)$$

subject to:

$$N_t + I_{d,t} + I_{k,t} + P_t \left(E_{h,t} + E_{f,t} \right) = Y_t \tag{6}$$

and equations (3), (4) and (5). We derive first order conditions in appendix A.

[8] Note that a representative household facing convex adjustment costs will make small adjustments in the stock of durables, while in the micro-data durable goods adjustments are infrequent and lumpy. We set the adjustment cost parameters to mimic the behavior of the aggregate economy or macro data rather than decision making at the individual level.

3 Calibration

3.1 Preference and technology parameters

One model period corresponds to one quarter in the data. We set $\alpha = 0.36$ and the time preference factor $\beta = 0.99$. These two parameters will remain unchanged for all the model specifications we consider in this paper. The data needed to calibrate the economy comes from the U.S. National Income and Product Accounts (NIPA) from the Department of Commerce, the Energy Information Administration (EIA) and the Flow of Funds Statistics of the Federal Reserve Board.

We retrieve series for energy use on the consumption side which corresponds to E_h in the model, consumption of nondurables and services excluding energy (N), consumption of durables (I_d) and investment in fixed capital (I_k) from NIPA data for years 1970 to 2005 at quarterly frequency.[9] For firm energy use we use data from EIA. The difference between total energy use and E_h is equal to E_f.[10] Output Y corresponds to $N + I_d + I_k + E_h + E_f$ given the definition of output in our model.[11] The durable goods stock (D) comes from the Flow of Funds Statistic of the Federal Reserve Board. We then compute the ratios $E_h/Y, I_d/Y, D/Y, E_f/Y$ based on the nominal data for the interval 1970 to 2005 that will serve as targets in the calibration stage. In addition, we target $K/Y = 12$ and $H = 0.3$ as is standard in the literature. The moments we use in our calibration are summarized in Table 1.[12]

Table 1: Targeted Moments

Moment	Value
E_h/Y	0.0456
I_d/Y	0.0932
D/Y	1.3668
E_f/Y	0.0517
K/Y	12.0000
H	0.3000

3.1.1 Elasticities

Notice that one cannot calibrate both the elasticity of substitution and the share parameter in a CES type production or utility function at the same time by just matching steady state values. Take the

[9] We combine BEA series 'Gasoline, Fuel Oil and Other Energy Goods' (part of nondurable consumption) and 'Electricity and Gas' (part of personal consumption expenditures on services).

[10] From Table 1.5 in the Annual Energy Review 2005, Energy Information Administration, we have annual data on total energy use from 1970 to 2001. We extrapolate total energy consumption for the years 2002 to 2005 by assuming the same growth rates in total energy consumption as in household energy consumption based on NIPA data. Without this extrapolation, that is, using only data until 2001, the share of energy would be very similar, 4.65% on the household and 5.33% on the firm side.

[11] Notice that our definition of output is lower than that observed in the NIPA. Had we used the NIPA definition of output the ratios E_h/Y and E_f/Y would have been 4.11% and 4.67%, respectively, a bit lower that the numbers we will use.

[12] Firm energy use differs slightly from Kim and Loungani's (1992). Our economy has a capital to firm energy ratio of about 232, while Kim and Loungani used a value of 200. Most of the difference can be accounted for by the fact that their estimate is based on data from 1947 to 1987, while we calibrate our economy to data from 1970 to 2005.

example of the CES utility function. The steady state ratio of household energy use and durable goods stock is

$$\frac{E_h}{D} = \left[\frac{1-\beta+\beta\delta_d}{\beta\theta P}(1-\theta)\right]^{\frac{1}{1-\rho}}. \tag{7}$$

Thus, we cannot calibrate both CES parameters ρ and θ at the same time from just the E_h/D ratio. An analogous result holds for the CES parameters ψ and η on the production side.

On the firm side we use the same CES parameter $\psi = -0.7$ as in Kim and Loungani (1992). We pick the parameter $\rho = -2.8748$ to match the volatility of quarterly household energy use in the model.[13] We will perform sensitivity analysis along these two elasticity parameters in Section 6 and confirm that our results are robust to a wide range of possible CES parameters.

3.1.2 Remaining parameters

The first order conditions in steady state pin down the six parameters $\theta, \gamma, \eta, \varphi, \delta_d$ and δ_k. Appendix A.3 details how to use the moments in Table 1 to derive these parameters. The model parameters are summarized in Table 2. Also notice that we can perform this calibration independent of the adjustment

Table 2: Model Parameters

ρ	θ	γ	ψ	η	δ_d	δ_k	φ
-2.8748	$1-2.42*10^{-5}$	0.8032	-0.7000	0.9963	0.0682	0.0156	0.3417

cost parameters, since adjustment costs are zero in the steady state.

We call this basic economy DE. For comparison purposes we will also compare this economy to the one without consumer durables and household energy use, labeled economy E, which approximates a Kim and Loungani type economy. We approximate that economy by having a zero weight on the CES part of the aggregator function (1) function, i.e., we use the same parameters as in Table 2 with the exception of γ set to equal to 1.

3.2 Calibration of the shocks

Just as Cooley and Prescott (1995), we assume that log-TFP follows an AR(1) process:

$$z_{y,t} = \rho_z z_{y,t-1} + \varepsilon_{z,t} \tag{8}$$

where $\rho_z = 0.95$ and $\varepsilon_{z,t} \overset{iid}{\sim} N(0, \sigma_z^2)$ with $\sigma_z = 0.007$. The fact that Cooley and Prescott (1995) used a Cobb-Douglas production function without any firm energy use warrants a check whether the Solow residuals that proxy TFP using our CES type production function also have the same AR(1) property. We constructed a quarterly series for the Solow residual using Cooley and Prescott's (1995)

[13] Specifically, we pin down the three parameters ρ, ω_{1d} and ω_{1k} to match the volatilities of household energy use, durables investment and fixed capital investment to those observed in the data, while keeping the calibration of shocks unchanged in this process.

assumptions: real GDP for Y, nonfarm hours worked from the Establishment Survey for H, and a constant capital stock K. Given that data on firm energy use is only at the annual frequency, we proxy this series at the quarterly frequency by substituting the household energy use as they tend to move similarly at the annual frequency. We found that ρ_z and σ_z of the Solow residuals obtained from our production function were 0.92 and 0.0062, which is not significantly different from the 0.95 and 0.007 values estimated by Cooley and Prescott, and commonly used in the DSGE literature. We will stick with the Cooley and Prescott value for ρ_z and σ_z as the simulation results were very similar.

We obtain the real energy price by deflating the price index of the energy series by the GDP deflator. We then estimate an energy price ARMA(1,1) process as in Kim and Loungani (1992) and Atkeson and Kehoe (1999).[14] We use quarterly log energy prices from 1970Q1-2005Q4 to estimate

$$p_t = \rho_p p_{t-1} + \varepsilon_{p,t} + \rho_\varepsilon \varepsilon_{p,t-1} \text{ with } \varepsilon_{p,t} \overset{iid}{\sim} N\left(0, \sigma_p^2\right) \tag{9}$$

via Maximum Likelihood.[15] We report the estimation results in Table 3.[16]

Table 3: ARMA(1,1) Maximum Likelihood Estimation Results

Parameter	Estimate	Standard Error
ρ_p	0.9753	0.0000
ρ_ε	0.4217	0.0818
σ_p	0.0308	0.0019

3.3 Adjustment costs

In the model without durable goods we abstract from adjustment cost. In the model with durable goods, we assume that the cost functions are quadratic ($\omega_{2d} = \omega_{2k} = 1$), as in Bruno and Portier (1995), and adjust the proportional part of adjustment costs ω_{1d} and ω_{1k} in order to match volatilities of durables and capital goods investments in the model to the data. We will call this model DEA.

[14] Notice that Rotemberg and Woodford (1996) use a different process for energy price shocks. They estimate a VAR with two variables, nominal oil price changes and real oil prices and study the effect of exogenous shocks to nominal price changes. As a robustness check we re-estimated their VAR and incorporated it in our model. We found that using their shock specification did not change our results.

[15] We use the Kalman Filter to write down the likelihood function as described in Hamilton (1994).

[16] We also checked for the independence of the two shocks $\varepsilon_{z,t}$ and $\varepsilon_{p,t}$ and found the correlation coefficient between the two to be only -0.0248. In order to check if there were any lagged responses of energy price changes on the TFP process we added contemporaneous and lagged ε_p to the TFP equation (8). The estimated equation is (t-stats are in parentheses):

$$z_{y,t} = \underset{(25.6081)}{0.9409}\, z_{y,t-1} - \underset{(-0.1994)}{0.0001}\, \varepsilon_{p,t} - \underset{(-1.6426)}{0.0009}\, \varepsilon_{p,t-1} - \underset{(-0.5677)}{0.0003}\, \varepsilon_{p,t-2} - \underset{(-0.7435)}{0.0004}\, \varepsilon_{p,t-3} + \tilde{\varepsilon}_{z,t}$$

There does not seem to be a negative effect of quarterly energy price innovation on TFP. While the coefficient for $\varepsilon_{p,t-1}$ is barely significant but negative, the size of the parameter estimate is economically insignificant. We therefore conclude that simulating the economy with independent innovations to the TFP and energy price is an appropriate assumption.

4 Solution Algorithm

We use the stochastic perturbation method, i.e., log-linearization around the deterministic steady state, to approximate the dynamics of our economy. From the first order conditions in Appendix A, we derive eleven conditions guiding the dynamic behavior of eleven variables N, D, E_h, H, W, E_f, K, R, Y, I_d, I_k plus two equations for the shocks. We then run the program Dynare Version 3.0 to generate a first order approximation for the policy function (see Collard and Juillard (2001) for the methodological details). To generate second order moments for each of the specifications considered, we simulate 1000 economies each 144 quarters long, which is the same length as the data series from 1970:Q1 to 2005:Q4.

5 Numerical Results

Table 4 details the percent standard deviations of HP-filtered series for both the data and the model simulations.[17] The first set of numbers are for simulations when both the TFP and energy shocks are present. The next panel is for only the energy shocks and the last panel for only the TFP shock. Since we do not have quarterly data for firm energy use we will report the volatility of annual total energy use $(E_h + E_f)$ at the annual frequency.[18]

Looking at the column for model E (simple DSGE model without durable goods), in the version with both shocks we generate output volatility close to that in the data, though consumption volatility is far below the data value and the investment volatility is slightly above its empirical target. Model simulations with only energy price shocks can account for only about 18 percent of output fluctuations. Most of output fluctuations are generated by TFP shocks alone. We thus replicate the main result from Kim and Loungani (1992), that energy price shocks do not play a major role in accounting for output fluctuations. Total factor productivity is still the driving force. Moreover, consumption volatility is well below its empirical target. The model accounts for only 31 percent of the target standard deviation of consumption. As previous research has pointed out, in this simple RBC type model, households are doing too good a job in smoothing consumption.[19] We also find that the volatility of total energy consumption (at the annual frequency) in the model is well above that observed in the data. Notice that since there is no household energy use in the model, the total energy use in the model is equal to firm energy use.

In the economy with durable goods we first report the results without adjustment costs (DE) in Table 4. With both shocks present, the model DE generates volatility for output slightly below the one observed in the data. Consumption volatility is higher than in the data, which is mainly driven by very high volatility of durables, while nondurable consumption is less volatile in the model than in the data. Moreover, the model generates excess volatility not only in durable goods but also in fixed

[17] For quarterly data we use an HP parameter $\lambda = 1600$.

[18] The empirical volatility is based on data from the Department of Energy (total energy use in BTU from table 2.1a). Both for the empirical and simulated data we detrend series with an HP-filter with a $\lambda = 100$.

[19] See for example Cooley and Prescott (1995).

Table 4: Percent standard deviation in the data versus model

Variable	Data	Model with both shocks			
		E	DE	DE^A	PC
Output	1.57	1.46	1.47	1.41	1.36
Consumption	1.26	0.39	1.46	0.80	1.70
NDS ex energy	0.82		0.38	0.43	0.38
HH energy use	2.10		2.36	2.10	1.81
Durables	4.55		11.36	4.55	13.78
Fixed Investment	5.37	6.31	7.95	5.37	8.57
Durables + Fixed Inv	4.80		4.60	4.26	4.59
Hours	1.51	0.79	0.81	0.72	0.75
Total Energy (annual)	2.45	4.64	3.90	3.82	1.79

Variable	Data	Model with energy shocks only			
		E	DE	DE^A	PC
Output	1.57	0.29	0.26	0.28	0.07
Consumption	1.26	0.07	1.39	0.63	1.65
NDS ex energy	0.82		0.07	0.08	0.13
HH energy use	2.10		2.35	2.08	1.79
Durables	4.55		11.32	4.39	13.75
Fixed Investment	5.37	1.87	5.17	1.31	6.18
Durables + Fixed Inv	4.80		1.72	1.77	1.91
Hours	1.51	0.18	0.18	0.18	0.10
Total Energy (annual)	2.45	4.47	3.80	3.72	1.67

Variable	Data	Model with TFP shocks only			
		E	DE	DE^A	PC
Output	1.57	1.43	1.44	1.38	1.36
Consumption	1.26	0.38	0.42	0.49	0.41
NDS ex energy	0.82		0.37	0.42	0.36
HH energy use	2.10		0.23	0.26	0.26
Durables	4.55		0.99	1.18	0.92
Fixed Investment	5.37	6.03	6.03	5.20	5.91
Durables + Fixed Inv	4.80		4.26	3.86	4.18
Hours	1.51	0.77	0.79	0.69	0.74
Total Energy (annual)	2.45	1.17	0.80	0.79	0.64

Data based on quarterly NIPA data from the BEA from 1970:Q1 to 2005:Q4. Notice that there are no quarterly data on firm energy use. Simulation results are averages over 1000 simulations each with length 144 quarters. Column "PC" is for the Atkeson and Kehoe Putty-Clay model, as detailed in Section 7.

investment. Notice that this happens despite the fact that the volatility of the sum of the two is below its target value of 4.80.

To explain this artifact, let's examine the impulse response function of investment variables to an energy price shock displayed in Figure 1. The top left panel displays a one time, one standard deviation positive shock to $\varepsilon_{p,t}$, i.e., an increase in energy prices. Notice that P_t increases for two periods which is due to the ARMA(1,1) structure of the energy price process. The sum of investment in durables and fixed capital $(I_d + I_k)$ in the top left panel reacts as expected, i.e., it falls for two periods mirroring the rise in energy prices followed by a reversion back to the steady state after period 2, which is the expected response of investment to an energy price shock.

Apart from the effect that energy prices have on total investment, in the first period after the shock there must be an additional effect because investment in durables (I_d) drops dramatically whereas investment in fixed capital (I_k) rises for one period before it falls below its steady state level. A look at the first order conditions explains why this happens. In the absence of adjustment costs, we can write the two Euler conditions (A-25) and (A-26) as:

$$\beta E \left\{ \frac{N_t}{N_{t+1}} \left[\left(R^D_{t+1} - \delta_d \right) - \left(R_{t+1} - \delta_k \right) \right] \right\} = 0 \tag{10}$$

where

$$R^D_{t+1} = \frac{(1-\gamma)\,\theta}{\gamma} N_{t+1} \left(\theta D^\rho_t + (1-\theta)\, E^\rho_{h,t+1} \right)^{-1} D^{\rho-1}_t \tag{11}$$

$$R_{t+1} = \alpha\eta Y_{t+1} \left(\eta K^\psi_t + (1-\eta)\, E^\psi_{f,t+1} \right)^{-1} K^{\psi-1}_t \tag{12}$$

Thus, the expected return on durable goods $(R^D_{t+1} - \delta_d)$ and fixed capital $(R_{t+1} - \delta_k)$, discounted by β and the pricing kernel $\frac{N_t}{N_{t+1}}$ has to be equal. Notice that in our calibration, the steady state share of energy to relevant asset is much smaller for the firm than for the household.[20] Thus, the percentage drop in R_{t+1} due to higher energy prices and lower firm energy use is smaller than the drop in R^D_{t+1}. In order to equalize the returns, the household rebalances its asset portfolio. It increases the fixed capital stock K and decreases the durables stock D. This leads to the large drop in durables investment and a one period increase in fixed capital investment that's large enough to offset the negative effect from higher energy prices on investment. In subsequent periods, both investment series are below their steady state values, i.e., the line for I_k also falls below zero. Since K is already high enough and D is low enough to align the returns of durables and fixed capital, the rebalancing in subsequent periods is small enough not to reverse the sign of the investment deviations from steady state, i.e., we observe the direct negative effect of an energy price hike on both investment series.

In the case of a productivity shock both investment series go up (see Figure 2). The response in durables investment is muted in the first period, which is due to the fact that productivity has a direct effect only on the production function and not the utility function. Thus, in order to satisfy the equation (10) the jump in fixed capital investment is larger than in durables investment.

[20]Specifically, $E_f/K = 0.0043$ and $E_h/D = 0.0334$.

Even in this basic durable goods model DE with excess volatility in investment, the proportion of output volatility explained with pure energy shocks is only 17 percent.[21] Despite the explicit modeling of durable goods, energy prices are not accounting for a sizeable share of output fluctuations. This is a surprising result, because the total energy use in the DE economy is about twice as high as in the economy without durable goods, yet the relevance of energy price shocks for output volatility has diminished.

As pointed out above, Model DE is off in some important dimensions. It has excess volatility in fixed investment and durables, which in turns causes excess volatility in consumption. Consequently, we make the parameters in the adjustment cost functions ω_{1d} and ω_{1k} non-zero to reduce volatility in investment. Our aim is to *exactly* match the two investment volatilities in the data. Specifically, we pick $\omega_{1d} = 0.8165$ and $\omega_{1k} = 15.9726$, while leaving all other parameters unchanged.[22] We call this new parametrization Model DE^{A} and the last column in Table 4 reports the simulation results. Most importantly, the model with adjustment costs still only generates a small fraction of about 18 percent of output fluctuations with energy price shocks alone. This is consistent with the output impulse response function in Figure 3, where an energy price shock by one standard deviation causes a drop in output by a mere 0.2 percent, while a one standard deviation shock to TFP increases output to more than 1 percent above the steady state.

Also notice that by construction the volatilities of I_d,I_k and E_h match their empirical target. The lower investment volatilities are consistent with the investment impulse response functions in Figure 3: the adjustment costs indeed muted the investment response to the energy shock. A one standard deviation shock to productivity has a smaller effect on durables investment than a one standard deviation shock to energy prices. The initial drop in I_d in response to a shock to energy price P is about four times larger than the increase in I_d in response to a shock to productivity. For fixed investment I_k it is the reverse: a shock to productivity generates an increase in fixed capital investment about four times larger than the drop in response to an energy price hike. The same mechanism that drove the investment variables impulse response functions in Model DE works here, too, though it is muted by the adjustment costs. Energy shocks still have a larger effect on durables investment, since household energy consumption has a larger share in the utility function than firm energy use has in the production function. Likewise, a productivity shock has a direct effect only on the production function, which creates a large response in the fixed capital investment series. The return to durables is only indirectly affected, thus the response in durables investment after a productivity shock is smaller than that of fixed capital investment.

In Table 4, consumption volatility is closer to the data in the new Model DE^{A} than in the model

[21] It is well known that energy's share in producing output has been falling over time, especially for firms. For example, E_f/Y was 5.9 percent in the 70's but is only 3.9 percent in the last five years. Using the higher value of E_f/Y still kept the contribution of energy price shocks to output volatility below 20 percent. Additionally, as Dhawan and Jeske (2007) show, changing the household energy share had an insignificant impact on the proportion of output volatility attributable to energy price shocks.

[22] We found that even though the adjustment costs reduce volatilities significantly, the costs are small relative to output. In the 1000 economies we simulated, adjustment costs were on average 0.0006 and 0.0021 percent of GDP for durable and fixed capital, respectively. Even the 99th percentile is at only 0.0041 percent of output for durables and 0.0143 percent for fixed capital.

E without durable goods. We achieve this by breaking the link between the consumption aggregator C^A, which is the series that consumers want to smooth, and measured consumption C^{NIPA}. Since the service flow of the stock of durables D_{t-1} enters the consumption aggregator, households can smooth C^A despite large fluctuations in measured consumption C^{NIPA} coming from fluctuations in durables investment I_d. Notice that model DE^{A} with only a TFP shock generates a mere 39 percent of the desired consumption fluctuation. The intuition for this result is that TFP does not directly affect the returns on durables in equation (11), which also explains the fact that durables goods investment volatility in economy DE^{A} without energy price shocks is only at 1.18.[23] Only the inclusion of energy price shocks drives the volatility of consumption to 0.80, which is 64 percent of its empirical target. This is again due to the rebalancing effect from energy price shocks that a have large effect on the return of durable goods.

6 Sensitivity analysis

We perform sensitivity analysis for the two parameters ρ and ψ that we were not able to pin down solely on the basis of simple steady state moments. Apart from the benchmark value of $\psi = -0.7000$, we pick $\psi = 0$ that Kim and Loungani (1992) used as their second alternative CES parameter. This value for ψ corresponds to unit elasticity of substitution between firm energy use and physical capital. We also pick a much smaller elasticity of 0.25 which corresponds to $\psi = -3.0000$, which was the lowest value that Kim and Loungani (1992) cited in their review of the empirical literature.

Along the dimension of the household CES parameter ρ we chose a higher value -0.7000, the same parameter as for the firm in the benchmark calibration. We also produce an estimate for ρ with the following procedure: We work with equation (A-19) in Appendix A, which is the the first order condition for household energy use. Log-linearizing and rearranging terms yields

$$e_{h,t} - d_{t-1} = \frac{\theta + (1-\theta)\left(\frac{E_h}{D}\right)^{\rho}}{\rho\theta}\left(p_t + e_{h,t} - n_t\right) \tag{13}$$

where lower-case letters refer to log-deviations from the steady state. We deflate the durable goods series from the Flow of Funds Statistics by the GDP deflator.

Table 5: Cross-Correlations

Cross-Correlation of $p_t + e_{h,t} - n_t$ with $e_{h,t+\tau} - d_{t-1+\tau}$						
$\tau = -3$	$\tau = -2$	$\tau = -1$	$\tau = 0$	$\tau = 1$	$\tau = 2$	$\tau = 3$
-0.2287	-0.3015	-0.3128	-0.2532	-0.3806	-0.3000	-0.1993

We find that in the data $e_{h,t} - d_{t-1}$ and $p_t + e_{h,t} - n_t$ are not coincident, i.e., the maximum correlation of the two (in absolute value) does not occur contemporaneously (see Table 5). Instead, there seems to be a one-quarter delay for households to reduce their $\frac{E_h}{D}$ in response to an increase in $\frac{PE_h}{N}$. Hence, we

[23] There is an indirect effect on the return R^D_{t+1} through fluctuations in the term N_{t+1}. However, due to consumption smoothing these fluctuations are lower than those in Z_{t+1}.

decided to run a regression of $\frac{E_h}{D}$ on one-quarter lagged $\frac{PE_h}{N}$ in order to account for the lag observed in the data. Specifically, we estimate

$$(1-\xi L)(e_{h,t}-d_{t-1}) = \mu\left(L-\xi L^2\right)(p_t+e_{h,t}-n_t)+\varepsilon_t \tag{14}$$

where $\varepsilon_t \overset{iid}{\sim} N(0,\sigma_\varepsilon^2)$. We introduced lag operator L and autocorelation parameter ξ after finding that the OLS error terms were correlated if one estimates the equation without the lag terms. Maximum Likelihood estimates are reported in Table 6.

Table 6: Maximum Likelihood Estimation Results

Parameter	Estimate	Standard Error
μ	-0.1600	0.0381
ξ	0.3880	0.0770
σ	0.0163	0.0010

With the estimate for μ we can back out the CES parameters ρ and θ as following. From equation (A-61) in the appendix, we solve for θ as a function of ρ. Thus, the parameters μ and ρ are related via

$$\mu = \frac{\theta(\rho)+(1-\theta(\rho))\left(\frac{E_h}{D}\right)^\rho}{\rho\theta(\rho)} \tag{15}$$

This is an equation in one unknown ρ as we know the value of β as well as δ_d and $\frac{E_h}{D}$. Given our estimate for μ, we find $\rho = -8.9260$, which implies a very small elasticity of substitution between energy and durables of about 0.1. Because of this low elasticity and the relatively large standard error around μ we also simulate the economy with $\rho = -6.0746$, which corresponds to the lower end of the 95 percent confidence interval of μ.

For each of the 12 possible combinations of CES parameters $\psi \in \{0.0000, -0.7000, -3.0000\}$ and $\rho \in \{-0.7000, -3.0000, -6.0746, -8.9260\}$ we report in Table 7 the proportion of output volatility accounted by energy price shocks only. We find that our results are quite robust to changing the CES parameters ρ and ψ. The share of output fluctuations accounted by energy price shocks is still rather low. Changing ρ does not have a sizable effect on this statistic and even for $\psi = 0$ this number is below 26 percent.

Table 7: Share of output volatility accounted by energy price shocks for different CES parameters

	$\rho=-0.7000$	$\rho=-2.8748$	$\rho=-6.0746$	$\rho=-8.9260$
$\psi=0.0000$	.2563	.2575	.2556	.2546
$\psi=-0.7000$	.1743	.1758	.1745	.1737
$\psi=-3.0000$	.1025	.1048	.1038	.1031

In Table 8 we report the volatility results of the twelve different simulated economies when both shocks are present. The investment volatilities hit their target by construction.[24] Consumption volatil-

[24]In the case of $\rho = -0.7$, when solving for the ω in order to match investment volatilities to the ones observed in the

Table 8: Percent standard deviation in the data versus model for different CES parameters

Variable	Data	$\psi = 0.0$: $\rho = -0.7000$	$\psi = 0.0$: $\rho = -2.8748$	$\psi = 0.0$: $\rho = -6.0746$	$\psi = 0.0$: $\rho = -8.9260$
Output	1.57	1.48	1.48	1.48	1.47
Consumption	1.26	0.92	0.88	0.85	0.84
NDS ex energy	0.82	0.45	0.46	0.46	0.46
HH energy use	2.10	3.74	2.13	1.51	1.30
Durables	4.55	4.50	4.55	4.55	4.55
Fixed Investment	5.37	5.37	5.37	5.37	5.37
Durables + Fixed Inv	4.80	4.12	4.22	4.24	4.25
Hours	1.51	0.75	0.74	0.74	0.73
Total Energy (annual)	2.45	6.27	5.38	5.00	4.87

Variable	Data	$\psi = -0.7$: $\rho = -0.7000$	$\psi = -0.7$: $\rho = -2.8748$	$\psi = -0.7$: $\rho = -6.0746$	$\psi = -0.7$: $\rho = -8.9260$
Output	1.57	1.41	1.41	1.40	1.40
Consumption	1.26	0.78	0.80	0.78	0.77
NDS ex energy	0.82	0.43	0.43	0.43	0.44
HH energy use	2.10	3.65	2.10	1.49	1.28
Durables	4.55	3.70	4.55	4.55	4.55
Fixed Investment	5.37	5.37	5.37	5.37	5.37
Durables + Fixed Inv	4.80	4.15	4.26	4.30	4.31
Hours	1.51	0.72	0.72	0.71	0.71
Total Energy (annual)	2.45	4.69	3.82	3.47	3.34

Variable	Data	$\psi = -3.0$: $\rho = -0.7000$	$\psi = -3.0$: $\rho = -2.8748$	$\psi = -3.0$: $\rho = -6.0746$	$\psi = -3.0$: $\rho = -8.9260$
Output	1.57	1.35	1.35	1.35	1.35
Consumption	1.26	0.67	0.74	0.72	0.71
NDS ex energy	0.82	0.42	0.43	0.43	0.43
HH energy use	2.10	3.58	2.07	1.47	1.27
Durables	4.55	3.00	4.55	4.55	4.56
Fixed Investment	5.37	5.37	5.37	5.37	5.37
Durables + Fixed Inv	4.80	4.16	4.30	4.34	4.35
Hours	1.51	0.69	0.69	0.69	0.68
Total Energy (annual)	2.45	3.32	2.48	2.15	2.03

Adjustment costs are such that we exactly match volatilities in I_D and I_K. Model DE^A corresponds to the case $\psi = -0.7$ and $\rho = -2.8748$.

ity actually improves somewhat if we make the parameter ρ less negative. For example, keeping the benchmark ψ at -0.7 and lowering ρ to -3.0 increases consumption volatility from 61 to 63 percent of the data value. The slight improvement is due to the sharp rise in the volatility of household energy use (E_h) to 2.07 percent, while keeping the other components at roughly the same level. The lesson from this exercise is that it will be hard to match the volatility of overall consumption ($C = N + E_h + I_d$) because consumers are doing too good a job at smoothing out the volatility of nondurables (N). Even those calibrations that match both the I_d and E_h volatility (both the benchmark and the $\rho = -0.7$ and $\psi = 0$) have a nondurables volatility stubbornly low at roughly one half of its empirical counterpart. An exercise for future research might be to decrease the elasticity of substitution between N and the energy-durables aggregator. That way, some of the volatility in energy prices and thus energy use E_h will spill over into N.

In Table 8 we also report the volatility of total energy use at annual frequency. The volatility in the model is higher than in the data for both $\psi = 0$ and $\psi = -0.7$. It appears that if energy is too substitutable on the firm level, then firm energy use is too volatile, which in turn creates excess volatility of total energy use. For $\psi = -3.0$, and the benchmark CES parameter value of $\rho = -2.8748$, we are able to closely match both the E_h volatility at quarterly frequency and the $E_h + E_f$ volatility at annual frequency.[25]

7 Extension: Putty-Clay Model

We study the business cycle properties of our model with a different setup of the production and utility function. Specifically, we rely on the seminal work of Atkeson and Kehoe (1999) to allow for a different short run and long run elasticity of substitution between energy and capital, both for the fixed capital stock and the durable stock. Specifically, their model assumes that a given airplane (washing-machine) requires a fixed energy input per unit of usage. The only way to alter the energy use is to either buy a new airplane (washing machine) or to walk instead of flying (wash by hand).

On the firm side we use the exact same setup as in Atkeson and Kehoe. There is a continuum of capital goods indexed by their capital to energy ratio V_k. This particular type of capital generates

$$f_k\left(V_k\right) \min\left\{\frac{K}{V_k}, E_f\right\} \tag{16}$$

units of capital services, where the function f_k satisfies $f_k\left(V_k\right), f_k'\left(V_k\right) \geq 0 > f''\left(V_k\right)$ for all types V_k.

Total capital services $X_{k,t}$ are aggregated via

$$X_k = \int_{V_k} f_k\left(V_k\right) \min\left\{\frac{K}{V_k}, E_f\right\} dV_k \tag{17}$$

data, we hit the lower bound $\omega_{1d} = 0$. Consequently, we are able to only match the I_k volatility, while the I_d is below its target even for zero adjustment costs. This is also the reason we don't consider the case $\rho = 0$: the I_d volatility would be even lower.

[25] In appendix C we report the volatilities if we *exactly* match both the investment volatilities (I_d and I_k) as well as those of quarterly household energy use (E_h) and the annual total energy use volatility ($E_h + E_f$).

and then combined with hours H into output

$$Y_t = Z_{y,t} X_{k,t}^{\alpha} H_t^{1-\alpha} \tag{18}$$

We set up the consumption aggregator in a similar fashion. Durable goods are indexed by their durables to energy ratio V_d. A stock of D durable goods of this type together with E_h units of energy generates

$$f_d\left(V_d\right) \min\left\{\frac{D}{V_d}, E_h\right\} \tag{19}$$

units of durables services. The total service flow of the durable goods $X_{d,t}$ is obtained by integrating over the different types

$$X_d = \int_{V_d} f_d\left(V_d\right) \min\left\{\frac{D}{V_d}, E_h\right\} dV_d \tag{20}$$

and the consumption aggregator C^A is

$$C_t^A = N_t^{\gamma} X_{d,t}^{1-\gamma} \tag{21}$$

The social planner then solves the following maximization problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\varphi\gamma \log N_t + \varphi\left(1-\gamma\right) \log X_{d,t-1} + \left(1-\varphi\right) \log\left(1-H_t\right)\right)$$

subject to:

$$\begin{aligned} Z_{y,t} X_{k,t-1}^{\alpha} H_t^{1-\alpha} &= N_t + \int_{V_d} I_{d,t}\left(V_d\right) dV_d + \int_{V_k} I_{k,t} dV_k \\ &\quad + P_t \left(\int_{V_d} E_{h,t-1} dV_d + \int_{V_k} E_{f,t-1} dV_d\right) \qquad (22) \\ I_{d,t}\left(V_d\right) &= D_t\left(V_d\right) - \left(1-\delta_d\right) D_{t-1}\left(V_d\right) \ \ \forall V_d \qquad (23) \\ I_{k,t}\left(V_k\right) &= K_t\left(V_k\right) - \left(1-\delta_k\right) K_{t-1}\left(V_k\right) \ \ \forall V_k \qquad (24) \end{aligned}$$

and equations (17) and (20).

In Appendix B we present more details, such as the first order conditions. We calibrate the model to match the same steady state moments as before. Simulation results are in Table 4 in the column "PC". The results are qualitatively similar to those in the economies DE and DE^A. Most importantly, energy price shocks alone account for a small share of output volatility. In fact, the share or output fluctuations explained by energy price shocks is even smaller than that in our benchmark economy.

The Atkeson and Kehoe model also displays a large volatility in the investment series, which implies that the rebalancing effect is at work here, too. Another difference is that the two investment variables are now very volatile, essentially the same magnitude as in our DE economy. The reason is the absence of adjustment costs in the our Putty-Clay setup. Without adjustment costs there is a strong rebalancing effect away from durables investment to fixed capital investment after an energy price hike, just like in our benchmark economy.

A key difference between the Putty-Clay model and our benchmark economy is that the total energy use has a very low volatility. Our investigation shows that this low volatility is primarily due to the low volatility of firm energy use. This sluggishness of firm energy use comes from the fact that energy is used in a fixed proportion to the existing capital stock. Thus, to change the firm energy use, the social planner has to change the fixed capital stock first, which takes a long time, due to the large capital to output ratio and the low depreciation rate of fixed capital.

For future research we might consider incorporating adjustment costs in the Putty-Clay model, but we doubt that will alter the main result of the paper, namely the low importance of energy price shocks in accounting for output volatility.

8 Conclusion

The main conclusion from our work is that energy price shocks are not a major factor for business cycle fluctuations even when incorporating three distinct categories of consumption: durables, non-durables and energy. With explicit modeling of durable goods we give the household an additional margin of adjustment in its aggregate investment decision. Consequently, in response to an exogenous shock, the household not only decides how much to invest in total, but also rebalances its portfolio mixture of durable goods and fixed capital. Energy shocks indeed cause a disruption in durable goods investment, but at the same time the disruption in fixed capital investment is smaller than in a Kim and Loungani (1992) type economy, which has only one margin for adjustment, namely fixed capital. Therefore, the household in our model can cushion the drop in output by adjusting on the durable goods margin instead of just fixed capital. This rebalancing ability keeps productivity (TFP) shocks as the driving force behind output fluctuations. The rebalancing effect is also responsible for generating a consumption volatility value that is closer to the one observed in the data. Our results extend also to an economy where the capital energy services are aggregated via a Putty-Clay technology as in Atkeson and Kehoe (1999).

For future research it will interesting to see how this rebalancing effect works in the presence of money and explicit monetary policy rules. The objective will be to find the optimal monetary policy following an energy shock given the state of the real economy. Another avenue of future research would be to explicitly introduce durable goods production in a separate sector as in Baxter (1996). Energy shocks may have larger effects if there are frictions in the movement of labor between sectors, as in Hamilton (1988). Additionally, the robustness of our rebalancing mechanism needs to be evaluated for non-standard specifications of the stochastic processes for TFP and the energy price.

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Figure 1: Investment variables: Impulse Response Functions to an energy price shock in Model DE. In percent.

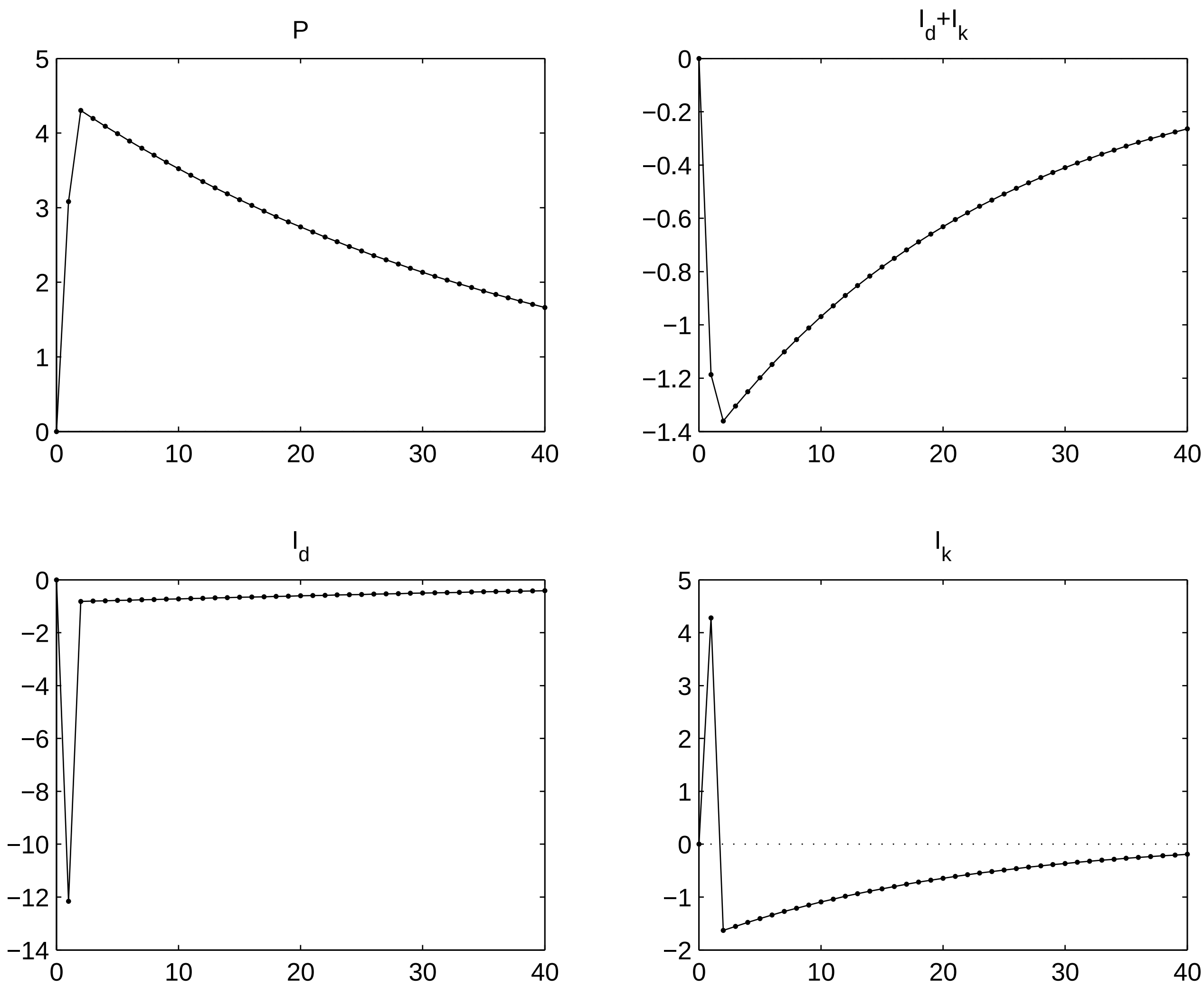


Figure 2: Investment variables: Impulse Response Functions to a TFP shock in Model DE. In percent.

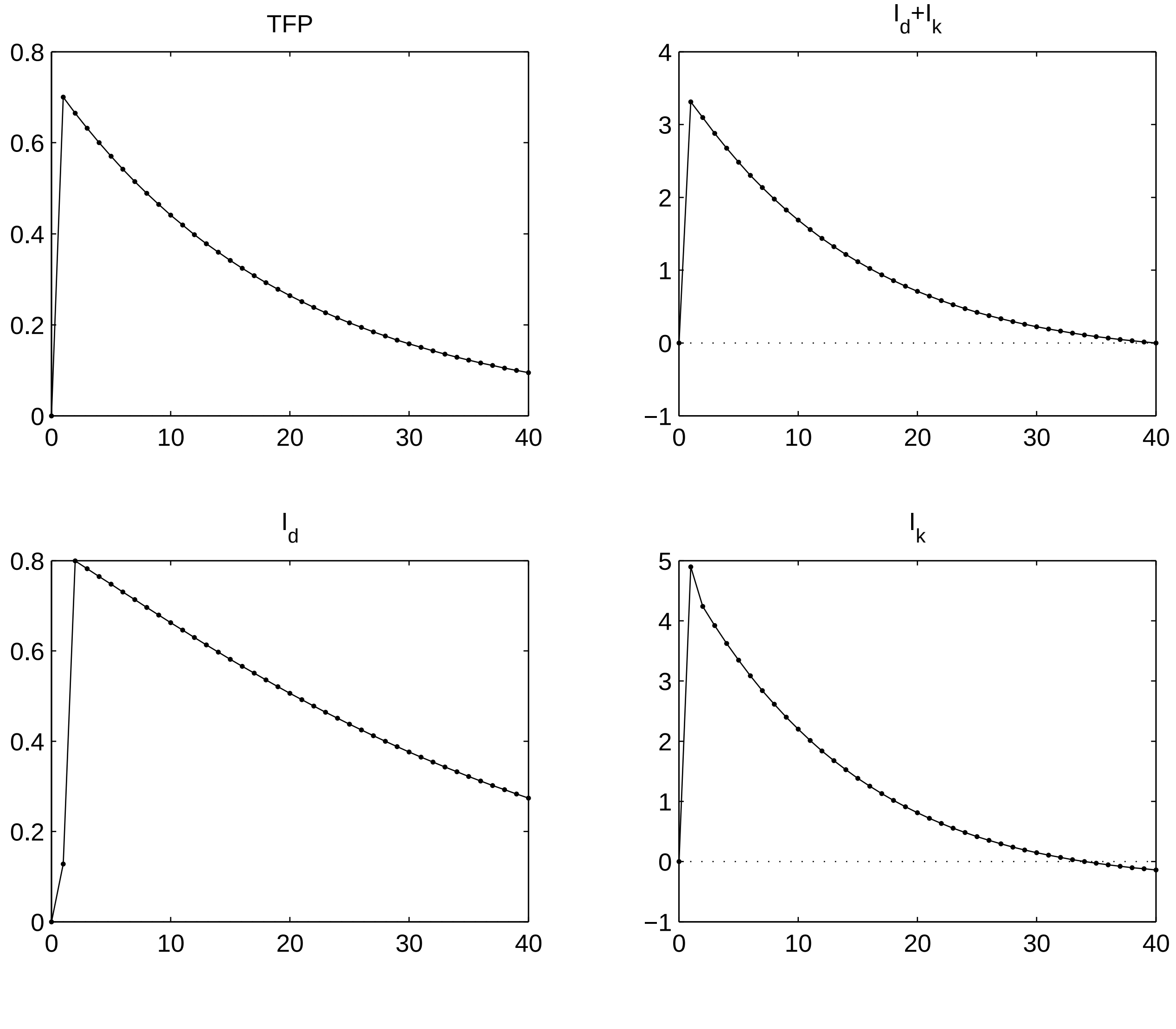
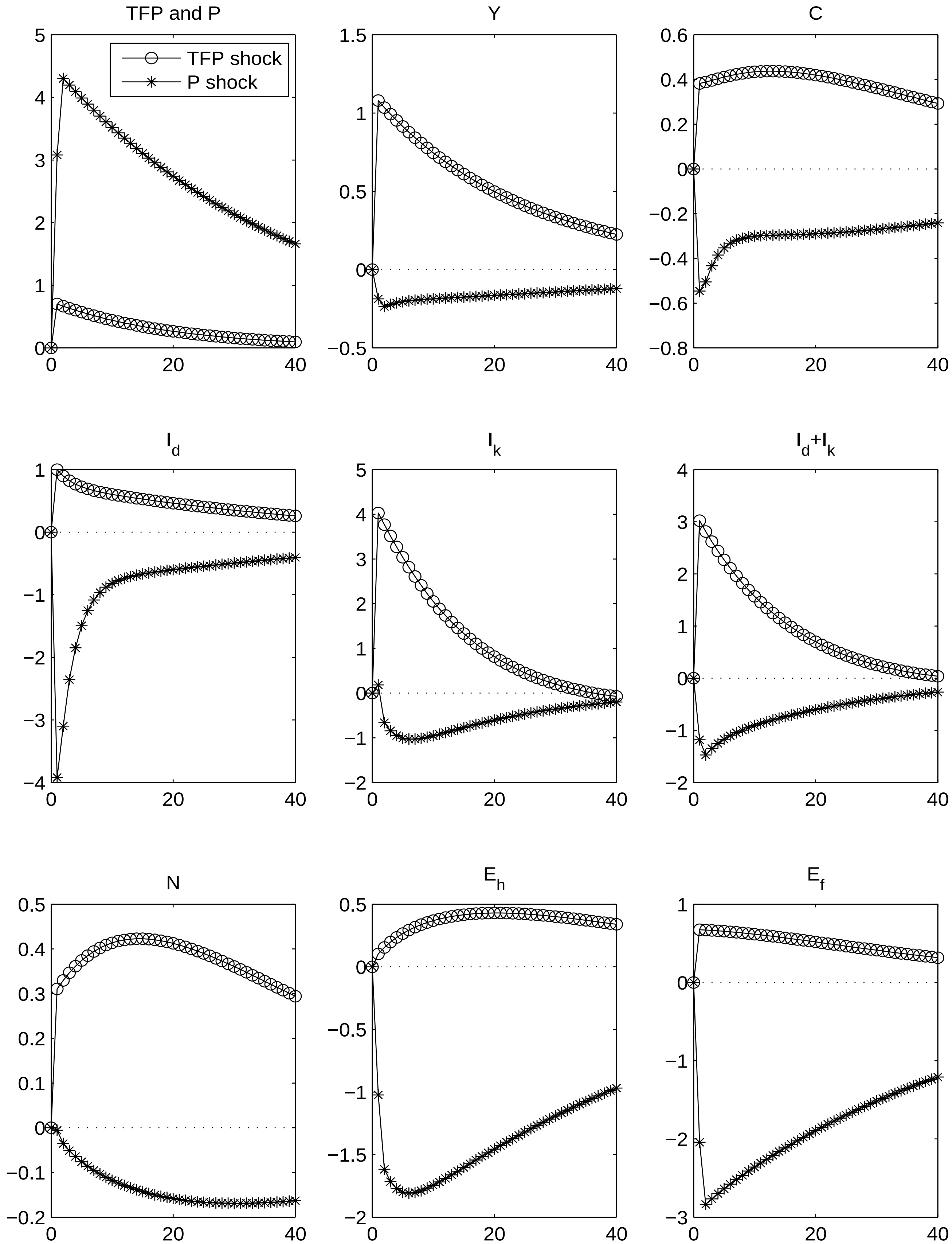


Figure 3: Impulse Response Functions in Model DE$^{\text{A}}$. In percent.



Appendix

A Benchmark model technical details

A.1 First order conditions

Let Λ_t be the Lagrange multiplier on the time t resource constraint. The first order conditions with respect to the decision variables are:

- Nondurables:

$$\beta^t u_1\left(C_t^A, H_t\right)\gamma N_t^{\gamma-1}\left(\theta D_{t-1}^{\rho}+(1-\theta)E_{h,t}^{\rho}\right)^{\frac{1-\gamma}{\rho}}=\beta^t\Lambda_t \tag{A-1}$$

where

$$u_1\left(C_t^A, H_t\right)=\frac{\varphi}{C_t^A} \tag{A-2}$$

Thus:

$$\Lambda_t=\varphi\gamma N_t^{-1} \tag{A-3}$$

- Durables:

$$\begin{aligned}\beta^t\Lambda_t\left[1+\frac{\omega_{1d}}{D_{t-1}}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right)^{\omega_{2d}}\right] &= E\beta^{t+1}u_1\left(C_{t+1}^A, H_{t+1}\right)N_{t+1}^{\gamma}\\ &\times\left(\theta D_t^{\rho}+(1-\theta)E_{h,t+1}^{\rho}\right)^{\frac{1-\gamma}{\rho}-1}(1-\gamma)\theta D_t^{\rho-1}\\ &+E\beta^{t+1}\Lambda_{t+1}\left[1-\delta_d+\omega_{1d}\left(\frac{D_{t+1}-D_t}{D_t}\right)^{\omega_{2d}}\frac{D_{t+1}}{D_t^2}\right]\end{aligned} \tag{A-4}$$

Plugging in for Λ and $u_1\left(C_{t+1}^A, H_{t+1}\right)$

$$\begin{aligned}1+\frac{\omega_{1d}}{D_{t-1}}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right)^{\omega_{2d}} &= E\beta\frac{1}{\varphi\gamma N_t^{-1}}\frac{\varphi}{C_{t+1}^A}N_{t+1}^{\gamma}\\ &\times\left(\theta D_t^{\rho}+(1-\theta)E_{h,t+1}^{\rho}\right)^{\frac{1-\gamma}{\rho}-1}(1-\gamma)\theta D_t^{\rho-1}\\ &+E\beta\frac{\varphi\gamma N_{t+1}^{-1}}{\varphi\gamma N_t^{-1}}\left[1-\delta_d+\omega_{1d}\left(\frac{D_{t+1}-D_t}{D_t}\right)^{\omega_{2d}}\frac{D_{t+1}}{D_t^2}\right]\end{aligned}$$

Thus:

$$\begin{aligned}1+\frac{\omega_{1d}}{D_{t-1}}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right)^{\omega_{2d}} &= E\beta\frac{(1-\gamma)\theta}{\gamma}N_t\left(\theta D_t^{\rho}+(1-\theta)E_{h,t+1}^{\rho}\right)^{-1}D_t^{\rho-1}\\ &+E\beta\frac{N_t}{N_{t+1}}\left[1-\delta_d+\omega_{1d}\left(\frac{D_{t+1}-D_t}{D_t}\right)^{\omega_{2d}}\frac{D_{t+1}}{D_t^2}\right]\end{aligned} \tag{A-5}$$

One can also rewrite this as:

$$1+\frac{\omega_{1d}}{D_{t-1}}\left(\frac{D_t-D_{t-1}}{D_{t-1}}\right)^{\omega_{2d}} = E\beta\frac{N_t}{N_{t+1}}\left[1+R_{t+1}^D-\delta_d+\omega_{1d}\left(\frac{D_{t+1}-D_t}{D_t}\right)^{\omega_{2d}}\frac{D_{t+1}}{D_t^2}\right] \tag{A-6}$$

where

$$R^D_{t+1} = \frac{(1-\gamma)\,\theta}{\gamma} N_{t+1} \left(\theta D^\rho_t + (1-\theta)\, E^\rho_{h,t+1}\right)^{-1} D^{\rho-1}_t \tag{A-7}$$

- Energy on consumer side:

$$\beta^t u_1 \left(C^A_t, H_t\right) N^\gamma_t \left(\theta D^\rho_{t-1} + (1-\theta)\, E^\rho_{h,t}\right)^{\frac{1-\gamma}{\rho}-1} (1-\gamma)(1-\theta)\, E^{\rho-1}_{h,t} = P_t \beta^t \Lambda_t \tag{A-8}$$

- Hours worked:

$$-\beta^t u_2 \left(C^A_t, H_t\right) = \beta^t \Lambda_t W_t \tag{A-9}$$

where

$$u_2 \left(C^A_t, H_t\right) = -\frac{1-\varphi}{1-H_t} \tag{A-10}$$

and

$$W_t = (1-\alpha)\, Z_{y,t} \left(\eta K^\psi_{t-1} + (1-\eta)\, E^\psi_{f,t}\right)^{\frac{\alpha}{\psi}} H^{-\alpha}_t \tag{A-11}$$

Thus:

$$1 = \frac{\varphi}{1-\varphi} \gamma W_t N^{-1}_t (1-H_t) \tag{A-12}$$

- Capital:

$$\begin{aligned} \beta^t \Lambda_t \left[1 + \frac{\omega_{1k}}{K_{t-1}} \left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)^{\omega_{2k}}\right] \quad &= \quad E\beta^{t+1}\Lambda_{t+1} \left[R_{t+1} + 1 - \delta_k \right. \\ &\quad \left. + \, \omega_{1k} \left(\frac{K_{t+1} - K_t}{K_t}\right)^{\omega_{2d}} \frac{K_{t+1}}{K^2_t}\right] \end{aligned} \tag{A-13}$$

where the real interest rate is given by:

$$R_{t+1} = Z_{y,t+1} \left(\eta K^\psi_t + (1-\eta)\, E^\psi_{f,t+1}\right)^{\frac{\alpha}{\psi}-1} H^{1-\alpha}_{t+1} \alpha \eta K^{\psi-1}_t \tag{A-14}$$

- Firm's energy use:

$$P_t = Z_{y,t} \left(\eta K^\psi_{t-1} + (1-\eta)\, E^\psi_{f,t}\right)^{\frac{\alpha}{\psi}-1} H^{1-\alpha}_t (1-\eta)\, \alpha E^{\psi-1}_{f,t} \tag{A-15}$$

We rearrange the above conditions and add the shock processes, the definitions of output and investment as well as the resource constraint to get 13 equations to be fed into Dynare:

- Resource constraint

$$N_t + I_{d,t} + I_{k,t} = Y_t - P_t\,(E_{h,t} + E_{f,t}) \tag{A-16}$$

- Investment in durables

$$I_{d,t} = D_t - (1-\delta_d)\, D_{t-1} + \frac{\omega_{1d}}{1+\omega_{2d}} \left(\frac{D_t - D_{t-1}}{D_{t-1}}\right)^{1+\omega_{2d}} \tag{A-17}$$

- Investment in capital

$$I_{k,t} = K_t - (1-\delta_k)\, K_{t-1} + \frac{\omega_{1k}}{1+\omega_{2k}} \left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)^{1+\omega_{2k}} \tag{A-18}$$

- Nondurables vs. Energy:

$$P_t = \frac{(1-\gamma)(1-\theta)}{\gamma} N_t \left(\theta D_{t-1}^{\rho} + (1-\theta) E_{h,t}^{\rho}\right)^{-1} E_{h,t}^{\rho-1} \tag{A-19}$$

- Labor supply:

$$N_t = \frac{\varphi\gamma}{1-\varphi} W_t (1-H_t) \tag{A-20}$$

- Wage equation:

$$W_t = (1-\alpha) Z_{y,t} \left(\eta K_{t-1}^{\psi} + (1-\eta) E_{f,t}^{\psi}\right)^{\frac{\alpha}{\psi}} H_t^{-\alpha} \tag{A-21}$$

- Interest rates:

$$R_t = Z_{y,t} \left(\eta K_{t-1}^{\psi} + (1-\eta) E_{f,t}^{\psi}\right)^{\frac{\alpha}{\psi}-1} H_t^{1-\alpha} \alpha\eta K_{t-1}^{\psi-1} \tag{A-22}$$

- Firm's energy use:

$$P_t = Z_{y,t} \left(\eta K_{t-1}^{\psi} + (1-\eta) E_{f,t}^{\psi}\right)^{\frac{\alpha}{\psi}-1} H_t^{1-\alpha} (1-\eta) \alpha E_{f,t}^{\psi-1} \tag{A-23}$$

- Output:

$$Y_t = Z_{y,t} \left(\eta K_{t-1}^{\psi} + (1-\eta) E_{f,t}^{\psi}\right)^{\frac{\alpha}{\psi}} H_t^{1-\alpha} \tag{A-24}$$

- Capital Euler equation

$$1 + \frac{\omega_{1k}}{K_{t-1}} \left(\frac{K_t - K_{t-1}}{K_{t-1}}\right)^{\omega_{2k}} = \beta E \left\{ \frac{N_t}{N_{t+1}} \left[1 + R_{t+1} - \delta_k + \omega_{1k} \left(\frac{K_{t+1} - K_t}{K_t}\right)^{\omega_{2d}} \frac{K_{t+1}}{K_t^2}\right]\right\} \tag{A-25}$$

- Durables Euler Equation:

$$\begin{aligned} 1 + \frac{\omega_{1d}}{D_{t-1}} \left(\frac{D_t - D_{t-1}}{D_{t-1}}\right)^{\omega_{2d}} &= \beta \frac{(1-\gamma)\theta}{\gamma} E \left\{ N_t \left(\theta D_t + (1-\theta) E_{h,t+1}^{\rho} D_t^{1-\rho}\right)^{-1} \right\} \\ &+ \beta E \left[1 - \delta_d + \omega_{1d} \left(\frac{D_{t+1} - D_t}{D_t}\right)^{\omega_{2d}} \frac{D_{t+1}}{D_t^2}\right] \frac{N_t}{N_{t+1}} \end{aligned} \tag{A-26}$$

- Productivity shock:

$$\log Z_{y,t} = \rho_z \log Z_{y,t-1} + \varepsilon_{z,t} \tag{A-27}$$

- Energy prices:

$$\log P_t = \rho_p \log P_{t-1} + \varepsilon_t + \rho_\varepsilon \varepsilon_{t-1} \tag{A-28}$$

A.2 Construct steady state

This section details how to derive the steady state values for all endogenous variables given the parameters.

- Resource Constraint

$$N + \delta_d D + \delta_k K = Y - P(E_h + E_f) \tag{A-29}$$

- Nondurables vs. Energy

$$P = \frac{(1-\gamma)(1-\theta)}{\gamma} N \left(\theta D^{\rho} + (1-\theta) E_h^{\rho}\right)^{-1} E_h^{\rho-1} \tag{A-30}$$

- Labor supply

$$N = \frac{\varphi\gamma}{1-\varphi} W (1-H) \tag{A-31}$$

- Wage equation

$$W = (1-\alpha) \frac{Y}{H} \tag{A-32}$$

- Interest rates

$$\begin{aligned} R &= Z_y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi}\right)^{\frac{\alpha}{\psi}-1} H^{1-\alpha} \alpha\eta K^{\psi-1} \\ &= Y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi}\right)^{-1} \alpha\eta K^{\psi-1} \end{aligned} \tag{A-33}$$

- Firm's energy use

$$\begin{aligned} P &= Z_y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi}\right)^{\frac{\alpha}{\psi}-1} H^{1-\alpha} (1-\eta) \alpha E_f^{\psi-1} \\ &= Y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi}\right)^{-1} (1-\eta) \alpha E_f^{\psi-1} \end{aligned} \tag{A-34}$$

- Output

$$Y = Z_y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi}\right)^{\frac{\alpha}{\psi}} H^{1-\alpha} \tag{A-35}$$

- Capital Euler Equation

$$1 = \beta (1 + R - \delta_k)$$

  and thus:

$$R = \frac{1}{\beta} - 1 + \delta_k \tag{A-36}$$

- Durables Euler Equation

$$1 = \beta \frac{(1-\gamma)\theta}{\gamma} N \left(\theta D^{\rho} + (1-\theta) E_h^{\rho}\right)^{-1} D^{1-\rho} + \beta (1-\delta_d) \tag{A-37}$$

Solve for steady state. As always:

$$R = \frac{1}{\beta} - 1 + \delta_k \tag{A-38}$$

From the interest rate and firm energy use equations we get:

$$\frac{R}{P} = \frac{\eta}{1-\eta} \left(\frac{K}{E_f}\right)^{\psi-1} \tag{A-39}$$

Let $\kappa_{ke} = \frac{K}{E_f}$ the capital to firm energy ratio then

$$\kappa_{ke} = \left(\frac{R}{P} \frac{1-\eta}{\eta} \right)^{\frac{1}{\psi-1}} \tag{A-40}$$

which is determined by parameters. Call $\kappa_{Ef} = \frac{E_f}{Y}$ the firm energy use to output ratio then

$$\begin{aligned} P &= Y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi} \right)^{-1} (1-\eta) \alpha E_f^{\psi-1} \\ &= \left(\eta \frac{(E_f \kappa_{ke})^{\psi} E_f^{1-\psi}}{Y} + (1-\eta) \frac{E_f}{Y} \right)^{-1} (1-\eta) \alpha \\ &= \left(\eta \kappa_{ke}^{\psi} \kappa_{Ef} + (1-\eta) \kappa_{Ef} \right)^{-1} (1-\eta) \alpha \\ &= \left(\eta \kappa_{ke}^{\psi} + (1-\eta) \right)^{-1} (1-\eta) \alpha \kappa_{Ef}^{-1} \end{aligned} \tag{A-41}$$

Thus:

$$\kappa_{Ef} = \frac{(1-\eta) \alpha}{P \left(\eta \kappa_{ke}^{\psi} + (1-\eta) \right)} \tag{A-42}$$

which is again only determined by parameters. Also notice that the capital output ratio $\kappa_K = \frac{K}{Y} = \frac{E_f \kappa_{ke}}{Y} = \kappa_{Ef} \kappa_{ke}$

From the output equation

$$Y = Z_y \left(\eta K^{\psi} + (1-\eta) E_f^{\psi} \right)^{\frac{\alpha}{\psi}} H^{1-\alpha} \tag{A-43}$$

Divide through by Y to get

$$1 = Z_y \left(\eta \left(\frac{K}{Y} \right)^{\psi} + (1-\eta) \left(\frac{E_f}{Y} \right)^{\psi} \right)^{\frac{\alpha}{\psi}} \left(\frac{H}{Y} \right)^{1-\alpha} \tag{A-44}$$

Thus:

$$1 = Z_y \left(\eta \kappa_K^{\psi} + (1-\eta) \kappa_{Ef}^{\psi} \right)^{\frac{\alpha}{\psi}} \kappa_H^{1-\alpha} \tag{A-45}$$

Thus:

$$\kappa_H = (Z_y)^{-\frac{1}{1-\alpha}} \left(\eta \kappa_K^{\psi} + (1-\eta) \kappa_{Ef}^{\psi} \right)^{-\frac{\alpha}{\psi(1-\alpha)}} \tag{A-46}$$

Also, the steady state wage rate is determined solely by parameters. It is the labor share times output to hours ratio:

$$W = \frac{1-\alpha}{\kappa_H} \tag{A-47}$$

From the consumer durables vs. nondurables equation:

$$1 = \beta \frac{(1-\gamma) \theta}{\gamma} N \left(\theta D^{\rho} + (1-\theta) E_h^{\rho} \right)^{-1} D^{\rho-1} + \beta (1-\delta_d) \tag{A-48}$$

Nondurables vs. energy:

$$P = \frac{(1-\gamma)(1-\theta)}{\gamma} N \left(\theta D^{\rho} + (1-\theta) E_h^{\rho} \right)^{-1} E_h^{\rho-1} \tag{A-49}$$

Solve for $\left(\theta D^{\rho}+(1-\theta)E_h^{\rho}\right)^{-1}$:

$$\left(\theta D^{\rho}+(1-\theta)E_h^{\rho}\right)^{-1}=P\frac{\gamma}{(1-\gamma)(1-\theta)}E_h^{1-\rho}N^{-1} \tag{A-50}$$

Plug into the previous equation

$$\begin{aligned} 1 &= \beta\frac{(1-\gamma)\theta}{\gamma}NP\frac{\gamma}{(1-\gamma)(1-\theta)}E_h^{1-\rho}N^{-1}D^{\rho-1}+\beta(1-\delta_d) \\ &= \beta P\frac{\theta}{(1-\theta)}\left(\frac{E_h}{D}\right)^{1-\rho}+\beta(1-\delta_d) \end{aligned} \tag{A-51}$$

Thus:

$$\frac{E_h}{D}=\left[\frac{1-\beta+\beta\delta_d}{\beta\theta P}(1-\theta)\right]^{\frac{1}{1-\rho}} \tag{A-52}$$

Next, write the Nondurables vs. Energy equation as:

$$\begin{aligned} P &= \frac{(1-\gamma)(1-\theta)}{\gamma}N\left(\theta D^{\rho}E_h^{1-\rho}+(1-\theta)E_h\right)^{-1} \\ &= \frac{(1-\gamma)(1-\theta)}{\gamma}\frac{N}{D}\left(\theta\left(\frac{E_h}{D}\right)^{1-\rho}+(1-\theta)\frac{E_h}{D}\right)^{-1} \end{aligned} \tag{A-53}$$

Thus:

$$\frac{N}{D}=\frac{\gamma P}{(1-\gamma)(1-\theta)}\left(\theta\left(\frac{E_h}{D}\right)^{1-\rho}+(1-\theta)\frac{E_h}{D}\right) \tag{A-54}$$

Also notice that

$$\frac{E_h}{D}=\frac{\kappa_{Eh}}{\kappa_D}\text{ and }\frac{N}{D}=\frac{\kappa_N}{\kappa_D} \tag{A-55}$$

Next, rewrite the budget constraint as:

$$\begin{aligned} 1-P\kappa_{Ef}-\delta_k\kappa_K &= \kappa_N+\delta_d\kappa_D+P\kappa_{Eh} \\ &= \kappa_D\left[\frac{\kappa_N}{\kappa_D}+\delta_d+P\frac{\kappa_{Eh}}{\kappa_D}\right] \end{aligned} \tag{A-56}$$

Then:

$$\kappa_D=\frac{1-P\kappa_{Ef}-\delta_k\kappa_K}{\frac{\kappa_N}{\kappa_D}+\delta_d+P\frac{\kappa_{Eh}}{\kappa_D}} \tag{A-57}$$

There is one equation left, we have not used so far: The labor supply equation. Divide it by Y to get:

$$\begin{aligned} \kappa_N &= \frac{\varphi\gamma}{1-\varphi}\frac{W}{Y}-\frac{\varphi\gamma}{1-\varphi}W\kappa_H \\ &= \frac{\varphi\gamma}{1-\varphi}\frac{1-\alpha}{H}-\frac{\varphi\gamma}{1-\varphi}W\kappa_H \end{aligned} \tag{A-58}$$

Thus:

$$\frac{\varphi\gamma}{1-\varphi}\frac{1-\alpha}{H}=\kappa_N+\frac{\varphi\gamma}{1-\varphi}W\kappa_H \tag{A-59}$$

Solve for H :

$$H=\frac{\varphi\gamma}{1-\varphi}\frac{1-\alpha}{\kappa_N+\frac{\varphi\gamma}{1-\varphi}W\kappa_H} \tag{A-60}$$

Via H and κ_H we determine Y which gives us all other steady state variables because we computed the output ratios for each variable.

A.3 Calibration

The targeted moments pin down the durable goods depreciation rate $\delta_d = \frac{I_d/Y}{D/Y} = 0.0682$. Moreover, in equation (A-52) we can solve for θ:

$$\theta = \frac{1-\beta(1-\delta_d)}{1-\beta(1-\delta_d)+\beta\left(\frac{D}{E_h}\right)^{\rho-1}} \tag{A-61}$$

From the firm energy use equation (A-34) we derive:

$$\eta = \frac{\alpha\left(\frac{E_f}{Y}\right)^{-1}-1}{\alpha\left(\frac{E_f}{Y}\right)^{-1}-1+\left(\frac{K}{E_f}\right)^{\psi}} \tag{A-62}$$

Next, from equation (A-33) we derive the steady state interest rate:

$$R = \left(\frac{K}{Y}\right)^{-1}\left(\eta+(1-\eta)\left(\frac{K}{E_f}\right)^{-\psi}\right)^{-1}\alpha\eta \tag{A-63}$$

Given interest rate R, the capital Euler equation (A-36) pins down depreciation of physical capital:

$$\delta_k = R - \frac{1}{\beta} + 1 \tag{A-64}$$

From the resource constraint (A-29) get the non-durables to output ratio:

$$\frac{N}{Y} = 1 - \left(\frac{E_h}{Y} + \frac{E_f}{Y} + \delta_d\frac{D}{Y} + \delta_k\frac{K}{Y}\right) \tag{A-65}$$

Next, solve equation (A-37) for γ :

$$\gamma = \frac{1-\theta}{1-\theta+\frac{E_h}{N}\left(\theta\left(\frac{D}{E_h}\right)^{\rho}+1-\theta\right)} \tag{A-66}$$

where the household energy to non-durable goods ratio is known: $E_h/N = \frac{E_h/Y}{N/Y}$. Next, from the labor supply equation (A-31) and the wage equation (A-32) we get:

$$\frac{N}{Y} = \frac{\varphi\gamma}{1-\varphi}\frac{1-\alpha}{H}(1-H) \tag{A-67}$$

Solve for φ :

$$\frac{1-\varphi}{\varphi} = \frac{Y}{N}\gamma\frac{1-\alpha}{H}(1-H) \tag{A-68}$$

Thus:

$$\varphi = \frac{1}{1+\left(\frac{N}{Y}\right)^{-1}\gamma(1-\alpha)\frac{1-H}{H}} \tag{A-69}$$

B Putty-Clay model technical details

B.1 First order conditions

Using the theorems in Atkeson and Kehoe (1999), we write the social planner's optimization problem as:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\varphi\gamma \log N_t + \varphi\left(1-\gamma\right) \log X_{d,t-1} + \left(1-\varphi\right) \log\left(1-H_t\right)\right)$$

subject to:

$$Z_{y,t} X_{k,t-1}^{\alpha} H_t^{1-\alpha} = N_t + I_{d,t} + I_{k,t} + P_t\left(M_{h,t-1} + M_{f,t-1}\right) \quad \text{(B-1)}$$

$$X_{d,t} \leq \left(1-\delta_d\right) X_{d,t-1} + \int_{V_{d,t}} I_{d,t}\left(V_{d,t}\right) \frac{f_d\left(V_{d,t}\right)}{V_{d,t}} dV_{d,t} \quad \text{(B-2)}$$

$$M_{h,t} \geq \left(1-\delta_d\right) M_{h,t-1} + \int_{V_{d,t}} I_{d,t}\left(V_{d,t}\right) \frac{1}{V_{d,t}} dV_{d,t} \quad \text{(B-3)}$$

$$X_{k,t} \leq \left(1-\delta_k\right) X_{k,t-1} + \int_{V_{k,t}} I_{k,t}\left(V_{k,t}\right) \frac{f_k\left(V_{k,t}\right)}{V_{k,t}} dV_{k,t} \quad \text{(B-4)}$$

$$M_{f,t} \geq \left(1-\delta_k\right) M_{f,t-1} + \int_{V_{k,t}} I_{k,t}\left(V_{k,t}\right) \frac{1}{V_{k,t}} dV_{k,t} \quad \text{(B-5)}$$

$$I_{d,t}\left(V_{d,t}\right), I_{k,t}\left(V_{d,t}\right) \geq 0 \quad \text{(B-6)}$$

Atkeson and Kehoe show that along an optimal path the social planner invests in at most one type of capital. The result applies here for both capital stocks, i.e., in every t there is exactly one $V_{d,t}$ and $V_{k,t}$ such that $I_{d,t}\left(V_{d,t}\right) > 0$ and $I_{k,t}\left(V_{k,t}\right) > 0$. Applying this result, the Lagrangian reads:

$$\begin{aligned} L = & \sum_{t=0}^{\infty} \beta^t \left(\varphi\gamma \log N_t + \varphi\left(1-\gamma\right) \log X_{d,t-1} + \left(1-\varphi\right) \log\left(1-H_t\right)\right) \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t^b \left[Z_{y,t} X_{k,t-1}^{\alpha} H_t^{1-\alpha} - N_t - I_{d,t} - I_{k,t} - P_t\left(M_{h,t-1} + M_{f,t-1}\right)\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t^{xd} \left[\left(1-\delta_d\right) X_{d,t-1} + I_{d,t} \frac{f_d\left(V_{d,t}\right)}{V_{d,t}} - X_{d,t}\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t^{mh} \left[M_{h,t} - \left(1-\delta_d\right) M_{h,t-1} - I_{d,t} \frac{1}{V_{d,t}}\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t^{xk} \left[\left(1-\delta_k\right) X_{k,t-1} + I_{k,t} \frac{f_k\left(V_{k,t}\right)}{V_{k,t}} - X_{k,t}\right] \\ & + \sum_{t=0}^{\infty} \beta^t \lambda_t^{mf} \left[M_{f,t} - \left(1-\delta_k\right) M_{f,t-1} - I_{k,t} \frac{1}{V_{k,t}}\right] \end{aligned} \quad \text{(B-7)}$$

We now derive first order conditions. Denote $\lambda_t^b, \lambda_t^{xd}, \lambda_t^{mh}, \lambda_t^{xk}, \lambda_t^{mf}$ the Lagrange multipliers for the first five constraints. First, choose functional forms for the f_k and f_h. As Atkeson and Kehoe we pick Cobb-Douglas functions for F_d and F_k :

$$F_d\left(D, E_h\right) = D^{\theta} E_h^{1-\theta} \quad \text{(B-8)}$$

$$F_k\left(K, E_f\right) = K^{\eta} E_f^{\eta-1} \quad \text{(B-9)}$$

Thus:

$$\begin{aligned} f_d(V_d) &= V_d^{\theta} && \text{(B-10)} \\ f_k(V_k) &= V_k^{\eta} && \text{(B-11)} \end{aligned}$$

Then first order necessary conditions are

1. Nondurable consumption:

$$\varphi\gamma\frac{1}{N_t} = \lambda_t^b \tag{B-12}$$

2. Durable aggregate:

$$\beta\varphi(1-\gamma)\frac{1}{X_{d,t}} - \lambda_t^{xd} + \beta\lambda_{t+1}^{xd}(1-\delta_d) = 0 \tag{B-13}$$

3. Durables investment

$$-\lambda_t^b + \lambda_t^{xd}\frac{f_d(V_{d,t})}{V_{d,t}} - \lambda_t^{mh}\frac{1}{V_{d,t}} = 0$$

   Thus

$$\lambda_t^{xd}V_{d,t}^{\theta} = V_{d,t}\lambda_t^b + \lambda_t^{mh} \tag{B-14}$$

4. Household energy

$$-P_{t+1}\beta\lambda_{t+1}^b + \lambda_t^{mh} - (1-\delta_d)\beta\lambda_{t+1}^{mh} = 0 \tag{B-15}$$

5. From Theorem 3 in Atkeson and Kehoe, the definition of $V_{d,t}$ is:

$$\begin{aligned} \lambda_t^{mh} &= \left(f_d(V_{d,t}) - f_d'(V_{d,t})V_{d,t}\right)\lambda_t^{xd} \\ &= (1-\theta)V_{d,t}^{\theta}\lambda_t^{xd} && \text{(B-16)} \end{aligned}$$

6. Hours worked:

$$\frac{1-\varphi}{1-H_t} = \lambda_t^b(1-\alpha)\frac{Y_t}{H_t} \tag{B-17}$$

7. Definition of output:

$$Y_t = Z_{y,t}X_{k,t-1}^{\alpha}H_t^{1-\alpha} \tag{B-18}$$

8. Fixed capital aggregate:

$$\beta\lambda_{t+1}^b\alpha\frac{Y_{t+1}}{X_{k,t}} - \lambda_t^{xk} + \beta(1-\delta_k)\lambda_{t+1}^{xk} = 0 \tag{B-19}$$

9. Fixed capital investment:

$$\lambda_t^{xk}V_k^{\eta} = \lambda_t^b V_{k,t} + \lambda_t^{mf} \tag{B-20}$$

10. Firm energy use:

$$-\beta\lambda_{t+1}^b P_{t+1} + \lambda_t^{mf} - \beta(1-\delta_k)\lambda_{t+1}^{mf} = 0 \tag{B-21}$$

11. From Theorem 3 in Atkeson and Kehoe, the definition of $V_{k,t}$ is:

$$\begin{aligned} \lambda_t^{mf} &= \lambda_t^{xk}\left(f_k(V_{k,t}) - f_k'(V_{k,t})V_{k,t}\right) && \text{(B-22)} \\ &= (1-\eta)V_k^{\eta}\lambda_t^{xk} && \text{(B-23)} \end{aligned}$$

12. Resource constraint

$$Y_t = N_t + I_{d,t} + I_{k,t} + P_t(M_{h,t-1} + M_{f,t-1}) \tag{B-24}$$

13. Evolution of durables service flow

$$X_{d,t} = (1 - \delta_d) X_{d,t-1} + I_{d,t} V_{d,t}^{\theta - 1} \tag{B-25}$$

14. Evolution of household energy use:

$$M_{h,t} = (1 - \delta_d) M_{h,t-1} + I_{d,t} V_{d,t}^{-1} \tag{B-26}$$

15. Evolution of fixed capital:

$$X_{k,t} = (1 - \delta_k) X_{k,t-1} + I_{k,t} V_{k,t}^{\eta - 1} \tag{B-27}$$

16. Evolution of firm energy use:

$$M_{f,t} = (1 - \delta_k) M_{f,t-1} + I_{k,t} V_{k,t}^{-1} \tag{B-28}$$

Counting variables:

Consumer:	$N, X_d, I_d, V_d, M_h, H_t$
Firm:	Y_t, X_k, I_k, V_k, M_f
Lagrange multipliers:	$\lambda^b, \lambda^{xd}, \lambda^{mh}, \lambda^{xk}, \lambda^{mf}$
Shocks:	Z_y, P

18 Variables, 15 first order conditions plus 2 for the shocks: 18 Variables and 18 Equations.

Cut down the number of equations and variables. Combining equations (B-14) and (B-16):

$$\begin{aligned} \lambda_t^{xd} V_d^{\theta} &= V_{d,t} \lambda_t^b + \lambda_t^{mh} \\ &= V_{d,t} \lambda_t^b + (1 - \theta) V_d^{\theta} \lambda_t^{xd} \end{aligned} \tag{B-29}$$

Thus

$$\lambda_t^{xd} = \frac{1}{\theta} V_{d,t}^{1-\theta} \lambda_t^b = \frac{\varphi\gamma}{\theta} \frac{V_{d,t}^{1-\theta}}{N_t} \tag{B-30}$$

$$\lambda_t^{mh} = \varphi\gamma \frac{1-\theta}{\theta} \frac{V_{d,t}}{N_t} \tag{B-31}$$

Likewise on the firm side:

$$\lambda_t^{xk} = \frac{\varphi\gamma}{\eta} \frac{V_{k,t}^{1-\eta}}{N_t} \tag{B-32}$$

$$\lambda_t^{mf} = \varphi\gamma \frac{1-\eta}{\eta} \frac{V_{k,t}}{N_t} \tag{B-33}$$

Now plug in for the Lagrange Multipliers to get the equations we feed into Dynare:

1. Euler Equation for X_d:

$$\frac{\varphi\gamma}{\theta} \frac{V_{d,t}^{1-\theta}}{N_t} = \beta\varphi (1 - \gamma) \frac{1}{X_{d,t}} + \beta \frac{\varphi\gamma}{\theta} \frac{V_{d,t+1}^{1-\theta}}{N_{t+1}} (1 - \delta_d)$$

Thus:

$$\frac{V_{d,t}^{1-\theta}}{N_t} = \beta\theta \left(\frac{1-\gamma}{\gamma} \right) \frac{1}{X_{d,t}} + \beta \frac{V_{d,t+1}^{1-\theta}}{N_{t+1}} (1 - \delta_d) \tag{B-34}$$

2. Euler equation for M_h:

$$\varphi\gamma\frac{1-\theta}{\theta}\frac{V_{d,t}}{N_t}=P_{t+1}\beta\varphi\gamma\frac{1}{N_{t+1}}+\left(1-\delta_d\right)\beta\varphi\gamma\frac{1-\theta}{\theta}\frac{V_{d,t+1}}{N_{t+1}}$$

Thus:

$$V_{d,t}\frac{N_{t+1}}{N_t}=P_{t+1}\beta\frac{\theta}{1-\theta}+\left(1-\delta_d\right)\beta V_{d,t+1} \tag{B-35}$$

3. Hours Worked:

$$\frac{1-\varphi}{\varphi\gamma}\frac{N_t}{1-H_t}=\left(1-\alpha\right)\frac{Y_t}{H_t} \tag{B-36}$$

4. Definition of output:

$$Y_t=Z_{y,t}X_{k,t-1}^{\alpha}H_t^{1-\alpha} \tag{B-37}$$

5. Euler Equation for X_k:

$$\frac{\varphi\gamma}{\eta}\frac{V_{k,t}^{1-\eta}}{N_t}=\beta\varphi\gamma\alpha\frac{Y_{t+1}}{N_{t+1}X_{k,t}}+\beta\left(1-\delta_k\right)\frac{\varphi\gamma}{\eta}\frac{V_{k,t+1}^{1-\eta}}{N_{t+1}}$$

Thus

$$V_{k,t}^{1-\eta}\frac{N_{t+1}}{N_t}=\beta\alpha\eta\frac{Y_{t+1}}{X_{k,t}}+\beta\left(1-\delta_k\right)V_{k,t+1}^{1-\eta} \tag{B-38}$$

6. Euler equation for firm energy use:

$$\varphi\gamma\frac{1-\eta}{\eta}\frac{V_{k,t}}{N_t}=\beta\varphi\gamma\frac{1}{N_{t+1}}P_{t+1}+\beta\left(1-\delta_k\right)\varphi\gamma\frac{1-\eta}{\eta}\frac{V_{k,t+1}}{N_{t+1}}$$

Thus

$$V_{k,t}\frac{N_{t+1}}{N_t}=\beta\frac{\eta}{1-\eta}P_{t+1}+\beta\left(1-\delta_k\right)V_{k,t+1} \tag{B-39}$$

7. Resource constraint

$$Y_t=N_t+I_{d,t}+I_{k,t}+P_t\left(M_{h,t-1}+M_{f,t-1}\right) \tag{B-40}$$

8. Evolution of durables service flow

$$X_{d,t}=\left(1-\delta_d\right)X_{d,t-1}+I_{d,t}V_{d,t}^{\theta-1} \tag{B-41}$$

9. Evolution of household energy use:

$$M_{h,t}=\left(1-\delta_d\right)M_{h,t-1}+I_{d,t}V_{d,t}^{-1} \tag{B-42}$$

10. Evolution of fixed capital:

$$X_{k,t}=\left(1-\delta_k\right)X_{k,t-1}+I_{k,t}V_{k,t}^{\eta-1} \tag{B-43}$$

11. Evolution of firm energy use:

$$M_{f,t}=\left(1-\delta_k\right)M_{f,t-1}+I_{k,t}V_{k,t}^{-1} \tag{B-44}$$

12. TFP process:

$$\log Z_{y,t}=\rho_z\log Z_{y,t-1}+\varepsilon_{z,t} \tag{B-45}$$

13. Energy price process
$$\log P_t = \rho_p^1 \log P_{t-1} + \varepsilon_{p,t} + \rho_p^2 \varepsilon_{p,t} \tag{B-46}$$

These are 13 equations in 13 variables, 11 of which are endogenous and 2 of which are the exogenous variables TFP and energy price.

B.2 Construct steady state

1. Euler Equation for X_d:
$$(1-\beta+\beta\delta_d)\, V_d^{1-\theta} = \beta\theta \frac{1-\gamma}{\gamma} \frac{N}{X_d} \tag{B-47}$$

2. Euler equation for M_h:
$$(1-\beta+\beta\delta_d)\, V_d = P\beta \frac{\theta}{1-\theta} \tag{B-48}$$

3. Hours Worked:
$$\frac{1-\varphi}{\varphi\gamma} \frac{N}{1-H} = (1-\alpha) \frac{Y}{H} \tag{B-49}$$

4. Definition of output:
$$Y = Z_y X_k^{\alpha} H^{1-\alpha} \tag{B-50}$$

5. Euler Equation for X_k:
$$(1-\beta+\beta\delta_k)\, V_k^{1-\eta} = \beta\alpha\eta \frac{Y}{X_k} \tag{B-51}$$

6. Euler equation for firm energy use:
$$(1-\beta+\beta\delta_k)\, V_k = \beta \frac{\eta}{1-\eta} P \tag{B-52}$$

7. Resource constraint
$$Y = N + I_d + I_k + P\,(M_h + M_f) \tag{B-53}$$

8. Evolution of durables service flow
$$\delta_d X_d = I_d V_d^{\theta-1} \tag{B-54}$$

9. Evolution of household energy use:
$$\delta_d M_h = I_d V_d^{-1} \tag{B-55}$$

10. Evolution of fixed capital:
$$\delta_k X_k = I_k V_k^{\eta-1} \tag{B-56}$$

11. Evolution of firm energy use:
$$\delta_k M_f = I_k V_k^{-1} \tag{B-57}$$

Start cranking: We can immediately read off the two steady state V

$$V_d = \frac{P\beta}{1-\beta+\beta\delta_d} \frac{\theta}{1-\theta} \tag{B-58}$$
$$V_k = \frac{\beta P}{1-\beta+\beta\delta_k} \frac{\eta}{1-\eta} \tag{B-59}$$

Define

$$\kappa_{id} = \frac{I_d}{Y}, \kappa_{xd} = \frac{X_d}{Y}, \kappa_{mh} = \frac{M_h}{Y} \quad \text{(B-60)}$$

$$\kappa_{ik} = \frac{I_k}{Y}, \kappa_{xk} = \frac{X_k}{Y}, \kappa_{mf} = \frac{M_f}{Y} \quad \text{(B-61)}$$

$$\kappa_h = \frac{H}{Y}, \kappa_n = \frac{N}{Y} \quad \text{(B-62)}$$

Then

$$\kappa_{id} = \delta_d V_d^{1-\theta} \kappa_{xd} \quad \text{(B-63)}$$

$$\kappa_{ik} = \delta_k V_k^{1-\eta} \kappa_{xk} \quad \text{(B-64)}$$

$$\kappa_{mh} = V_d^{-\theta} \kappa_{xd} \quad \text{(B-65)}$$

$$\kappa_{mf} = V_k^{-\eta} \kappa_{xk} \quad \text{(B-66)}$$

From the resource constraint

$$\begin{aligned} 1 &= \kappa_N + \kappa_{id} + \kappa_{ik} + P\left(\kappa_{mh} + \kappa_{mf}\right) \\ &= \kappa_N + \delta_d V_d^{1-\theta} \kappa_{xd} + \delta_k V_k^{1-\eta} \kappa_{xk} + P\left(V_d^{-\theta} \kappa_{xd} + V_k^{-\eta} \kappa_{xk}\right) \\ &= \kappa_N + \left(\delta_d V_d^{1-\theta} + P V_d^{-\theta}\right) \kappa_{xd} + \left(\delta_k V_k^{1-\eta} + P V_k^{-\eta}\right) \kappa_{xk} \end{aligned} \quad \text{(B-67)}$$

From the X_k Euler equation:

$$\kappa_{xk} = \frac{\beta \alpha \eta}{1 - \beta + \beta \delta_k} V_k^{\eta - 1} \quad \text{(B-68)}$$

From the X_d Euler equation:

$$V_d^{1-\theta} \left(1 - \beta + \beta \delta_d\right) = \beta \theta \frac{1-\gamma}{\gamma} \frac{\kappa_n}{\kappa_{xd}} \quad \text{(B-69)}$$

Solve for κ_{xd}

$$\kappa_{xd} = \frac{\beta \theta}{1 - \beta + \beta \delta_d} \frac{1-\gamma}{\gamma} V_d^{\theta - 1} \kappa_n \quad \text{(B-70)}$$

plug this κ_{xd} into equation (B-67)

$$1 = \left[1 + \left(\delta_d V_d^{1-\theta} + P V_d^{-\theta}\right) \frac{\beta \theta}{1 - \beta + \beta \delta_d} \frac{1-\gamma}{\gamma} V_d^{\theta - 1}\right] \kappa_n + \left(\delta_k V_k^{1-\eta} + P V_k^{-\eta}\right) \kappa_{xk} \quad \text{(B-71)}$$

and solve for κ_n:

$$\kappa_n = \frac{1 - \left(\delta_k V_k^{1-\eta} + P V_k^{-\eta}\right) \kappa_{xk}}{1 + \left(\delta_d V_d^{1-\theta} + P V_d^{-\theta}\right) \frac{\beta \theta}{1 - \beta + \beta \delta_d} \frac{1-\gamma}{\gamma} V_d^{\theta - 1}} \quad \text{(B-72)}$$

From the definition of output

$$\kappa_h = Z_y^{-\frac{1}{1-\alpha}} \kappa_{xk}^{-\frac{\alpha}{1-\alpha}} \quad \text{(B-73)}$$

From the hours equation:

$$\frac{H}{Y} = (1-\alpha) \frac{\varphi \gamma}{1 - \varphi} \frac{1 - H}{N} \quad \text{(B-74)}$$

Thus

$$\kappa_h = (1-\alpha) \frac{\varphi \gamma}{1 - \varphi} \frac{\frac{1}{Y} - \kappa_h}{\kappa_n} \quad \text{(B-75)}$$

Thus

$$Y = \left(\kappa_h \kappa_n \frac{1}{1-\alpha} \frac{1-\varphi}{\varphi\gamma} + \kappa_h\right)^{-1} \tag{B-76}$$

From Y we can back out all the other variables through their output ratios. For example:

$$H = \kappa_h Y \tag{B-77}$$

B.3 Calibration

As before, we specify six moments to pin down the six parameters $\gamma, \varphi, \theta, \eta, \delta_d, \delta_k$. We use the same moments as in the benchmark. From the calibration targets E_h/Y and D/Y we compute the target for $V_d = \frac{D/Y}{E_h/Y}$. Likewise, the steady state capital to energy ratio for the firm is $V_k = \frac{K/Y}{E_f/Y}$.

First from the evolution of X_k and M_f:

$$\frac{X_k}{Y} = \frac{M_f}{Y} V_k^{\eta} \tag{B-78}$$

Then combining the two Euler equation for the firm yields

$$\beta \frac{\eta}{1-\eta} P = \beta\alpha\eta \left(\frac{M_f}{Y}\right)^{-1} \Rightarrow \eta = 1 - \frac{P}{\alpha}\frac{M_f}{Y} \tag{B-79}$$

Next, derive δ_k from the firm energy use Euler equation

$$\delta_k = \frac{\eta}{1-\eta}\frac{P}{V_k} - \frac{1}{\beta} + 1 \tag{B-80}$$

From the evolution of household energy use we find

$$\delta_d = \frac{I_d/Y}{M_h/Y} V_d^{-1} \tag{B-81}$$

From the household energy Euler equation:

$$\theta = \left[\frac{P\beta}{(1-\beta+\beta\delta_d) V_d} + 1\right]^{-1} \tag{B-82}$$

From the resource constraint:

$$\kappa_n = 1 - \kappa_{id} - \kappa_{ik} - P\left(\kappa_{mh} + \kappa_{mf}\right) \tag{B-83}$$

Also, from the evolution X_d and M_h we find

$$\kappa_{xd} = \kappa_{mh} V_k^{\theta} \tag{B-84}$$

Combining the two consumer Euler equations yields

$$\gamma = \left[P \frac{1}{1-\theta} \frac{\kappa_{mh}}{\kappa_n} + 1\right]^{-1} \tag{B-85}$$

Finally, solve for φ in the hours equation

$$\varphi = \left[\gamma\left(1-\alpha\right) \frac{1-H}{H} \kappa_n^{-1} + 1\right]^{-1} \tag{B-86}$$

C Targeting four moments

In Table 9 we display the results in economy DE^{A} when we match the volatilities of I_k, I_d, E_h on a quarterly basis and $E_h + E_f$ on the annual basis by picking the four parameters ω_{1d}, ω_{1k}, ρ and ψ. This would imply the following parameters:

$$\begin{aligned} \omega_{1d} &= 0.5600 \\ \omega_{1k}, &= 17.4095 \\ \rho &= -2.7870 \\ \psi &= -3.2149 \end{aligned}$$

The results are similar to those in Table 4: most of the output volatility comes from TFP shocks, while only about 10 percent of volatility is due to energy price shocks.

Table 9: Percent standard deviation in the data versus model

Variable	Data	Both shocks	Energy only	TFP only
Output	1.57	1.35	0.16	1.34
Consumption	1.26	0.74	0.55	0.49
NDS ex energy	0.82	0.43	0.10	0.42
HH energy use	2.10	2.10	2.08	0.26
Durables	4.55	4.55	4.38	1.20
Fixed Investment	5.37	5.37	1.50	5.15
Durables + Fixed Inv	4.80	4.30	1.91	3.83
Hours	1.51	0.69	0.17	0.67
Total Energy (annual)	2.45	2.45	2.35	0.62