Оценка инфляционных последствий инфраструктурных инвестиций с помощью калиброванной DSGE-модели российской экономики

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1 Main blocks

1.1 Firms

$$y_t = \left[\int_0^1 y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1}$$

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}} \tag{2}$$

$$y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} y_t \tag{3}$$

$$y_t(i) = x_t \tilde{k}_t(i)^{\alpha} l_t(i)^{1-\alpha} (k_t^g)^{\xi} \tag{4}$$

$$\tilde{k}_t(i) = k_t(i) u_t \tag{5}$$

Firm's problem:

$$\min \frac{R_t^K}{P_t}\tilde{k}_t(i) + \frac{W_t}{P_t} l_t(i) \tag{6}$$

$$\text{s.t } y_t(i) = x_t \tilde{k}_t(i)^{\alpha} l_t(i)^{1-\alpha} (k_t^g)^{\xi} \tag{7}$$

$$L = \frac{R_t^K}{P_t}\tilde{k}_t(i) + \frac{W_t}{P_t} l_t(i) - \lambda_t (x_t \tilde{k}_t(i)^{\alpha} l_t(i)^{1-\alpha} (k_t^g)^{\xi} - y_t(i)) \tag{8}$$

$$\frac{\partial L}{\partial \tilde{k}_t(i)} = \frac{R_t^K}{P_t} - \lambda_t \alpha x_t \tilde{k}_t(i)^{\alpha-1} l_t(i)^{1-\alpha} (k_t^g)^{\xi} = 0 \tag{9}$$

$$\frac{\partial L}{\partial l_t(i)} = \frac{W_t}{P_t} - \lambda_t (1-\alpha) x_t \tilde{k}_t(i)^{\alpha} l_t(i)^{-\alpha} (k_t^g)^{\xi} = 0 \tag{10}$$

The FOC's can be rewritten such that:

$$\frac{R_t^K}{P_t} = \lambda_t \alpha \frac{y_t(i)}{\tilde{k}_t(i)} \Leftrightarrow \frac{R_t^K}{P_t}\tilde{k}_t(i) = \lambda_t \alpha y_t(i) \tag{11}$$

$$\frac{W_t}{P_t} = \lambda_t (1-\alpha) \frac{y_t(i)}{l_t(i)} \Leftrightarrow \frac{W_t}{P_t} l_t = \lambda_t (1-\alpha) y_t(i) \tag{12}$$

$$\tag{13}$$

Thus, λ_t can be interpreted as real marginal cost, because:

$$TC_t = \frac{R_t^K}{P_t} + \frac{W_t}{P_t} l_t = \lambda_t (1-\alpha) y_t(i) + \lambda_t \alpha y_t(i) = \lambda_t y_t(i) \tag{14}$$

$$mc_t = \frac{\partial TC_t}{\partial y_t(i)} = \lambda_t \tag{15}$$

Hence, FOCs are:

$$\frac{R_t^K}{P_t} = mc_t \alpha x_t \tilde{k_t}(i)^{\alpha-1} l_t(i)^{1-\alpha} (k_t^g)^{\xi} \tag{16}$$

$$\frac{W_t}{P_t} = mc_t (1-\alpha) x_t \tilde{k_t}(i)^{\alpha} l_t(i)^{-\alpha} (k_t^g)^{\xi} \tag{17}$$

The price setting follows Calvo (1983). The price level evolves according to:

$$P_t = \left[(1-\chi)(P_t^*)^{1-\varepsilon} + \chi(P_{t-1})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{18}$$

$$P_t^{1-\varepsilon} = (1-\chi)(P_t^*)^{1-\varepsilon} + \chi(P_{t-1})^{1-\varepsilon} \tag{19}$$

$$1 = (1-\chi)(p_t^*)^{1-\varepsilon} + \chi \pi_t^{\varepsilon-1} \tag{20}$$

The producer solves:

$$\max \ E_t \sum_{s=0}^{\infty} (\beta\chi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(\frac{P_t^*(i)}{P_{t+s}} - mc_{t+s}) y_{t+s}(i) \right] = \tag{21}$$

$$= E_t \sum_{s=0}^{\infty} (\beta\chi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(\frac{P_t^*(i)}{P_{t+s}} - mc_{t+s}) \left[\frac{P_t^*(i)}{P_{t+s}}\right]^{-\varepsilon} y_{t+s} \right] = \tag{22}$$

$$= E_t \sum_{s=0}^{\infty} (\beta\chi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[\left(\frac{P_t^*(i)}{\Pi_{t,t+s} P_t}\right)^{1-\varepsilon} - \left(\frac{P_t^*(i)}{\Pi_{t,t+s} P_t}\right)^{-\varepsilon} mc_{t+s} \right] y_{t+s} \tag{23}$$

The FOC (we drop i as we consider a symmetric equilibrium):

$$E_t \sum_{s=0}^{\infty} (\beta\chi)^s \lambda_{t+s} \left[(1-\varepsilon)(p_t^*(i))^{-\varepsilon} \Pi_{t,t+s}^{\varepsilon-1} + \varepsilon (p_t^*(i))^{-\varepsilon-1} \Pi_{t,t+s}^{\varepsilon} mc_{t+s}\right] y_{t+s} = 0 \tag{24}$$

$$E_t \sum_{s=0}^{\infty} (\beta\chi)^s \lambda_{t+s} \left[(1-\varepsilon) p_t^*(i) \Pi_{t,t+s}^{\varepsilon-1} + \varepsilon \Pi_{t,t+s}^{\varepsilon} mc_{t+s}\right] y_{t+s} = 0 \tag{25}$$

$$p_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{E_t \sum_{s=0}^{\infty} (\beta\chi)^s \lambda_{t+s} \Pi_{t,t+s}^{\varepsilon} mc_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\chi)^s \lambda_{t+s} \Pi_{t,t+s}^{\varepsilon-1} y_{t+s}} = \frac{\varepsilon}{\varepsilon-1} \frac{g_t^1}{g_t^2} \tag{26}$$

$$g_t^1 = \lambda_t mc_t y_t + \beta\chi E_t \pi_{t+1}^{\varepsilon} g_{t+1}^1 \tag{27}$$

$$g_t^2 = \lambda_t y_t + \beta\chi E_t \pi_{t+1}^{\varepsilon-1} g_{t+1}^2 \tag{28}$$

1.2 Households

1.3 Household choice

The utility function:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t^p(j) + \rho c_t^g(j))^{1-\sigma}}{1-\sigma} + \frac{l_t(j)^{1+\psi}}{1+\psi} + \frac{\kappa}{1-\mu} \left(\frac{M_t(j)}{P_t}\right)^{1-\mu} \right] \tag{29}$$

The budget constraint:

$$P_t c_t(j) + P_t i_t(j) + M_t(j) + B_t(j) = M_{t-1}(j) + R_t B_{t-1}(j) + R_t^k k_t(j) u_t(j) + W_t^*(j) l_t(j) + \Lambda_t(j) - P_t t_t(j) \tag{30}$$

Capital accumulation follows:

$$k_{t+1} = \omega_t i_t + (1-\delta(u_t)) k_t - \frac{\phi}{2} (\frac{i_t}{k_t} - \frac{\delta}{\omega})^2 k_t \tag{31}$$

$$\delta(u_t) = \delta + \phi_1 (u_t - 1) + \frac{\phi_2}{2} (u_t - 1)^2 \tag{32}$$

where ω_t is the MEI.

The Lagrangian for household optimization is:

$$L = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\frac{(c_t^p(j) + \rho c_t^g(j))^{1-\sigma}}{1-\sigma} - \frac{l_t(j)^{1+\psi}}{1+\psi} + \frac{\kappa}{1-\mu} \left(\frac{M_t(j)}{P_t}\right)^{1-\mu}$$
$$- \frac{\lambda_t}{P_t} \left(P_t c_t(j) + P_t i_t(j) + M_t(j) + B_t(j) - M_{t-1}(j) - R_t B_{t-1}(j) - R_t^k k_t(j) u_t(j) - W_t^*(j) l_t(j) - \Lambda_t(j) + P_t t_t(j)\right)$$
$$- \mu_t \left(k_{t+1}(j) - \omega_t i_t(j) - (1 - \delta(u_t)) k_t(j) + \frac{\phi}{2} (\frac{i_t(j)}{k_t(j)} - \frac{\delta}{\omega})^2 k_t(j) \right) \Bigg] \quad (33)$$

FOCs are:

$$\frac{\partial L}{\partial c_t(j)} = 0 \Leftrightarrow (c_t^p(j) + \rho c_t^g(j))^{-\sigma} - \lambda_t = 0 \quad (34)$$
$$\frac{\partial L}{\partial M_t(j)} = 0 \Leftrightarrow \kappa \left(\frac{M_t(j)}{P_t}\right)^{-\mu} \frac{1}{P_t} - \frac{\lambda_t}{P_t} + \beta \frac{\lambda_{t+1}}{P_{t+1}} = 0 \quad (35)$$
$$\frac{\partial L}{\partial B_t(j)} = 0 \Leftrightarrow -\frac{\lambda_t}{P_t} + \beta R_t \frac{\lambda_{t+1}}{P_{t+1}} = 0 \quad (36)$$
$$\frac{\partial L}{\partial u_t(j)} = 0 \Leftrightarrow \frac{\lambda_t}{P_t} R_t^k k_t(j) - \mu_t k_t(j) \delta'(u_t) = 0 \quad (37)$$
$$\frac{\partial L}{\partial i_t(j)} = 0 \Leftrightarrow -\lambda_t + \mu_t \omega_t - \mu_t \phi (\frac{i_t(j)}{k_t(j)} - \delta) = 0 \quad (38)$$
$$\frac{\partial L}{\partial k_{t+1}(j)} = 0 \Leftrightarrow \frac{\lambda_{t+1}}{P_{t+1}} R_{t+1}^k u_{t+1} + \mu_{t+1} \left((1 - \delta(u_{t+1})) + \phi (\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega}) \frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\phi}{2} (\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})^2 \right) - \frac{1}{\beta} \mu_t = 0 \quad (39)$$

Using 34 we get:

$$\lambda_t = (c_t^p(j) + \rho c_t^g(j))^{-\sigma} = c_t(j)^{-\sigma} \quad (40)$$
$$\frac{\lambda_{t+1}}{\lambda_t} = (\frac{c_{t+1}(j)}{c_t(j)})^{-\sigma} \quad (41)$$

From 36:

$$\frac{\lambda_t}{P_t} = \beta R_t \frac{\lambda_{t+1}}{P_{t+1}} \quad (42)$$
$$\lambda_{t+1} = \frac{\lambda_t}{\beta R_t} \pi_{t+1} \quad (43)$$
$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\pi_{t+1}}{\beta R_t} \quad (44)$$

Combining 41 and 44 yields the Euler equation:

$$\frac{\pi_{t+1}}{\beta R_t} = (\frac{c_{t+1}(j)}{c_t(j)})^{-\sigma} \quad (45)$$
$$c_t(j)^{-\sigma} = \beta \frac{R_t}{\pi_{t+1}} c_{t+1}(j)^{-\sigma} \quad (46)$$

35 can be rewritten as:

$$\kappa\left(\frac{M_t(j)}{P_t}\right)^{-\mu} = \lambda_t - \beta\frac{\lambda_{t+1}P_t}{P_{t+1}} \tag{47}$$

$$\kappa\left(\frac{M_t(j)}{P_t}\right)^{-\mu} = \lambda_t - \beta\frac{\frac{\pi_{t+1}\lambda_t}{\beta R_t}P_t}{P_{t+1}} \tag{48}$$

$$\kappa\left(\frac{M_t(j)}{P_t}\right)^{-\mu} = \lambda_t(1-\frac{1}{R_t}) \tag{49}$$

$$(m_t)^{-\mu} = \frac{1}{\kappa}c_t(j)^{\sigma}\frac{R_t}{R_t-1} \tag{50}$$

From 38:

$$-\lambda_t + \mu_t(\omega_t - \phi(\frac{i_t(j)}{k_t(j)} - \delta)) = 0 \tag{51}$$

$$q_t \equiv \frac{\mu_t}{\lambda_t} = (\omega_t - \phi(\frac{i_t(j)}{k_t(j)} - \frac{\delta}{\omega})) \tag{52}$$

From 39:

$$\mu_t = \beta\lambda_{t+1}\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} + \mu_{t+1}\left((1-\delta(u_{t+1})) + \phi(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\phi}{2}(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})^2\right) \tag{53}$$

$$\frac{\mu_t}{\lambda_t} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} + \frac{\mu_{t+1}}{\lambda_t}\frac{\lambda_{t+1}}{\lambda_{t+1}}\left((1-\delta(u_{t+1})) + \phi(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\phi}{2}(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})^2\right) \tag{54}$$

$$q_t = \beta(\frac{c_{t+1}(j)}{c_t(j)})^{-\sigma}\left[\frac{R^k_{t+1}}{P_{t+1}}u_{t+1} + q_{t+1}\left((1-\delta(u_{t+1})) + \phi(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\phi}{2}(\frac{i_{t+1}(j)}{k_{t+1}(j)} - \frac{\delta}{\omega})^2\right)\right] \tag{55}$$

From 37 and 32:

$$\frac{\lambda_t}{P_t}R^k_t k_t(j) = \mu_t k_t(j)\delta'(u_t) \tag{56}$$

$$\frac{R^k_t}{P_t} = q_t\delta'(u_t) \tag{57}$$

$$r^k_t = \frac{R^k_t}{P_t} = q_t(\phi_1 + \phi_2(u_t - 1)) \tag{58}$$

1.3.1 Wage setting

Labour demand:

$$l_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\varepsilon_w} l^D_t \tag{59}$$

$$\tag{60}$$

Household maximizes:

$$E_t\sum_{s=0}^{\infty}(\beta\chi_w)^s\left[-\frac{l_{t+s}(j)^{1+\psi}}{1+\psi} + \lambda_{t+s}w^*_t(j)\Pi^{-1}_{t,t+s}l_{t+s}(j)\right] \tag{61}$$

$$E_t\sum_{s=0}^{\infty}(\beta\chi_w)^s\left[-\frac{1}{1+\psi}(\left[\frac{w^*_t(j)\Pi^{-1}_{t,t+s}}{w_{t+s}}\right]^{-\varepsilon_w} l^D_{t+s})^{1+\psi} + \lambda_{t+s}w^*_t(j)\Pi^{-1}_{t,t+s}\left[\frac{w^*_t(j)\Pi^{-1}_{t,t+s}}{w_{t+s}}\right]^{-\varepsilon_w} l^D_{t+s}\right] \tag{62}$$

$$\tag{63}$$

FOC:

$$0 = E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[\varepsilon_w (\left[\frac{w_t^*(j)}{w_{t+s}}\right]^{-\varepsilon_w} l_{t+s}^D)^{\psi} l_{t+s}^D \left[\frac{w_t^*(j)}{w_{t+s}}\right]^{-\varepsilon_w - 1} \frac{1}{w_{t+s}} \Pi_{t,t+s}^{\varepsilon_w(1+\psi)} + (1-\varepsilon_w) \left[\frac{w_t^*(j)\Pi_{t,t+s}^{-1}}{w_{t+s}}\right]^{-\varepsilon_w} l_{t+s}^D \Pi_{t,t+s}^{-1} \lambda_{t+s} \right] \tag{64}$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[\varepsilon_w (l_{t+s}^D)^{1+\psi} \left[\frac{w_t^*(j)}{w_{t+s}}\right]^{-\varepsilon_w(1+\psi)-1} \frac{1}{w_{t+s}} \Pi_{t,t+s}^{\varepsilon_w(1+\psi)} + (1-\varepsilon_w) \left[\frac{w_t^*(j)}{w_{t+s}}\right]^{-\varepsilon_w} l_{t+s}^D c_{t+s}^{-\sigma} \Pi_{t,t+s}^{\varepsilon_w - 1} \right] \tag{65}$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[\varepsilon_w (l_{t+s}^D)^{1+\psi} w_t^*(j)^{-\varepsilon_w(1+\psi)-1} w_{t+s}^{\varepsilon_w(1+\psi)} \Pi_{t,t+s}^{\varepsilon_w(1+\psi)} + (1-\varepsilon_w) w_t^*(j)^{-\varepsilon_w} w_{t+s}^{\varepsilon_w} l_{t+s}^D c_{t+s}^{-\sigma} \Pi_{t,t+s}^{\varepsilon_w - 1} \right] \tag{66}$$

$$\varepsilon_w w_t^*(j)^{-\varepsilon_w\psi - 1} E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[(l_{t+s}^D)^{1+\psi} w_{t+s}^{\varepsilon_w(1+\psi)} \Pi_{t,t+s}^{\varepsilon_w(1+\psi)} \right] = (\varepsilon_w - 1) E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[w_{t+s}^{\varepsilon_w} l_{t+s}^D c_{t+s}^{-\sigma} \Pi_{t,t+s}^{\varepsilon_w - 1} \right] \tag{67}$$

$$w_t^*(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[(l_{t+s}^D)^{1+\psi} w_{t+s}^{\varepsilon_w(1+\psi)} \Pi_{t,t+s}^{\varepsilon_w(1+\psi)} \right]}{E_t \sum_{s=0}^{\infty} (\beta\chi_w)^s \left[w_{t+s}^{\varepsilon_w} l_{t+s}^D c_{t+s}^{-\sigma} \Pi_{t,t+s}^{\varepsilon_w - 1} \right]} \tag{68}$$

This can be simplified to yield:

$$w_t^*(j)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{g_t^3}{g_t^4} \tag{69}$$

$$g_t^3 = (l_t^D)^{1+\psi} w_t^{\varepsilon_w(1+\psi)} + \beta\chi_w E_t \pi_{t+1}^{\varepsilon_w(1+\psi)} g_{t+1}^3 \tag{70}$$

$$g_t^4 = c_t^{-\sigma} l_t^D w_t^{\varepsilon_w} + \beta\chi_w E_t \pi_{t+1}^{\varepsilon_w - 1} g_{t+1}^4 \tag{71}$$

1.4 Policy

1.4.1 Fiscal policy

The government has the following budget constraint:

$$P_t g_t + B_{t-1} R_t = P_t t_t + B_t + (M_t - M_{t-1}) \tag{72}$$

$$g_t + \frac{R_t}{P_t} B_{t-1} = t_t + \frac{B_t}{P_t} + \underbrace{\frac{M_t - M_{t-1}}{P_t}}_{s_t = m_t - \frac{m_{t-1}}{\pi_t}} \tag{73}$$

$$g_t + \frac{R_t}{\pi_t} b_{t-1} = t_t + b_t + s_t \tag{74}$$

$$b_t = \frac{R_t}{\pi_t} b_{t-1} + g_t - t_t - s_t \tag{75}$$

Lump-sum taxes t_t adjust so that the budget constraint is satisfied:

$$t_t = \psi_b b_{t-1}, \quad \psi_b > \frac{1}{\beta} \tag{76}$$

Then:

$$b_t = (\frac{R_t}{\pi_t} - \psi_b) b_{t-1} + g_t - s_t \tag{77}$$

1.4.2 Monetary policy

We consider two alternative settings:

1. the Central bank targets inflation and adjusts the interst rate via a Taylor rule

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\right]^{1-\rho_i} \tag{78}$$

2. the Central bank allows partial or full monetary financing of government spending, thus deviating from the conventional Taylor rule

$$s_t = \bar{s}\left(\frac{g_t}{g}\right)^{\phi_{MF}} \tag{79}$$

1.5 Aggregation

Goods production (note that $\int_0^1 k_t(i) = k_t u_t$ and $\int_0^1 l_t(i) = l_t^D$, and the capital-labour ratio is constant across firms):

$$y_{it} = x_t l_t(i)^{1-\alpha} k_t(i)^\alpha (k_t^G)^\xi \tag{80}$$

$$\left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} y_t = x_t l_t(i)^{1-\alpha} k_t(i)^\alpha (k_t^G)^\xi \tag{81}$$

$$y_t \int_0^1 \left[\frac{P_t(i)}{P_t}\right]^{-\varepsilon} di = x_t (k_t^G)^\xi \int_0^1 l_t(i) \left(\frac{k_t(i)}{l_t(i)}\right)^\alpha di \tag{82}$$

$$y_t v_t = x_t (l_t^D)^{1-\alpha} (k_t u_t)^\alpha (k_t^G)^\xi \tag{83}$$

$$y_t v_t = x_t (l_t^D)^{1-\alpha} (\tilde{k}_t)^\alpha (k_t^G)^\xi \tag{84}$$

where v_t is the price dispersion term which evolves according to:

$$v_t = (1-\chi)(p_t^*)^{-\varepsilon} + \chi \pi_t^\varepsilon v_{t-1} \tag{85}$$

Labour market:

$$l_t(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\varepsilon_w} l_t^D \tag{86}$$

$$\int_0^1 l_t(j) dj = l_t^D \int_0^1 \left[\frac{w_t(j)}{w_t}\right]^{-\varepsilon_w} dj \tag{87}$$

$$l_t = l_t^D \int_0^1 \left[\frac{w_t(j)}{w_t}\right]^{-\varepsilon_w} dj \tag{88}$$

$$l_t = v_t^w l_t^D \tag{89}$$

where v_t^w is the wage dispersion term which evolves according to:

$$v_t^w = (1-\chi_w)(\frac{w_t^*}{w_t})^{-\varepsilon_w} + \chi_w \pi_t^{\varepsilon_w} (\frac{w_{t-1}}{w_t})^{-\varepsilon_w} v_{t-1} \tag{90}$$

Wage evolves according to:

$$w_t^{1-\varepsilon_w} = (1-\chi_w)(w_t^*)^{1-\varepsilon_w} + \chi_w \pi_t^{\varepsilon_w - 1} w_{t-1}^{1-\varepsilon_w} \tag{91}$$

2 Equilibrium conditions

$$y_t = c_t + i_t + g_t \tag{92}$$
$$y_t v_t = x_t \tilde{k_t}^{\alpha} (l_t^D)^{1-\alpha} (k_t^g)^{\xi} \tag{93}$$
$$v_t = (1-\chi)(p_t^*)^{-\varepsilon} + \chi \pi_t^{\varepsilon} v_{t-1} \tag{94}$$
$$l_t = v_t^w l_t^D \tag{95}$$
$$v_t^w = (1-\chi_w)(\frac{w_t^*}{w_t})^{-\varepsilon_w} + \chi_w \pi_t^{\varepsilon_w} (\frac{w_{t-1}}{w_t})^{-\varepsilon_w} v_{t-1}^w \tag{96}$$
$$\tilde{k_t} = k_t u_t \tag{97}$$
$$r_t^k = mc_t \alpha x_t \tilde{k_t}^{\alpha-1} (l^D)_t^{1-\alpha} (k_t^g)^{\xi} \tag{98}$$
$$w_t = mc_t (1-\alpha) x_t \tilde{k_t}^{\alpha} (l^D)_t^{-\alpha} (k_t^g)^{\xi} \tag{99}$$
$$1 = (1-\chi)(p_t^*)^{1-\varepsilon} + \chi \pi_t^{\varepsilon-1} \tag{100}$$
$$p_t^* = \frac{\varepsilon}{\varepsilon-1} \frac{g_t^1}{g_t^2} \tag{101}$$
$$g_t^1 = c_t^{-\sigma} mc_t y_t + \beta \chi E_t \pi_{t+1}^{\varepsilon} g_{t+1}^1 \tag{102}$$
$$g_t^2 = c_t^{-\sigma} y_t + \beta \chi E_t \pi_{t+1}^{\varepsilon-1} g_{t+1}^2 \tag{103}$$
$$k_{t+1} = \omega_t i_t + (1-\delta(u_t))k_t - \frac{\phi}{2}(\frac{i_t}{k_t} - \delta)^2 k_t \tag{104}$$
$$\delta(u_t) = \delta + \phi_1(u_t - 1) + \frac{\phi_2}{2}(u_t-1)^2 \tag{105}$$
$$(w_t^*)^{1+\varepsilon_w \psi} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{g_t^3}{g_t^4} \tag{106}$$
$$g_t^3 = (l_t^D)^{1+\psi} w_t^{\varepsilon_w(1+\psi)} + \beta \chi_w E_t \pi_{t+1}^{\varepsilon_w(1+\psi)} g_{t+1}^3 \tag{107}$$
$$g_t^4 = c_t^{-\sigma} l_t^D w_t^{\varepsilon_w} + \beta \chi_w E_t \pi_{t+1}^{\varepsilon_w - 1} g_{t+1}^4 \tag{108}$$
$$w_t^{1-\varepsilon_w} = (1-\chi_w)(w_t^*)^{1-\varepsilon_w} + \chi_w \pi_t^{\varepsilon_w - 1} w_{t-1}^{1-\varepsilon_w} \tag{109}$$
$$c_t = c_t^p + \rho c_t^g \tag{110}$$
$$c_t^{-\sigma} = \beta \frac{R_t}{\pi_{t+1}} c_{t+1}^{-\sigma} \tag{111}$$
$$(m_t)^{-\mu} = \frac{1}{\kappa} c_t^{\sigma} \frac{R_t}{R_t - 1} \tag{112}$$
$$q_t = (\omega_t - \phi(\frac{i_t}{k_t} - \frac{\delta}{\omega})) \tag{113}$$
$$q_t = \beta(\frac{c_{t+1}}{c_t})^{-\sigma} \left[r_{t+1}^k u_{t+1} + q_{t+1} \left((1-\delta(u_{t+1})) + \phi(\frac{i_{t+1}}{k_{t+1}} - \frac{\delta}{\omega})\frac{i_{t+1}}{k_{t+1}} - \frac{\phi}{2}(\frac{i_{t+1}}{k_{t+1}} - \frac{\delta}{\omega})^2 \right) \right] \tag{114}$$
$$r_t^k = q_t(\phi_1 + \phi_2(u_t - 1)) \tag{115}$$
$$b_t = (\frac{R_t}{\pi_t} - \psi_b) b_{t-1} + g_t - s_t \tag{116}$$
$$s_t = m_t - \frac{m_{t-1}}{\pi_t} \tag{117}$$
$$k_{t+1}^g = i_{t-4}^g + (1-\delta^g) k_t^g \tag{118}$$
$$g_t = c_t^g + \sum_{s=0}^{4} \omega^i i_{t-s}^g \tag{119}$$
$$\frac{i_t^g - i^g}{y} = \rho_{i,g} \frac{i_{t-1}^g - i^g}{y} + \varepsilon_{t,ig} \tag{120}$$
$$\frac{c_t^g - c^g}{y} = \rho_{c,g} \frac{c_{t-1}^g - c^g}{y} + \varepsilon_{t,cg} \tag{121}$$
$$\log(x_t) = \rho_x \log(x_{t-1}) + \varepsilon_{t,x} \tag{122}$$
$$\log(\omega_t) = \rho_\omega \log(\omega_{t-1}) + (1-\rho_\omega)\log(\omega) + \varepsilon_{t,\omega} \tag{123}$$

This system is complemented with either the Taylor rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_i} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\right]^{1-\rho_i} \tag{124}$$

$$\tag{125}$$

or an expression defining the level of segniorage allowed by the Central bank:

$$s_t = \bar{s}\left(\frac{g_t}{g}\right)^{\phi_{MF}} \tag{126}$$

The system has 33 equations, 4 exogenous processes $x_t, i_t^g, c_t^g, \omega_t$, and 29 endogenous variables

$$y_t, c_t, i_t, g_t, v_t, v_t^w, k_t, u_t, \tilde{k}_t, l_t, l_t^D, k_t^g, r_t^k, w_t, w_t^*, g_t^3, g_t^4, mc_t, p_t^*, \pi_t, g_t^1, g_t^2, \delta_t, c_t^p, m_t, R_t, q_t, b_t, s_t$$

3 Steady-state solution

For exogenous processes (equations 121, 120, 123, 122):

$$\frac{i^g}{y} = y_{ig} \tag{127}$$

$$\frac{i^g}{y} = y_{cg} = y_g - y_{ig} \tag{128}$$

$$x = 1 \tag{129}$$

$$\omega = \bar{\omega} \tag{130}$$

Steady-state nominal interest rate (equation 111):

$$R = \frac{\pi}{\beta} \tag{131}$$

Normalize capital utilization to 1 in steady state (equation 105):

$$u = 1 \tag{132}$$

$$\delta = \bar{\delta} \tag{133}$$

This implies that (from equation 115):

$$r^k = q(\phi_1 + \phi_2(u-1)) \tag{134}$$

$$\phi_1 = \frac{r^k}{q} \tag{135}$$

From capital accumulation (equation 104 and equation 118) it follows that:

$$\frac{i}{k} = \frac{\delta}{\omega} \tag{136}$$

$$\frac{i^g}{k^g} = \delta^g \tag{137}$$

$$k^g = \frac{y_{ig} y}{\delta^g} \tag{138}$$

Investment FOC (equation 113) yields:

$$q = \omega \tag{139}$$

Capital FOC (equation 114) yields:

$$q = \beta(r^k + q(1-\delta)) \tag{140}$$
$$q(1-\beta(1-\delta)) = \beta r^k \tag{141}$$
$$r^k = \frac{q}{\beta} - q(1-\delta) \tag{142}$$

Steady-state seigniorage is positive as long as there is positive trend inflation (equation 117):

$$s = m(1 - \frac{1}{\pi}) \tag{143}$$

Optimal reset price (equation 100):

$$p^* = \left(\frac{1-\chi\pi^{\varepsilon-1}}{1-\chi}\right)^{\frac{1}{1-\varepsilon}} \tag{144}$$

Price dispersion (equation 94):

$$v = \frac{1-\chi}{1-\chi\pi^{\epsilon}}(p^*)^{-\epsilon} \tag{145}$$

From equations 102 and 103:

$$g^1 = yc^{-\sigma}mc + \beta\chi\pi^{\varepsilon}g^1 \tag{146}$$
$$g^1 = \frac{yc^{-\sigma}mc}{1-\beta\chi\pi^{\varepsilon}} \tag{147}$$
$$g^2 = c^{-\sigma}y + \beta\chi\pi^{\varepsilon-1}g^2 \tag{148}$$
$$g^2 = \frac{c^{-\sigma}y}{1-\beta\chi\pi^{\varepsilon-1}} \tag{149}$$
$$\tag{150}$$

Hence,

$$\frac{g^1}{g^2} = \frac{yc^{-\sigma}mc}{c^{-\sigma}y}\frac{1-\beta\chi\pi^{\varepsilon-1}}{1-\beta\chi\pi^{\varepsilon}} \tag{151}$$
$$\frac{g^1}{g^2} = mc\frac{1-\beta\chi\pi^{\varepsilon-1}}{1-\beta\chi\pi^{\varepsilon}} \tag{152}$$
$$\tag{153}$$

Then, from 101:

$$p^* = \frac{\varepsilon}{\varepsilon-1}\frac{g^1}{g^2} = mc\frac{\varepsilon}{\varepsilon-1}\frac{1-\beta\chi\pi^{\varepsilon-1}}{1-\beta\chi\pi^{\varepsilon}} \tag{154}$$
$$mc = p^*\frac{\varepsilon-1}{\varepsilon}\frac{1-\beta\chi\pi^{\varepsilon}}{1-\beta\chi\pi^{\varepsilon-1}} \tag{155}$$

Capital-labour ratio can be derived by combining equations 99 and 98:

$$\frac{r^k}{w} = \frac{\alpha}{1-\alpha}\frac{l^D}{\tilde{k}} \tag{156}$$
$$\frac{l^D}{\tilde{k}} = \frac{1-\alpha}{\alpha}\frac{r^k}{w} \tag{157}$$

Express marginal cost from 98:

$$mc = \frac{r^k}{\alpha x \tilde{k}^{\alpha-1} l^{1-\alpha} (k^g)^{\xi}} \tag{158}$$

$$mc = \alpha^{-1} r^k \left(\frac{l}{\tilde{k}}\right)^{\alpha-1} (k^g)^{-\xi} = \alpha^{-1} r^k \left(\frac{1-\alpha}{\alpha}\frac{r^k}{w}\right)^{\alpha-1} (k^g)^{-\xi} \tag{159}$$

$$= \alpha^{-\alpha}(1-\alpha)^{\alpha-1}(r^k)^{\alpha} w^{1-\alpha} (k^g)^{-\xi} \tag{160}$$

This expression allows to pin down steady-state wage:

$$w = \left(\alpha^{\alpha}(1-\alpha)^{1-\alpha}(r^k)^{-\alpha}(k^g)^{\xi} mc\right)^{\frac{1}{1-\alpha}} \tag{161}$$

From equation 109:

$$w^{1-\varepsilon_w} = (1-\chi_w)(w^*)^{1-\varepsilon_w} + \chi_w \pi^{\varepsilon_w - 1} w^{1-\varepsilon_w} \tag{162}$$

$$w^{1-\varepsilon_w} = \frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}} (w^*)^{1-\varepsilon_w} \tag{163}$$

$$w = \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1}{1-\varepsilon_w}} w^* \tag{164}$$

$$w^* = \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1}{\varepsilon_w-1}} w \tag{165}$$

From equations 107 and 108:

$$g^3 = (l^D)^{1+\psi} w^{\varepsilon_w(1+\psi)} + \beta\chi_w \pi^{\varepsilon_w(1+\psi)} g^3 \tag{166}$$

$$g^3 = \frac{(l^D)^{1+\psi} w^{\varepsilon_w(1+\psi)}}{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}} \tag{167}$$

$$g^4 = c^{-\sigma} l^D w^{\varepsilon_w} + \beta\chi_w \pi^{\varepsilon_w-1} g^4 \tag{168}$$

$$g^4 = \frac{c^{-\sigma} l^D w^{\varepsilon_w}}{1-\beta\chi_w \pi^{\varepsilon_w-1}} \tag{169}$$

$$\frac{g^3}{g^4} = \frac{(l^D)^{1+\psi} w^{\varepsilon_w(1+\psi)}}{c^{-\sigma} l^D w^{\varepsilon_w}} \frac{1-\beta\chi_w \pi^{\varepsilon_w-1}}{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}} = \frac{(l^D)^{\psi} w^{\varepsilon_w \psi}}{c^{-\sigma}} \frac{1-\beta\chi_w \pi^{\varepsilon_w-1}}{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}} \tag{170}$$

Plug this result into equation 106:

$$(w^*)^{1+\varepsilon_w\psi} = \frac{\varepsilon_w}{\varepsilon_w-1}\frac{g^3}{g^4} = \frac{\varepsilon_w}{\varepsilon_w-1}\frac{(l^D)^{\psi} w^{\varepsilon_w \psi}}{c^{-\sigma}} \frac{1-\beta\chi_w \pi^{\varepsilon_w-1}}{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}} \tag{171}$$

Use the result for w^*:

$$\left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1+\varepsilon_w\psi}{\varepsilon_w-1}} w = \frac{\varepsilon_w}{\varepsilon_w-1}\frac{(l^D)^{\psi}}{c^{-\sigma}} \frac{1-\beta\chi_w \pi^{\varepsilon_w-1}}{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}} \tag{172}$$

$$(l^D)^{\psi} = c^{-\sigma} w \frac{\varepsilon_w-1}{\varepsilon_w} \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1+\varepsilon_w\psi}{\varepsilon_w-1}} \frac{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}}{1-\beta\chi_w \pi^{\varepsilon_w-1}} \tag{173}$$

$$(l^D)^{\psi} = (y\cdot y_c)^{-\sigma} w \frac{\varepsilon_w-1}{\varepsilon_w} \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1+\varepsilon_w\psi}{\varepsilon_w-1}} \frac{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}}{1-\beta\chi_w \pi^{\varepsilon_w-1}} \tag{174}$$

$$(l^D)^{\psi} = ((\frac{\tilde{k}}{l^D})^{\alpha} l^D (k^g)^{\xi} v^{-1}\cdot y_c)^{-\sigma} w \frac{\varepsilon_w-1}{\varepsilon_w} \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1+\varepsilon_w\psi}{\varepsilon_w-1}} \frac{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}}{1-\beta\chi_w \pi^{\varepsilon_w-1}} \tag{175}$$

$$l^D = \left(((\frac{\tilde{k}}{l^D})^{\alpha} (k^g)^{\xi} v^{-1}\cdot y_c)^{-\sigma} w \frac{\varepsilon_w-1}{\varepsilon_w} \left(\frac{1-\chi_w}{1-\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1+\varepsilon_w\psi}{\varepsilon_w-1}} \frac{1-\beta\chi_w \pi^{\varepsilon_w(1+\psi)}}{1-\beta\chi_w \pi^{\varepsilon_w-1}}\right)^{\frac{1}{\psi+\sigma}} \tag{176}$$

Then steady-state output, as well as consumption, investment, and government spending can be determined from equation 93 and the respective shares y_c, y_i and y_g:

$$y = v^{-1}(k^g)^{\xi}(\frac{\tilde{k}}{l^D})^{\alpha} l^D \tag{177}$$

$$c = y_c y \tag{178}$$

$$i = y_i y \tag{179}$$

$$g = y_g y \tag{180}$$

Since we know y, c and l^D, we also know g^1, g^2, g^3 and g^4.

From equation 112 steady-state money demand is:

$$m = \left(\frac{1}{\kappa} c^{\sigma} \frac{R}{R-1}\right)^{-\frac{1}{\mu}} \tag{181}$$

To calibrate money-to-GDP ratio κ has to be set:

$$\frac{m}{y} = \kappa^{\frac{1}{\mu}} \left(c^{\sigma} \frac{R}{R-1}\right)^{-\frac{1}{\mu}} \frac{1}{y} \tag{182}$$

$$\kappa^{\frac{1}{\mu}} = \frac{m}{y} y \left(c^{\sigma} \frac{R}{R-1}\right)^{\frac{1}{\mu}} \tag{183}$$

$$\kappa = (\frac{m}{y} y)^{\mu} \left(c^{\sigma} \frac{R}{R-1}\right) \tag{184}$$

Government debt in the steady state has to satisfy:

$$b = (\frac{R}{\pi} - \psi_b) b + g - s \tag{185}$$

$$b = \frac{g - s}{1 - \frac{R}{\pi} + \psi_b} = \frac{y_g y - m(1 - \frac{1}{\pi})}{1 - \frac{R}{\pi} + \psi_b} \tag{186}$$

To ensure a pre-specified debt-to-GDP ratio in the steady state, ψ_b should be set:

$$\frac{b}{y} = \frac{y_g - \frac{m}{y}(1 - \frac{1}{\pi})}{1 - \frac{R}{\pi} + \psi_b} \tag{187}$$

$$\frac{b}{y}(1 - \frac{R}{\pi} + \psi_b) = y_g - \frac{m}{y}(1 - \frac{1}{\pi}) \tag{188}$$

$$\frac{b}{y}(1 - \frac{R}{\pi}) + \psi_b \frac{b}{y} = y_g - \frac{m}{y}(1 - \frac{1}{\pi}) \tag{189}$$

$$\psi_b = \frac{y_g - \frac{m}{y}(1 - \frac{1}{\pi}) - \frac{b}{y}(1 - \frac{R}{\pi})}{\frac{b}{y}} \tag{190}$$