For Online Publication: Appendix for Assessing the Stabilizing Effects of Unemployment Benefit Extensions

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Appendix A presents derivations of the model equilibrium conditions, as well as the full equilibrium system. It also presents the derivation of the condition we use to calibrate the disutility of work. Finally, it discusses key aspects of the model that underlie its dynamics. Appendix B presents further proofs, derivations and results related to the transmission channels of unemployment insurance. Appendix C evaluates the ability of the model to track unemployment over a long sample when productivity shocks drive fluctuations. Appendix D reports additional tables and figures.

A Model Derivations

A.1 Household FOCs

Let λ_t^B, λ_t^{CN}, λ_t^{CUR}, λ_t^{CUN}, λ_t^{BC}, λ_t^A, λ_t^N, be the multipliers associated with the following constraints in the main text: the household budget constraint (equation (8)), the liquidity constraint for employed (equation (9)), the liquidity constraint for benefit recipients (equation (10)), the liquidity constraint for non-recipients (equation (11)), the borrowing constraint (equation (12)), the end-of-period asset constraint (equation (13)), the employment accumulation constraint (equation (14)). The household first-order conditions are:

w.r.t. x_t:

$$\lambda_t^B - \lambda_t^A + \lambda_t^{CN} + \lambda_t^{CUR} + \lambda_t^{CUN} = 0 \tag{A.1}$$

w.r.t. c_t^n:

$$n_t u'\left(c_t^n\right) - \lambda_t^{CN} + n_t \lambda_t^A = 0 \tag{A.2}$$

with

$$\lambda_t^{CN}\left(x_t + (1-\tau_t)\, w_t + (1-\tau_t)\, d_t - c_t^n\right) = 0 \tag{A.3}$$

w.r.t. c_t^{ur}:

$$(1-n_t)\, \nu_t u'\left(c_t^{ur}\right) - \lambda_t^{CUR} + (1-n_t)\, \nu_t \lambda_t^A = 0 \tag{A.4}$$

with

$$\lambda_t^{CUR}\left(x_t + \tau_t^u - c_t^{ur}\right) = 0 \tag{A.5}$$

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w.r.t. c_t^{un}:

$$(1-n_t)(1-\nu_t)\,u'\left(c_t^{un}\right)-\lambda_t^{CUN}+(1-n_t)(1-\nu_t)\,\lambda_t^A=0 \tag{A.6}$$

with

$$\lambda_t^{CUN}\left(x_t+\tau^s-c_t^{un}\right)=0 \tag{A.7}$$

w.r.t. b_{t+1}:

$$-\frac{1}{p_t}\lambda_t^B-\lambda_t^{BC}+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial b_{t+1}}\right\}=0 \tag{A.8}$$

with

$$\lambda_t^{BC}\left(p_t\overline{b}_t-b_{t+1}\right)=0 \tag{A.9}$$

w.r.t. a_{t+1}:

$$\frac{1}{p_t}\lambda_t^A+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial a_{t+1}}\right\}=0 \tag{A.10}$$

w.r.t. σ_t:

$$-\varsigma'\left(\sigma_t\right)(1-n_{t-1})+\lambda_t^N f_t^s\,(1-n_{t-1})\left(\varphi_{t-1}+\overline{\sigma}\left(1-\varphi_{t-1}\right)\right)=0 \tag{A.11}$$

w.r.t. n_t:

$$\left(u\left(c_t^n\right)-\chi\right)-\left(\nu_t u\left(c_t^{ur}\right)+(1-\nu_t)\,u(c_t^{un})\right)+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial n_t}\right\}-\lambda_t^N \tag{A.12}$$
$$-(1-\tau_t)\,D_t n_t^{-2}\lambda_t^{CN}-\lambda_t^A\left((1-\tau_t)\,w_t-\tau_t^u\nu_t-\tau^s(1-\nu_t)-\left(c_t^n-\nu_t c_t^{ur}-(1-\nu_t)\,c_t^{un}\right)\right)=0$$

The envelope conditions are:

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial a_t}=-\frac{(1+i_t)}{p_t}\lambda_t^B \tag{A.13}$$

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial b_t}=\frac{(1+i_t)}{p_t}\lambda_t^B \tag{A.14}$$

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial n_{t-1}}=\varsigma\left(\sigma_t\right)+\lambda_t^N\left(\rho_t-f_t^s\sigma_t\left(\varphi_{t-1}+\overline{\sigma}\left(1-\varphi_{t-1}\right)\right)\right) \tag{A.15}$$

We next solve for the multipliers. In general, which among the inequality constraints are binding will depend on the calibration of the model. We are interested in the solution of the model that implies different consumption levels by employment states. In particular, we calibrate the model to have $\overline{c}^n>\overline{c}^{ur}>\overline{c}^{un}$ and a positive borrowing limit $\overline{b}$.[1] In that case, the liquidity constraints of unemployed workers are binding, while the liquidity constraint of employed is not. This implies $\lambda_t^{CN}=0$. Then, from (A.2), we get

$$\lambda_t^A=-u'\left(c_t^n\right), \tag{A.16}$$

from (A.4) we get

$$\lambda_t^{CUR}=(1-n_t)\,\nu_t\left(u'\left(c_t^{ur}\right)-u'\left(c_t^n\right)\right), \tag{A.17}$$

[1] We also check that the order of consumption levels is preserved in the dynamic simulations.

and from (A.6) we get

$$\lambda_t^{CUN} = (1-n_t)(1-\nu_t)\left(u'\left(c_t^{un}\right) - u'\left(c_t^n\right)\right). \tag{A.18}$$

Substitute these into (A.1) to obtain:

$$\begin{aligned}
\lambda_t^B &= \lambda_t^A - \left(\lambda_t^{CUR} + \lambda_t^{CUN}\right) \\
&= -u'\left(c_t^n\right) - (1-n_t)\left[\nu_t\left(u'\left(c_t^{ur}\right) - u'\left(c_t^n\right)\right) + (1-\nu_t)\left(u'\left(c_t^{un}\right) - u'\left(c_t^n\right)\right)\right] \\
&= -n_t u'\left(c_t^n\right) - (1-n_t)\left(\nu_t u'\left(c_t^{ur}\right) + (1-\nu_t)\, u'\left(c_t^{un}\right)\right)
\end{aligned} \tag{A.19}$$

To solve for λ_t^{BC}, sum (A.8) and (A.10), using also (A.13) and (A.14), to obtain:

$$\begin{aligned}
\lambda_t^{BC} &= \frac{1}{p_t}\lambda_t^A - \frac{1}{p_t}\lambda_t^B \\
&= -\frac{1}{p_t}u'\left(c_t^n\right) + \frac{1}{p_t}\left[n_t u'\left(c_t^n\right) + (1-n_t)\left(\nu_t u'\left(c_t^{ur}\right) + (1-\nu_t)\, u'\left(c_t^{un}\right)\right)\right] \\
&= \frac{1}{p_t}(1-n_t)\left(\nu_t u'\left(c_t^{ur}\right) + (1-\nu_t)\, u'\left(c_t^{un}\right) - u'\left(c_t^n\right)\right) > 0
\end{aligned} \tag{A.20}$$

Because the multiplier λ_t^{BC} is positive, the borrowing constraint must be binding.

To derive the Euler equation, combine (A.10) with (A.13) and use previous results:

$$\begin{aligned}
\frac{1}{p_t}\lambda_t^A &= -\beta E_t\left\{\frac{\partial W\left(n_t, a_{t+1}, b_{t+1}\right)}{\partial a_{t+1}}\right\} \\
\frac{1}{p_t}\lambda_t^A &= \beta E_t\left\{\frac{1+i_{t+1}}{p_{t+1}}\lambda_{t+1}^B\right\} \\
\lambda_t^A &= \beta E_t\left\{\frac{1+i_{t+1}}{\pi_{t+1}}\lambda_{t+1}^B\right\} \\
u'\left(c_t^n\right) &= \beta E_t\left\{\frac{1+i_{t+1}}{\pi_{t+1}}\left[n_{t+1}u'\left(c_{t+1}^n\right) + (1-n_{t+1})\left(\nu_{t+1}u'\left(c_{t+1}^{ur}\right) + (1-\nu_{t+1})\, u'\left(c_{t+1}^{un}\right)\right)\right]\right\}
\end{aligned} \tag{A.21}$$

To derive the optimal search condition, we first solve (A.11) for λ_t^N as:

$$\lambda_t^N = \frac{\varsigma'\left(\sigma_t\right)}{f_t^s\left(\varphi_{t-1} + \overline{\sigma}\left(1-\varphi_{t-1}\right)\right)} \tag{A.22}$$

We then use it in (A.12) together with expressions for other multipliers obtained above and the envelope condition (A.15):

$$\begin{aligned}
&\left(u\left(c_t^n\right) - \chi\right) - \left(\nu_t u\left(c_t^{ur}\right) + (1-\nu_t)\, u(c_t^{un})\right) \\
&+\beta E_t\left\{\varsigma\left(\sigma_{t+1}\right) + \frac{\varsigma'\left(\sigma_{t+1}\right)}{f_{t+1}^s\left(\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right)}\left(\rho_{t+1} - f_{t+1}^s\sigma_{t+1}\left(\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right)\right)\right\} \\
&-\frac{\varsigma'\left(\sigma_t\right)}{f_t^s\left(\varphi_{t-1} + \overline{\sigma}\left(1-\varphi_{t-1}\right)\right)} \\
&+u'\left(c_t^n\right)\left(\left(1-\tau_t\right)w_t - \tau_t^u\nu_t - \tau^s\left(1-\nu_t\right) - \left(c_t^n - \nu_t c_t^{ur} - (1-\nu_t)\, c_t^{un}\right)\right) = 0
\end{aligned} \tag{A.23}$$

$$\beta E_t \left\{ \frac{\varsigma'(\sigma_{t+1})}{f^s_{t+1}(\varphi_t + \overline{\sigma}(1-\varphi_t))} \left(\rho_{t+1} - f^s_{t+1}\sigma_{t+1}(\varphi_t + \overline{\sigma}(1-\varphi_t))\right) \right\} \\ - \frac{\varsigma'(\sigma_t)}{f^s_t(\varphi_{t-1} + \overline{\sigma}(1-\varphi_{t-1}))} \\ + u'(c^n_t)((1-\tau_t) w_t - \xi_t) = 0$$

where the second step uses the definition of ξ_t

We finally derive the discount factor $\Lambda_{t,t+1}$ and the value of an additional employed member to the household $W_{n,t}$, equations (15) and (19) in the main text.

The discount factor is obtained as follows:

$$\begin{aligned} \Lambda_{t,t+1} &\equiv \beta E_t \left\{ \frac{\partial W(n_t, a_{t+1}, b_{t+1})}{\partial D_{t+1}} \Big/ \frac{\partial W(n_{t-1}, a_t, b_t)}{\partial D_t} \right\} \\ &= \beta E_t \left\{ \frac{\frac{(1-\tau_{t+1})}{n_{t+1}} \lambda^{CN}_{t+1} - (1-\tau_{t+1}) \lambda^A_{t+1}}{\frac{(1-\tau_t)}{n_t} \lambda^{CN}_t - (1-\tau_t) \lambda^A_t} \right\} \\ &= \beta E_t \left\{ \frac{(1-\tau_{t+1}) u'(c^n_{t+1})}{(1-\tau_t) u'(c^n_t)} \right\} \end{aligned} \tag{A.24}$$

The value of $W_{n,t}$ is obtained via the following steps:

$$\begin{aligned} W_{n,t} &\equiv \frac{\partial (W(n_{t-1}, a_t, b_t) + (1-n_{t-1}) \varsigma(\sigma_t))}{\partial n_t} \\ &= u(c^n_t) - \chi - (\nu_t u(c^{ur}_t) + (1-\nu_t) u(c^{un}_t)) - \lambda^N_t - (1-\tau_t) D_t n_t^{-2} \lambda^{CN}_t \\ &\quad - [(1-\tau_t) w_t - \tau^u_t \nu_t - \tau^s (1-\nu_t) - c^n_t + \nu_t c^{ur}_t + (1-\nu_t) c^{un}_t] \lambda^A_t \\ &\quad + \beta E_t \left\{ \frac{\partial W(n_t, a_{t+1}, b_{t+1})}{\partial n_t} \right\} \\ &= u(c^n_t) - \chi - (\nu_t u(c^{ur}_t) + (1-\nu_t) u(c^{un}_t)) \\ &\quad + [(1-\tau_t) w_t - \tau^u_t \nu_t - \tau^s (1-\nu_t) - c^n_t + \nu_t c^{ur}_t + (1-\nu_t) c^{un}_t] u'(c^n_t) \\ &\quad + \beta E_t \left\{ \frac{\partial (W(n_t, a_{t+1}, b_{t+1}) + (1-n_t) \varsigma(\sigma_{t+1}) - (1-n_t) \varsigma(\sigma_{t+1}))}{\partial n_t} \right\} \\ &= u'(c^n_t) [(1-\tau_t) w_t - \xi_t] + \beta E_t \left\{ W_{n,t+1} \frac{\partial n_{t+1}}{\partial n_t} \right\} \\ &= u'(c^n_t) [(1-\tau_t) w_t - \xi_t] + \beta E_t \left\{ [\rho_{t+1} - [\varphi_t + \overline{\sigma}(1-\varphi_t)] f^s_{t+1} \sigma_{t+1}] W_{n,t+1} \right\} \end{aligned} \tag{A.25}$$

where we have used:

$$\begin{aligned} \frac{\partial n_{t+1}}{\partial n_t} &= \frac{\partial \left(\rho_{t+1} n_t + f^s_{t+1}(1-n_t) \sigma_{t+1}(\varphi_t + \overline{\sigma}(1-\varphi_t))\right)}{\partial n_t} \\ &= \rho_{t+1} - f^s_{t+1} \sigma_{t+1}(\varphi_t + \overline{\sigma}(1-\varphi_t)) \end{aligned} \tag{A.26}$$

RA Version of the Model

To obtain the representative agent version of our model we remove the liquidity constraints and have the household pool its members' incomes before taking consumption/saving decisions. The problem becomes:

$$W_t\left(n_{t-1},a_t,b_t\right) = \max\left\{n_t\left(u\left(c_t^n\right)-\chi\right)+\left(1-n_t\right)\left(\nu_t u\left(c_t^{ur}\right)+\left(1-\nu_t\right)u(c_t^{un})\right) - \left(1-n_{t-1}\right)\varsigma(\sigma_t)+\beta E_t\left\{W_{t+1}\left(n_t,a_{t+1},b_{t+1}\right)\right\}\right\} \tag{A.27}$$

Subject to:

$$x_t=\frac{b_{t+1}}{p_t}+\left(1+i_t\right)\frac{a_t}{p_t}-\left(1+i_t\right)\frac{b_t}{p_t} \tag{A.28}$$

$$b_{t+1}\leq p_t\overline{b}_t \tag{A.29}$$

$$\frac{a_{t+1}}{p_t} = x_t+\left(1-\tau_t\right)w_t n_t+\left(1-\tau_t\right)d_t n_t+\tau_t^u\left(1-n_t\right)\nu_t+\tau^s\left(1-n_t\right)\left(1-\nu_t\right) - \left(n_t c_t^n+\left(1-n_t\right)\nu_t c_t^{ur}+\left(1-n_t\right)\left(1-\nu_t\right)c_t^{un}\right) \tag{A.30}$$

$$n_t=\rho_t n_{t-1}+f_t^s s_t \tag{A.31}$$

The FOCs are:

w.r.t. x_t:

$$\lambda_t^B-\lambda_t^A=0 \tag{A.32}$$

w.r.t. c_t^n:

$$n_t u'\left(c_t^n\right)+n_t\lambda_t^A=0 \tag{A.33}$$

w.r.t. c_t^{ur}:

$$\left(1-n_t\right)\nu_t u'\left(c_t^{ur}\right)+\left(1-n_t\right)\nu_t\lambda_t^A=0 \tag{A.34}$$

w.r.t. c_t^{un}:

$$\left(1-n_t\right)\left(1-\nu_t\right)u'\left(c_t^{un}\right)+\left(1-n_t\right)\left(1-\nu_t\right)\lambda_t^A=0 \tag{A.35}$$

w.r.t. b_{t+1}:

$$-\frac{1}{p_t}\lambda_t^B-\lambda_t^{BC}+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial b_{t+1}}\right\}=0 \tag{A.36}$$

with

$$\lambda_t^{BC}\left(p_t\overline{b}_t-b_{t+1}\right)=0 \tag{A.37}$$

w.r.t. a_{t+1}:

$$\frac{1}{p_t}\lambda_t^A+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial a_{t+1}}\right\}=0 \tag{A.38}$$

w.r.t. σ_t:

$$-\varsigma'\left(\sigma_t\right)\left(1-n_{t-1}\right)+\lambda_t^N f_t^s\left(1-n_{t-1}\right)\left(\varphi_{t-1}+\overline{\sigma}\left(1-\varphi_{t-1}\right)\right)=0 \tag{A.39}$$

w.r.t. n_t:

$$\left(u\left(c_t^n\right)-\chi\right)-\left(\nu_t u\left(c_t^{ur}\right)+\left(1-\nu_t\right)u(c_t^{un})\right)+\beta E_t\left\{\frac{\partial W\left(n_t,a_{t+1},b_{t+1}\right)}{\partial n_t}\right\}-\lambda_t^N -\lambda_t^A\left(\left(1-\tau_t\right)w_t-\tau_t^u\nu_t-\tau^s\left(1-\nu_t\right)-\left(c_t^n-\nu_t c_t^{ur}-\left(1-\nu_t\right)c_t^{un}\right)\right)=0 \tag{A.40}$$

The envelope conditions are:

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial a_t} = -\frac{\left(1+i_t\right)}{p_t}\lambda_t^B \tag{A.41}$$

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial b_t} = \frac{\left(1+i_t\right)}{p_t}\lambda_t^B \tag{A.42}$$

$$\frac{\partial W\left(n_{t-1},a_t,b_t\right)}{\partial n_{t-1}} = \varsigma\left(\sigma_t\right) + \lambda_t^N\left(\rho_t - f_t^s\sigma_t\left(\varphi_{t-1}+\overline{\sigma}\left(1-\varphi_{t-1}\right)\right)\right) \tag{A.43}$$

The solution implies that consumption in individual states is equalized (since $u'\left(c_t^n\right) = u'\left(c_t^{ur}\right) = u'\left(c_t^{ur}\right) = -\lambda_t^A$), the borrowing constraint is not binding (since $\lambda_t^{BC} = 0$), and a similar optimal search condition subject to a different definition of ξ_t.

A.2 Nash Bargained Wage

Here we derive the expression for the Nash bargained wage in equation (36) in the main text.

The wage bargaining problem reads:

$$w_t^* = \arg\max\left(W_{n,t}\right)^{\eta}\left(F_{n,t}\right)^{1-\eta}, \tag{A.44}$$

where

$$F_{n,t} = q_t z_t - w_t + E_t\left\{\rho_{t+1}\Lambda_{t,t+1}F_{n,t+1}\right\}, \tag{A.45}$$

and

$$W_{n,t} = u'\left(c_t^n\right)\left(1-\tau_t\right)\left(w_t - \frac{\xi_t}{1-\tau_t}\right) + \beta E_t\left\{\left[\rho_{t+1} - \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right]f_{t+1}^s\sigma_{t+1}\right]W_{n,t+1}\right\}. \tag{A.46}$$

The solution of the bargaining problem implies the following sharing rule:

$$\left(1-\tau_t\right)u'\left(c_t^n\right)\eta F_{n,t} = \left(1-\eta\right)W_{n,t}. \tag{A.47}$$

Substitute the expressions for $F_{n,t}$ and $W_{n,t}$ and divide both sides by $\left(1-\tau_t\right)u'\left(c_t^n\right)$:

$$\begin{aligned}&\eta\left(q_t z_t - w_t^* + E_t\left\{\rho_{t+1}\Lambda_{t,t+1}F_{n,t+1}\right\}\right) \\ &= \left(1-\eta\right)\left(\left(w_t^* - \frac{\xi_t}{1-\tau_t}\right) + \frac{1}{\left(1-\tau_t\right)u'\left(c_t^n\right)}\beta E_t\left\{\left[\rho_{t+1} - \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right]f_{t+1}^s\sigma_{t+1}\right]W_{n,t+1}\right\}\right).\end{aligned} \tag{A.48}$$

Use next period sharing rule, given by $W_{n,t+1} = \left(1-\tau_{t+1}\right)u'\left(c_{t+1}^n\right)\frac{\eta}{1-\eta}F_{n,t+1}$:

$$\begin{aligned}&\eta\left(q_t z_t - w_t^* + E_t\left\{\rho_{t+1}\Lambda_{t,t+1}F_{n,t+1}\right\}\right) \\ &= \left(1-\eta\right)\left(\left(w_t^* - \frac{\xi_t}{1-\tau_t}\right) + \beta E_t\left\{\left[\rho_{t+1} - \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right]f_{t+1}^s\sigma_{t+1}\right]\frac{\left(1-\tau_{t+1}\right)u'\left(c_{t+1}^n\right)}{\left(1-\tau_t\right)u'\left(c_t^n\right)}\frac{\eta}{1-\eta}F_{n,t+1}\right\}\right).\end{aligned} \tag{A.49}$$

Use the expression of the discount factor, given by $\Lambda_{t,t+1} = \beta \frac{(1-\tau_{t+1})u'\left(c_{t+1}^n\right)}{(1-\tau_t)u'(c_t^n)}$:

$$\eta \left(q_t z_t - w_t^* + E_t \left\{\rho_{t+1} \Lambda_{t,t+1} F_{n,t+1}\right\}\right) \tag{A.50}$$
$$= (1-\eta) \left(\left(w_t^* - \frac{\xi_t}{1-\tau_t} \right) + E_t \left\{ \left[\rho_{t+1} - \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right] f_{t+1}^s \sigma_{t+1}\right] \frac{\eta}{1-\eta} \Lambda_{t,t+1} F_{n,t+1} \right\} \right).$$

Solve for w_t^* and simplify, using also the firm's FOC at time $t+1$, given by $\kappa = f_{t+1}^v F_{n,t+1}$:

$$w_t^* = \eta \left(q_t z_t + E_t \left\{ \Lambda_{t,t+1} \kappa \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right] \frac{f_{t+1}^s \sigma_{t+1}}{f_{t+1}^v} \right\} \right) + (1-\eta) \frac{\xi_t}{1-\tau_t}, \tag{A.51}$$

which gives equation (36) in the text.

A.3 Equilibrium System

Households:

Euler:

$$u'\left(c_t^n\right) = \beta E_t \left\{ \frac{1+i_{t+1}}{\pi_{t+1}} \left[n_{t+1} u'\left(c_{t+1}^n\right) + \left(1-n_{t+1}\right) \left(\nu_{t+1} u'\left(c_{t+1}^{ur}\right) + \left(1-\nu_{t+1}\right) u'\left(c_{t+1}^{un}\right) \right) \right] \right\} \tag{A.52}$$

Constraints:

$$x_t = \frac{b_{t+1}}{p_t} + (1+i_t) \frac{a_t}{p_t} - (1+i_t) \frac{b_t}{p_t} \tag{A.53}$$

$$c_t^{ur} = x_t + \tau_t^u \tag{A.54}$$

$$c_t^{un} = x_t + \tau^s \tag{A.55}$$

$$\frac{a_{t+1}}{p_t} = x_t + (1-\tau_t) w_t n_t + (1-\tau_t) d_t n_t + \tau_t^u (1-n_t) \nu_t + \tau^s (1-n_t)(1-\nu_t) \tag{A.56}$$
$$- \left(n_t c_t^n + (1-n_t) \nu_t c_t^{ur} + (1-n_t)(1-\nu_t) c_t^{un} \right)$$

Employment accumulation:

$$n_t = \rho_t n_{t-1} + f_t^s s_t \tag{A.57}$$

Total efficiency units of search:

$$s_t = (1-n_{t-1}) \sigma_t \left[\varphi_{t-1} + \overline{\sigma}\left(1-\varphi_{t-1}\right)\right] \tag{A.58}$$

Optimal search effort:

$$\beta E_t \left\{ \frac{\varsigma'\left(\sigma_{t+1}\right)}{f_{t+1}^s \left(\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right)} \left(\rho_{t+1} - f_{t+1}^s \sigma_{t+1} \left(\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right) \right) \right\} \tag{A.59}$$
$$- \frac{\varsigma'\left(\sigma_t\right)}{f_t^s \left(\varphi_{t-1} + \overline{\sigma}\left(1-\varphi_{t-1}\right)\right)}$$
$$+ u'\left(c_t^n\right) \left((1-\tau_t) w_t - \xi_t \right) = 0$$

Assets market equilibrium:

$$\frac{b_{t+1}}{p_t} = \frac{a_{t+1}}{p_t} = \bar{b}_t \tag{A.60}$$

Firms:

Optimal hiring:

$$q_t z_t - w_t + E_t \left\{ \Lambda_{t,t+1} \rho_{t+1} \frac{\kappa}{f^v_{t+1}} \right\} = \frac{\kappa}{f^v_t} \tag{A.61}$$

Dividends definition:

$$d^w_t = q_t z_t n_t - w_t n_t - \kappa v_t \tag{A.62}$$

Desired price:

$$\frac{p^*_t}{p_t} = \frac{p^A_t}{p^B_t} \tag{A.63}$$

with

$$p^A_t = \frac{\epsilon}{(\epsilon - 1)} q_t Y_t + E \left\{ \Lambda_{t,t+1} (1-\theta) (\pi_{t+1})^{\epsilon} p^A_{t+1} \right\} \tag{A.64}$$

and

$$p^B_t = Y_t + E \left\{ \Lambda_{t,t+1} (1-\theta) (\pi_{t+1})^{\epsilon - 1} p^B_{t+1} \right\} \tag{A.65}$$

Inflation:

$$\pi_t = \left(\frac{1-\theta}{1 - \theta \left(\frac{p^*_t}{p_t} \right)^{1-\epsilon}} \right)^{\frac{1}{1-\epsilon}} \tag{A.66}$$

Output:

$$\varsigma_t Y_t = z_t n_t \tag{A.67}$$

Output loss due to price dispersion:

$$\varsigma_t = (1-\theta) s_{t-1} \pi^{\epsilon}_t + \theta \left(\frac{p^*_t}{p_t} \right)^{-\epsilon} \tag{A.68}$$

Total dividends:

$$D_t = Y_t - q_t z_t n_t + d^w_t \tag{A.69}$$

Government:

Government budget constraint:

$$\tau^u_t (1 - n_t) \nu_t + \tau^s (1 - n_t)(1 - \nu_t) = \tau_t w_t n_t + \tau_t d_t n_t \tag{A.70}$$

Taylor rule:

$$1 + i_{t+1} = (1 + \bar{i}) \left(\frac{p_t}{p_{t-1}} \right)^{\phi} e^{\varepsilon_{it}} \tag{A.71}$$

UI rules:

$$\nu_t = \nu^r_t + \nu^e_t \tag{A.72}$$

$$\nu_t^e = \overline{\nu}^e + \Gamma_\nu \log \frac{u_{t-1}}{\overline{u}} + \varepsilon_{\nu,t} \tag{A.73}$$

$$\nu_t^r = \overline{\nu}^r + \Gamma_{r,\varphi} \log \left(\frac{\varphi_{t-1}}{\overline{\varphi}} \right) + \Gamma_{r,u} \log \left(\frac{u_{t-1}}{\overline{u}} \right) + \varepsilon_{r,t} \tag{A.74}$$

$$\tau_t^u = \overline{\tau}^u + \Gamma_\tau \log \frac{u_{t-1}}{\overline{u}} + \varepsilon_{\tau,t} \tag{A.75}$$

Labor Market:

Job finding rate:

$$f_t^s = \alpha_m \left(\frac{v_t}{s_t} \right)^{1-\alpha} \tag{A.76}$$

Job filling rate:

$$f_t^v = \alpha_m \left(\frac{v_t}{s_t} \right)^{-\alpha} \tag{A.77}$$

Share of short-term unemployed:

$$\varphi_t = \frac{u_t^{ST}}{u_t^{LT} + u_t^{ST}} \tag{A.78}$$

Short- and long-term unemployed:

$$u_t^{ST} = u_{t-1}^{ST} (1 - f_t^s)(1 - \delta_t) + n_{t-1} (1 - \rho_t) \tag{A.79}$$

$$u_t^{LT} = u_{t-1}^{LT} (1 - f_t^s \overline{\sigma}) + u_{t-1}^{ST} (1 - f_t^s) \delta_t \tag{A.80}$$

Wages:

Bargained wage:

$$w_t^* = \eta \left(q_t z_t + E_t \left\{ \Lambda_{t,t+1} \kappa \left[\varphi_t + \overline{\sigma} (1 - \varphi_t) \right] \frac{f_{t+1}^s \sigma_{t+1}}{f_{t+1}^v} \right\} \right) + (1 - \eta) \frac{\xi_t}{(1 - \tau_t)} \tag{A.81}$$

Wage schedule:

$$w_t = \gamma w_t^* + (1 - \gamma) \overline{w} \tag{A.82}$$

Shocks:

Productivity:

$$\log(z_t) = (1 - \rho_z) \log(\overline{z}) + \rho_z \log(z_{t-1}) + \sigma_z \epsilon_{zt} \tag{A.83}$$

Separation:

$$\log(\rho_t) = (1 - \rho_\rho) \log(\overline{\rho}) + \rho_\rho \log(\rho_{t-1}) + \sigma_\rho \epsilon_{\rho t} \tag{A.84}$$

Borrowing:

$$\overline{b}_t = (1 - \rho_b) \overline{b} + \rho_b \overline{b}_{t-1} + \sigma_b \epsilon_{bt} \tag{A.85}$$

LTU:

$$\log(\delta_t) = (1 - \rho_\delta) \log(\overline{\delta}) + \rho_\delta \log(\delta_{t-1}) + \sigma_\delta \epsilon_{\delta t} \tag{A.86}$$

Benefits:

$$\varepsilon_{\nu t} = \rho_\nu \varepsilon_{\nu t-1} + \sigma_\nu \epsilon_{\nu t} \tag{A.87}$$

$$\varepsilon_{rt} = \rho_r \varepsilon_{rt-1} + \sigma_r \epsilon_{rt} \tag{A.88}$$

$$\varepsilon_{\tau t} = \rho_\tau \varepsilon_{\tau t-1} + \sigma_\tau \epsilon_{\tau t} \tag{A.89}$$

Monetary policy:

$$\varepsilon_{it} = \rho_i \varepsilon_{it-1} + \sigma_i \epsilon_{it} \tag{A.90}$$

A.4 Calibration of the Disutility of Work χ

The implicit first-order condition for the choice of hours is obtained by augmenting the setup with variable hours of work, h_t, and choosing them to maximize the total surplus. This gives

$$\max_{h_t} \left\{ W_{n,t}\left(h_t\right) + F_{n,t}\left(h_t\right) \right\}, \tag{A.91}$$

where $F_{n,t}\left(h_t\right)$ is given by

$$F_{n,t}\left(h_t\right) = q_t z_t h_t - w_t + E_t \left\{ \rho_{t+1} \Lambda_{t,t+1} F_{n,t+1}\left(h_{t+1}\right) \right\}, \tag{A.92}$$

and $W_{n,t}\left(h_t\right)$ is given by

$$W_{n,t}\left(h_t\right) = u'(c_t^n)\left(1-\tau_t\right)\left(w_t - \frac{\xi_t(h_t)}{1-\tau_t}\right) + \beta E_t \left\{ \left[\rho_{t+1} - \left[\varphi_t + \overline{\sigma}\left(1-\varphi_t\right)\right] f_{t+1}^s \sigma_{t+1}\right] W_{n,t+1}\left(h_{t+1}\right) \right\}, \tag{A.93}$$

with

$$\begin{aligned} \xi_t(h_t) \;\; = \;\; & \nu_t \tau_t^u + \left(1-\nu_t\right)\tau^s + \left[c_t^n - \left(\nu_t c_t^{ur} + \left(1-\nu_t\right) c_t^{un}\right)\right] \\ & + \left(\lambda_t^n\right)^{-1} \left[\left(\nu_t u\left(c_t^{ur}\right) + \left(1-\nu_t\right) u\left(c_t^{un}\right)\right) - U\left(c_t^n, h_t\right)\right] - \left(\lambda_t^n\right)^{-1} \beta E_t \left\{ \varsigma(\sigma_{t+1}) \right\} \end{aligned} \tag{A.94}$$

and $U\left(c_t^n, h_t\right) = u\left(c_t^n\right) - \chi(h_t)$.

The first-order condition reads

$$q_t z_t + \frac{\partial U\left(c_t^n, h_t\right)}{\partial h_t} = 0.$$

Assuming a labor disutility of the form

$$\chi\left(h_t\right) = \frac{\psi \tilde{\chi}}{1+\psi} h_t^{\frac{1+\psi}{\psi}},$$

and evaluating the first-order condition at steady state, gives

$$\tilde{\chi} \overline{h}^{\frac{1}{\psi}} = \overline{q},$$

which can be simplified to $\tilde{\chi} = \overline{q}$, after normalizing $\overline{h}$ to 1. Combining, we finally obtain $\chi = \frac{\psi \overline{q}}{1+\psi}$.