The Smets-Wouters Model

Monetary and Fiscal Policy

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Outline

Source and Impact

- **Source:** Smets, Frank and Raf Wouters, “An estimated dynamic stochastic general equilibrium model of the Euro area,” Journal of the European Economic Association, September 2003, 1(5), 1123-1175.
- **Related:** Christiano, Lawrence, Martin Eichenbaum and Charlie Evans, “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” Journal of Political Economy, 2005, vol. 113, no. 1, 1-45.
- **Impact:** The Smets-Wouters model have become a modern workhorse and benchmark model for analyzing monetary and fiscal policy in European central banks, and is spreading to policy institutions in the US as well.

Overview

- This is an elaborate New Keynesian model.
- There is a continuum of households, who supply household-specific labor in monopolistic competition. They set wages. Wages are Calvo-sticky.
- There is a continuum of intermediate good firms, who supply intermediate goods in monopolistic competition. They set prices. Prices are Calvo-sticky.
- Final goods use intermediate goods and are produced in perfect competition.
- There is habit formation, adjustment costs to investment, variable capital utilization.
- The monetary authority follows a Taylor-type rule.
- There are many sources of shocks - enough to make sure the data can be matched to the model.

Outline

Households

Utility of household τ:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t U_t^\tau$$

where

$$U_t^\tau = \epsilon_t^b \left(\frac{(C_t^\tau - H_t)^{1-\sigma_c}}{1-\sigma_c} - \epsilon_t^L \frac{(\ell_t^\tau)^{1+\sigma_l}}{1+\sigma_l} \right)$$

- C_t^τ: consumption.
- H_t: external habit / catching up with the Joneses.
- ℓ_t^τ: labor
- ϵ_t^b: intertemporal substitution shock
- ϵ_t^L: labor supply shock

Preference Shocks and Habit

Preference Shocks:

$$\begin{aligned}\epsilon_t^b &= \rho_b \epsilon_{t-1}^b + \eta_t^b \\ \epsilon_t^L &= \rho_L \epsilon_{t-1}^L + \eta_t^L\end{aligned}$$

Habits:

$$H_t = hC_{t-1}$$

where C_{t-1} is aggregate consumption in $t-1$.

Money

- ... is left unmodelled, but implicitely assumed to be there.
- Easiest solution: assume that households have **money in the utility**, i.e. enjoy holding real balances,...
- ... and the monetary authority influences nominal rates per **helicopter drops** of money on households,...
- ... but that otherwise money does not influence budget constraints etc.
- See Woodford for details on how this can be done.
- Later on, we shall formulate an interest-rate setting rule for the monetary authority. Thus money does not need to be modelled explicitely (hopefully... but subtle and possibly crucial issues may be overlooked this way!!)

Intertemporal budget constraint and income

- Intertemporal budget constraint:

$$b_t \frac{B_t^\tau}{P_t} = \frac{B_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau$$

  where B_t^τ are nominal discount bonds with market price b_t.
- Real income

$$Y_t^\tau = (w_t^\tau \ell_t^\tau + A_t^\tau) + (r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau)K_{t-1}^\tau) + \mathrm{Div}_t^\tau - \mathrm{tax}_t$$

- $w_t^\tau \ell_t^\tau + A_t^\tau$: Labor income plus state contingent security payoffs.
- $r_t^k z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau)K_{t-1}^\tau$: return on real capital stock minus costs from capital utilization z_t^τ. Assume $\Psi(1) = 0$.
- Div_t^τ: dividends from imperfectly competitive firms.
- tax_t: real lump-sum tax.

Imperfect substitutability of labor

- Individual households supply different types of labor, which is not perfectly substitutable,

$$L_t = \left(\int_0^1 (\ell_t^\tau)^{1/(1+\lambda_{w,t})} \, d\tau \right)^{1+\lambda_{w,t}}$$

- The degree of substitutability is random,

$$\lambda_{w,t} = \lambda_w + \eta_t^w$$

- In the flexible-wage economy, $1 + \lambda_{w,t}$ will be the markup of real wages over the usual ratio of the marginal disutility of labor to the marginal utility of consumption. Thus, η_t^w is a **wage markup shock**.

Wage setting

- Households are monopolistically competitive suppliers of labor and wage setters, offering their labor in the quantity demanded at their current wage W_t^τ.
- Wages are Calvo-sticky.
- Each period, the household has probability $1 - \xi_w$ that it is allowed to freely adjust its wage, choosing a new nominal wage

$$W_t^\tau = \tilde{w}_t^\tau$$

- If not, wages are adjusted according to the indexation rule

$$W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}^\tau$$

- $\gamma_w = 0$: no indexation. $\gamma_w = 1$: perfect indexation.

Perfect insurance markets

Perfect insurance:

- labor income of an individual household equals aggregate labor income.
- Thus, the consumption of an individual household equals aggregate consumption, $C_t^\mathcal{T} = C_t$, ...
- ... and marginal utility $\Lambda_t^\mathcal{T} \equiv \Lambda_t$ of consumption is equal across households.
- As a consequence, capital holdings $K_t^\mathcal{T} \equiv K_t$, bond holdings $B_t^\mathcal{T} = B_t$ as well as firm dividends $\text{Div}_t^\mathcal{T} \equiv \text{Div}_t$ will be identical across different types of households.
- Also possible: these remain in constant proportions forever across different households.

Outline

All sectors

- **Final goods production**: homogenous final good, produced with a continuum of imperfectly substitutable intermediate goods.
- A continuum of **intermediate goods**, produced with capital and labor.
- **Labor**, in turn, is a "composite" of individual household labor.
- New **capital** is produced with old capital and investment, subject to an **investment adjustment cost**.
- Depreciation varies with **capital utilization**.

Final goods production

- $$Y_t = \left(\int_0^1 \left(y_{j,t} \right)^{1/(1+\lambda_{p,t})} dj \right)^{1+\lambda_{p,t}}$$
- The degree of substitutability is random,
$$\lambda_{p,t} = \lambda_p + \eta_t^p$$
- It will turn out that $1 + \lambda_{p,t}$ is the markup of prices over marginal costs at the intermediate goods level. Thus, η_t^p is a **goods markup shock** or a **cost-push shock**.

Intermediate goods production

- Intermediate goods production is

$$y_{j,t} = \epsilon_t^a \tilde{K}_{j,t}^{\alpha} L_{j,t}^{1-\alpha} - \Phi$$

- Φ: a fixed cost.
- $\tilde{K}_{j,t}$: effective utilization of the capital stock,

$$\tilde{K}_{j,t} = z_t K_{j,t-1}$$

- ϵ_t^a: aggregate productivity shock,

$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a$$

- The profits of intermediate goods firms are paid as dividends Div_t.

Price setting

- Intermediate good firms are monopolistically competitive, offering their good in the quantity demanded at their current price $P_{j,t}$.
- Prices are Calvo-sticky.
- Each period, the firm has probability $1 - \xi_p$ that it is allowed to freely adjust its price, choosing a new nominal price

$$P_{j,t} = \tilde{p}_{j,t}$$

- If not, prices are adjusted according to the indexation rule

$$P_{j,t} = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} P_{j,t-1}$$

- $\gamma_p = 0$: no indexation. $\gamma_p = 1$: perfect indexation.

Capital evolution

- New capital is produced from old capital and investment goods,

$$K_t = (1-\tau)K_{t-1} + \left(1 - S\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right)\right) I_t$$

- I_t: gross investment
- τ: depreciation rate
- $S(\cdot)$: cost for changing the level of investment, with $S(1) = 0, S'(1) = 0, S''(1) > 0$.
- ϵ_t^I: shock to investment cost,

$$\epsilon_t^I = \rho_I \epsilon_{t-1}^I + \eta_t^I$$

Outline

Government

- Government consumption: G_t, following government spending rule, financed by lump sum taxation,

$$G_t = \text{tax}_t$$

- Monetary authority: sets nominal interest rate

$$R_t = 1 + i_t = 1/b_t$$

  following some interesting setting rule.
- We shall specify these rules in the log-linearized version of the model.

Outline

Market clearing

- Labor market:

$$\int_0^1 L_{j,t} dj = L_t = \left(\int_0^1 \left(\ell_t^\tau \right)^{1/(1+\lambda_{w,t})} d\tau \right)^{1+\lambda_{w,t}}$$

- Final goods market:

$$C_t + I_t + G_t + I_t + \psi(z_t) K_{t-1} = Y_t$$

- Capital rental market:

$$\int K_{j,t-1} dj = K_{t-1}$$

Wage and price aggregation

Dixit-Stiglitz aggregation:

- Aggregate wages are

$$W_t = \left(\int_0^1 (W_t^\tau)^{-1/\lambda_{w,t}} \, d\tau \right)^{-\lambda_{w,t}}$$

- Aggregate prices are

$$P_t = \left(\int_0^1 (P_{j,t})^{-1/\lambda_{p,t}} \, d\tau \right)^{-\lambda_{p,t}}$$

- One can derive these formulas from first-order conditions of producing aggregate output or aggregate labor.

Equilibrium

Definition

Given policy rules for G_t and R_t and thus tax_t, an equilibrium is an allocation $(B_t, C_t, H_t, (\ell_t^\tau)_{\tau \in [0,1]}, (L_{i,t})_{i \in [0,1]}, L_t, (\tilde{K}_{i,t})_{i \in [0,1]}, (K_{i,t})_{i \in [0,1]}, K_t, z_t, I_t, (y_{i,t})_{i \in [0,1]}, Y_t, \text{Div}_t)$ and prices $(b_t, r_t^k, (W_t^\tau)_{\tau \in [0,1]}, W_t, (P_{i,t})_{i \in [0,1]})$, so that

1. Given prices and the demand function for labor ℓ_t^τ, the allocation maximizes the utility of the household, subject to the Calvo-sticky wages.
2. Given prices and the demand function for $y_{t,i}$, the allocation maximizes the profits of the firms, subject to the Calvo-sticky prices.
3. Markets clear.
4. The policy rules are consistent with allocation and prices.

Outline

Intertemporal optimization 1

- Lucas asset pricing equation for bonds:

$$E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t P_t}{P_{t+1}} \right] = 1$$

where

$$\Lambda_t = U_{C,t} = \epsilon_t^b \left(C_t - H_t \right)^{-\sigma_c}$$

- Lucas asset pricing equation for capital:

$$Q_t = E_t \left[\beta \frac{\Lambda_{t+1}}{\Lambda_t} \left(Q_{t+1}(1-\tau) + z_{t+1} r_{t+1}^k - \psi(z_{t+1}) \right) \right]$$

Intertemporal optimization 2

- Optimal investment:

$$Q_t \left(1 - S\left(\frac{\epsilon_t^I I_t}{I_{t-1}}\right)\right) = Q_t S'\left(\frac{\epsilon_t^I I_t}{I_{t-1}}\right)\frac{\epsilon_t^I I_t}{I_{t-1}} + 1$$
$$-E_t\left[\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}S'\left(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}\right)\left(\frac{\epsilon_{t+1}^I I_{t+1}}{I_t}\right)\frac{I_{t+1}}{I_t}\right]$$

  copied from Smets-Wouters, equation (16). Is it correct?
- For capital utilization:

$$r_t^k = \psi'(z_t)$$

Wage setting

- Demand curve for labor:

$$\ell_t^\tau = \left(\frac{W_t^\tau}{W_t} \right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t$$

- Optimality condition for setting a new wage $\tilde{w}_t$:

$$\begin{aligned} \frac{\tilde{w}_t}{P_t} E_t \left[\sum_{i=0}^{\infty} \beta^i \xi_w^i \left(\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+i}/P_{t+i-1}} \right) \frac{\ell_{t+i}^\tau U_{C,t+i}}{1+\lambda_{w,t+i}} \right] \\ = \quad E_t \left[\sum_{i=0}^{\infty} \beta^i \xi_w^i \ell_{t+i}^\tau U_{l,t+i} \right] \end{aligned}$$

where $U_{C,t}, U_{l,t}$ denote marginal utility of consumption and marginal disutility of labor.

Evolution of wages

Per aggregation of wages,

$$(W_t)^{-1/\lambda_{w,t}} = \xi_w \left(W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} \right)^{-1/\lambda_{w,t}} + (1 - \xi_w) (\tilde{w}_t)^{-1/\lambda_{w,t}}$$

Outline

Labor and capital

- Cost minimization gives

$$\frac{W_t L_{j,t}}{r_t^k \tilde{K}_{j,t}} = \frac{1-\alpha}{\alpha}$$

- Thus, marginal costs for producing one extra unit of intermediate goods output is

$$\mathrm{MC}_t = \frac{1}{\epsilon_t^a} W_t^{1-\alpha} \left(r_t^k\right)^{\alpha} \left(\alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)}\right)$$

Demand and profits

- The demand function $y_t^j = D(P_{j,t}; P_t, Y_t)$ is given by

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-(1-\lambda_{p,t})/\lambda_{p,t}} Y_t$$

- Nominal profits are

$$\pi_{j,t} = \left(P_{j,t} - \mathrm{MC}_t\right)\left(\frac{P_{j,t}}{P_t}\right)^{-(1-\lambda_{p,t})/\lambda_{p,t}} Y_t - \mathrm{MC}_t \Phi$$

Price setting

Optimality condition for setting a new price $\tilde{p}_t$:

$$E_t\left[\sum_{i=0}^{\infty}\beta^i\xi_p^i\Lambda_{t+i}y_{j,t+i}\left(\frac{\tilde{p}_t}{P_t}\left(\frac{(P_{t-1+i}/P_{t-1})^{\gamma_p}}{P_{t+i}/P_t}\right)\right)\right]$$

$$= \quad E_t\left[\sum_{i=0}^{\infty}\beta^i\xi_p^i\Lambda_{t+i}y_{j,t+i}(1+\lambda_{p,t+i})\mathrm{mc}_{t+i}\right]$$

where

$$\mathrm{mc}_t = \frac{MC_t}{P_t}$$

are the real marginal costs.

Evolution of prices

Per aggregation of prices,

$$(P_t)^{-1/\lambda_{p,t}} = \xi_p \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_p} \right)^{-1/\lambda_{p,t}} + (1 - \xi_p) (\tilde{p}_t)^{-1/\lambda_{p,t}}$$

Outline

Remark

- The equations in the published version of the paper do not appear to be entirely correct (this can easily happen), ...
- ... and they do not appear to be consistent with the code either, even once corrected.
- We therefore spent considerable time clarifying the differences.
- We believe we got everything correct now.
- Very special thanks go to Wenjuan Chen and Matthieu Droumaguet for doing this and to Stefan Ried for supervising it! This was a lot of work...
- Details are in a "SmetsWouters-"Manual.

Equations 1,2 and 3

- The capital accumulation equation:

$$\widehat{K}_t = (1-\tau)\widehat{K}_{t-1} + \tau\widehat{I}_{t-1} \tag{1}$$

- The labour demand equation:

$$\widehat{L}_t = -\widehat{w}_t + (1+\psi)\widehat{r}_t^k + \widehat{K}_{t-1} \tag{2}$$

- The goods market equilibrium condition:

$$\widehat{Y}_t = (1-\tau k_y - g_y)\widehat{C}_t + \tau k_y \widehat{I}_t + \epsilon_t^G \tag{3}$$

Equations 4,5

- The production function:

$$\widehat{Y}_t = \phi\epsilon_t^a + \phi\alpha\widehat{K}_{t-1} + \phi\alpha\psi\widehat{r}_t^k + \phi(1-\alpha)\widehat{L}_t \qquad (4)$$

- The monetary policy reaction function: a Taylor-type rule

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left\{\bar{\pi}_t + r_\pi(\widehat{\pi}_{t-1} - \bar{\pi}_t) + r_Y(\widehat{Y}_t - \widehat{Y}_t^P)\right\}$$
$$+r_{\Delta\pi}(\widehat{\pi}_t - \widehat{\pi}_{t-1}) + r_{\Delta Y}\left[\widehat{Y}_t - \widehat{Y}_t^P - (\widehat{Y}_{t-1} - \widehat{Y}_{t-1}^P)\right] + \eta_t^R \qquad (5)$$

where Y_t^P refers to a hypothetical “frictionless economy” and **potential output**. The difference $\widehat{Y}_t - \widehat{Y}_t^P$ is the **output gap**.

Equation 6

- The consumption equation:

$$\begin{aligned}\widehat{C}_t &= \frac{h}{1+h}\widehat{C}_{t-1} + \frac{1}{1+h}E_t\widehat{C}_{t+1} - \frac{1-h}{(1+h)\sigma_c}(\widehat{R}_t - E_t\widehat{\pi}_{t+1}) \\ &\quad + \frac{1-h}{(1+h)\sigma_c}\widehat{\epsilon}_t^b \end{aligned} \tag{6}$$

Equations 7,8

- The investment equation:

$$\widehat{I}_t = \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1} + \frac{\varphi}{1+\beta}\widehat{Q}_t + \widehat{\epsilon}^I_t \qquad (7)$$

- The Q equation:

$$\widehat{Q}_t = -(\widehat{R}_t - E_t\widehat{\pi}_{t+1}) + \frac{1-\tau}{1-\tau+\bar{r}^k}E_t\widehat{Q}_{t+1} + \frac{\bar{r}^k}{1-\tau+\bar{r}^k}E_t\widehat{r}^k_{t+1} + \eta^Q_t \qquad (8)$$

Equation 9

- The inflation equation:

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\gamma_p} E_t \widehat{\pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p} \widehat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p} \frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p} [\alpha \widehat{r}_t^k + (1-\alpha)\widehat{w}_t - \widehat{\epsilon}_t^a] + \eta_t^p \tag{9}$$

Equation 10

- The wage equation is given as follows. Pay attention that the sign before the labour supply shock shall be positive instead of negative, which is confirmed by the authors.

$$\begin{aligned}\widehat{w}_t = & \frac{\beta}{1+\beta}E_t\widehat{w}_{t+1} + \frac{1}{1+\beta}\widehat{w}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{\pi}_{t+1} \\ & - \frac{1+\beta\gamma_w}{1+\beta}\widehat{\pi}_t + \frac{\gamma_w}{1+\beta}\widehat{\pi}_{t-1} \\ & - \frac{1}{1+\beta}\frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w} * \ldots \\ & \left[\widehat{w}_t - \sigma_L\widehat{L}_t - \frac{\sigma_c}{1-h}(\widehat{C}_t - h\widehat{C}_{t-1}) + \widehat{\epsilon}_t^L\right] + \eta_t^w \end{aligned} \tag{10}$$

Differences to published version

Equation in Smets-Wouters (2003)	Here
28	take out ϵ^b_{t+1}
29	take out ϵ^I_{t+1}
29	ϵ^b_t rescaled to equal 1
35a	ϵ^G_t rescaled to equal 1
32	η^p_t rescaled to equal 1
33	η^w_t rescaled to equal 1

Calculating results

- In order to calculate results, we have to create two systems. One is the flexible system where there is no price stickiness, wage stickiness or three cost-push shocks.
- The other one is the sticky system where prices and wages are set following a Calvo mechanism.
- We use the potential output produced in the flexible system to calculate the output gap in the Taylor rule.
- In each system, there are 8 endogenous variables and 2 state variables. The 8 endogenous variables are capital, consumption, investment, inflation, wages, output, interest rate, and real capital stock. The 2 state variables are labour and return on capital.

Outline

Calibration 1

Parameters	Value	Description
β	0.99	discount factor
τ	0.025	depreciation rate of capital
α	0.3	capital output ratio
ψ	1/0.169	inverse elasticity of cap. util. cost
γ_p	0.469	degree of partial indexation of price
γ_w	0.763	degree of partial indexation of wage
λ_w	0.5	mark up in wage setting
ξ_s^p	0.908	Calvo price stickiness
ξ_s^w	0.737	Calvo wage stickiness
σ_L	2.4	inverse elasticity of labor supply

Calibration 1

Parameters	Value	Description
σ_c	1.353	coeff. of relative risk aversion
h	0.573	habit portion of past consumption
ϕ	1.408	1 + share of fixed cost in prod.
φ	1/6.771	inverse of inv. adj. cost
$\bar{r}_k$	$1/\beta - 1 + \tau$	steady state return on capital
k_y	8.8	capital output ratio
inv_y	0.22	share of investment to GDP
c_y	0.6	share of consumption to GDP
k_y	inv_y/τ	capital income share, inv. share
g_y	$1 - c_y - inv_y$	government expend. share in GDP
r_π^Δ	0.14	inflation growth coeff.
r_y	0.099	output gap coeff

Calibration 1

Parameter	Value	Description
r_y^{Δ}	0.159	output gap growth coefficient
ρ	0.961	AR for lagged interest rate
r_π	1.684	inflation coefficient
ρ_{ϵ_L}	0.889	AR for labour supply shock
ρ_{ϵ_a}	0.823	AR for productivity shock
ρ_{ϵ_b}	0.855	AR for f preference shock
ρ_G	0.949	AR for government expenditure shock
$\rho_{\overline{\pi}}$	0.924	AR for inflation objective schock
ρ_{ϵ_i}	0.927	AR for investment shock
ρ_{ϵ_r}	0	AR for interest rate shock,IID
ρ_{λ_w}	0	AR for wage markup,IID

Calibration 1

Parameter	Value	Description
ρ_q	0	AR for return on equity,IID
ρ_{λ_p}	0	AR for price mark-up schock,IID
σ_{ϵ_L}	3.52	stand. dev. of labour supply shock
σ_{ϵ_a}	0.598	stand. dev. of productivity shock
σ_{ϵ_b}	0.336	stand. dev. of preference shock
σ_G	0.325	stand. dev. of goverment expenditure shock
$\sigma_{\overline{\pi}}$	0.017	stand. dev. inflation objective shock
σ_{ϵ_r}	0.081	stand. dev. of interest rate shock
σ_{ϵ_i}	0.085	stand. dev. of investment shock
σ_{λ_p}	0.16	stand. dev. of mark-up shock
σ_{λ_w}	0.289	stand. dev. of wage mark-up shock
σ_{ϵ_q}	0.604	stand. dev. of equity premium shock