Appendix To "Real Business Cycles in Emerging Countries?" *

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1 Optimality Conditions of the Household's Problem

Letting $\lambda_t X_{t-1}^{-\gamma}$ denote the Lagrange multiplier associated with the sequential budget constraint, the optimality conditions associated with this problem are (??), (??), the no-Ponzi-game constraint holding with equality, and

$$\left[C_t/X_{t-1} - \theta\omega^{-1}h_t^{\omega}\right]^{-\gamma} = \lambda_t$$

$$\left[C_t/X_{t-1} - \theta\omega^{-1}h_t^{\omega}\right]^{-\gamma}\theta h_t^{\omega-1} = (1-\alpha)a_t\left(\frac{K_t}{X_{t-1}h_t}\right)^{\alpha}\left(\frac{X_t}{X_{t-1}}\right)^{1-\alpha}\lambda_t$$

$$\lambda_t = \beta\frac{1+r_t}{g_t^{\gamma}}E_t\lambda_{t+1}$$

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$$\left[1+\phi\left(\frac{K_{t+1}}{K_t}-g\right)\right]\lambda_t = \frac{\beta}{g_t^{\gamma}}E_t\lambda_{t+1}\left[1-\delta+\alpha a_{t+1}\left(\frac{X_{t+1}h_{t+1}}{K_{t+1}}\right)^{1-\alpha}\right.$$
$$\left.+\phi\left(\frac{K_{t+2}}{K_{t+1}}\right)\left(\frac{K_{t+2}}{K_{t+1}}-g\right)-\frac{\phi}{2}\left(\frac{K_{t+2}}{K_{t+1}}-g\right)^2\right]$$

2 Equilibrium Conditions in Stationary Form

Define $y_t = Y_t/X_{t-1}$, $c_t = C_t/X_{t-1}$, $d_t = D_t/X_{t-1}$, and $k_t = K_t/X_{t-1}$. Then, a stationary competitive equilibrium is give by a set of processes stationary solution to the following equations:

$$[c_t - \theta\omega^{-1}h_t^{\omega}]^{-\gamma} = \lambda_t$$

$$\theta h_t^{\omega-1} = (1-\alpha)a_t g_t^{1-\alpha}\left(\frac{k_t}{h_t}\right)^{\alpha}$$

$$\lambda_t = \frac{\beta}{g_t^{\gamma}}\left[1+r^*+\psi\left(e^{d_t-\bar{d}}-1\right)\right]E_t\lambda_{t+1}$$

$$\left[1+\phi\left(\frac{k_{t+1}}{k_t}g_t-g\right)\right]\lambda_t = \frac{\beta}{g_t^{\gamma}}E_t\lambda_{t+1}\left[1-\delta+\alpha a_{t+1}\left(\frac{g_{t+1}h_{t+1}}{k_{t+1}}\right)^{1-\alpha}\right.$$
$$\left.+\phi\frac{k_{t+2}}{k_{t+1}}g_{t+1}\left(\frac{k_{t+2}}{k_{t+1}}g_{t+1}-g\right)-\frac{\phi}{2}\left(\frac{k_{t+2}}{k_{t+1}}g_{t+1}-g\right)^2\right]$$

$$\frac{d_{t+1}}{1+r_t}g_t = d_t - y_t + c_t + i_t + \frac{\phi}{2}\left(\frac{k_{t+1}}{k_t}g_t-g\right)^2 k_t,$$

$$r_t = r^* + \psi\left(e^{d_t-\bar{d}}-1\right),$$

$$k_{t+1}g_t = (1-\delta)k_t + i_t$$

$$y_t = a_t k_t^{\alpha}(g_t h_t)^{1-\alpha}$$

3 GMM Estimation Procedure

Let $b \equiv [g\,\sigma_g\,\rho_g\,\sigma_a\,\rho_a\,\phi]'$ be the 6×1 vector of structural parameters to be estimated. We write the moment conditions as:[1]

[1]The estimation results are little changed if in writing the moment conditions we replace the empirical moments $\bar{g}^Y$, $\bar{g}^C$, and $\bar{g}^I$ by their theoretical counterpart $E_{gy}(b)$, and the empirical moment $\overline{tby}$ by its theoretical counterpart $E_{tby}(b)$. Specifically, the parameter estimates using annual Mexican data from 1900 to 2005 are

$$
u_t(b) = \begin{bmatrix}
E_{gy}(b) - g_t^Y \\
\sigma_{gy}^2(b) - (g_t^Y - \bar{g}^Y)^2 \\
\sigma_{gc}^2(b) - (g_t^C - \bar{g}^C)^2 \\
\sigma_{gi}^2(b) - (g_t^I - \bar{g}^I)^2 \\
\sigma_{tby}^2(b) - (tby_t - \overline{tby})^2 \\
\rho_{gy,gc} - \frac{(g_t^Y - \bar{g}^Y)(g_t^C - \bar{g}^C)}{\sigma_{gy}(b)\sigma_{gc}(b)} \\
\rho_{gy,gi} - \frac{(g_t^Y - \bar{g}^Y)(g_t^I - \bar{g}^I)}{\sigma_{gy}(b)\sigma_{gi}(b)} \\
\rho_{gy,tby} - \frac{(g_t^Y - \bar{g}^Y)(tby_t - \overline{tby})}{\sigma_{gy}(b)\sigma_{tby}(b)} \\
\rho_{gy1}(b) - \frac{(g_t^Y - \bar{g}^Y)(g_{t-1}^Y - \bar{g}^Y)}{\sigma_{gy}^2(b)} \\
\rho_{gy2}(b) - \frac{(g_t^Y - \bar{g}^Y)(g_{t-2}^Y - \bar{g}^Y)}{\sigma_{gy}^2(b)} \\
\rho_{gc1}(b) - \frac{(g_t^C - \bar{g}^C)(g_{t-1}^C - \bar{g}^C)}{\sigma_{gc}^2(b)} \\
\rho_{gc2}(b) - \frac{(g_t^C - \bar{g}^C)(g_{t-2}^C - \bar{g}^C)}{\sigma_{gc}^2(b)} \\
\rho_{gi1}(b) - \frac{(g_t^I - \bar{g}^I)(g_{t-1}^I - \bar{g}^I)}{\sigma_{gi}^2(b)} \\
\rho_{gi2}(b) - \frac{(g_t^I - \bar{g}^I)(g_{t-2}^I - \bar{g}^I)}{\sigma_{gi}^2(b)} \\
\rho_{tby1}(b) - \frac{(tby_t - \overline{tby})(tby_{t-1} - \overline{tby})}{\sigma_{tby}^2(b)} \\
\rho_{tby2}(b) - \frac{(tby_t - \overline{tby})(tby_{t-2} - \overline{tby})}{\sigma_{tby}^2(b)}
\end{bmatrix},
$$

where $Ex(b)$ denotes the expected value of the variable x_t implied by the theoretical model, $\sigma_x(b)$ denotes the standard deviation of x_t implied by the theoretical model, $\rho_{xy}(b)$ denotes the correlation between x_t and y_t implied by the theoretical model, and ρ_{xj} denotes the autocorrelation of order j of x_t implied by the theoretical model. All of these statistics are functions of the vector b of structural parameters. We denote by $\bar{x} \equiv T^{-1} \sum_{t=1}^{T} x_t$ the sample mean of x_t, where T is the sample size. We compute moments implied by the theoretical model by solving a linearized version of the system of equilibrium conditions with respect to the logarithm of all variables except the trade-balance share in GDP, which we keep in levels.

Define $J(b, W) = \bar{u}' W \bar{u}$, where $\bar{u}(b)$ denotes the sample mean of $u_t(b)$ and W is a sym-

Parameter	Point Estimate	Standard Deviation
g	1.0023	0.0011
σ_g	0.0195	0.0026
ρ_g	0.2507	0.0728
σ_a	0.0186	0.0009
ρ_a	-0.0367	0.0281
ϕ	0.0000	0.0018
Overidentifying Restrictions Test	p value	0.0698

We thank Masao Ogaki for suggesting to us to carry out this robustness exercise.

Table 1: Mexico 1980:Q1-2003:Q3: Estimated Structural Parameters

Parameter	Point Estimate	Standard Error
g	1.000	0.001
σ_g	0.008	0.001
ρ_g	0.715	0.038
σ_a	0.006	0.001
ρ_a	0.508	0.147
ϕ	1.150	0.128
Overidentifying Restrictions Test	p value	0.189

metric positive definite matrix compatible with $\bar{u}(b)$. The GMM estimate of b, denoted $\hat{b}$, is given by

$$\hat{b} = \underset{b}{\operatorname{argmin}}\, J(b, W).$$

The matrix W is estimated in two steps. For more details see Burnside (1999).[2]

4 GMM Estimation: Mexico 1980:Q1 2003:Q2

The estimation of the RBC model using quarterly Mexican data from 1980:1 to 2003:2 is thown in table Table 1. The fit of the model, as measured by the p value of the test of overidentifying restrictions is much better than the one obtained using the long sample 1900-2005. This is reflected in a better matching of the second moments of interest, as shown in table 2 and figure 1.

[2]Burnside, Craig, "Real Business Cycle Models: Linear Approximation and GMM Estimation," manuscript, The World Bank, May 1, 1999.

Table 2: Mexico 1980:Q1-2003:Q2

Statistic	g^Y	g^C	g^I	tby
Standard Deviation				
—Model	1.6	1.6	7.4	4.0
—Data	1.5	1.9	5.7	3.7
	(0.3)	(0.2)	(1.0)	(0.4)
Correlation with g^Y				
—Model		0.91	0.65	-0.41
—Data		0.76	0.75	-0.19
		(0.07)	(0.07)	(0.09)
Correlation with tby				
—Model		-0.45	-0.32	
—Data		-0.23	-0.14	
		0.07	0.11	
Serial Correlation				
—Model	0.07	0.08	-0.06	0.89
—Data	0.25	0.19	0.44	0.95
	(0.10)	(0.14)	(0.10)	(0.03)

Note: Standard deviations are reported in percentage points. Standard errors of sample-moment estimates are shown in parenthesis.

Figure 1: Mexico 1980:Q1-2003:Q2: The Autocorrelation Function of the Trade Balance-to-Output Ratio