**EXERCISE.**

# 1 The model

Consider the following model.

Households allocate their resources between consumption $C_t$, investments $I_t$ and government-issued bonds $B_t$. They receive income from labor services $W_t h_t$, from profits $D_t$, from renting capital services $K_t$ at the rate $R_t^k$ and from holding government bonds.

Households' utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

Households' budget constraint (in nominal terms) is:

$$P_t C_t + P_t I_t + B_t = R_{t-1} B_{t-1} + W_t h_t + D_t + R_t^k K_{t-1} - P_t T_t \tag{2}$$

Capital accumulation equation:

$$K_t = (1-\delta) K_{t-1} + I_t \tag{3}$$

$P_t$ is the consumption price index $R_t$ is the (gross) nominal interest rate, $K_t$ is the physical capital stock and $T_t$ are lump-sum taxes. $W_t$ is the nominal.

The final good $Y_t$ is produced under perfect competition. Intermediate firms are monopolistically competitive and use as inputs capital and labor services, $K_t$ and $h_t$ respectively. The production technology is:

$$Y_t = K_{t-1}^{\alpha} h_t^{1-\alpha}$$

Firms maximize their (nominal) profits $D_t = P_t Y_t - W_t h_t - R_t^k K_{t-1}$ subject to the technology constraint, choosing how much inputs $K_t$ and $h_t$ to use.

Following Calvo (1983), each firm may reset its price only with probability $1-\xi_p$ in any given period, independently of the time elapsed since the last adjustment. Thus, each period a measure $1-\xi_p$ of producers reset their prices, while a fraction keep their prices unchanged. As a result, the average duration of a price is given by $(1-\xi_p)^{-1}$. In this context, $\xi_p$ becomes a natural index of price stickiness.

The government budget constraint is:

$$R_{t-1} B_{t-1} = B_t + P_t T_t$$

The monetary authority sets the nominal interest rate according to a Taylor rule.

## 2 First order conditions

The first order conditions obtained from this model are the following.

From the households' problem, we obtain:

$$C_t^{-\sigma} = \lambda_t \tag{4}$$

$$R_t = \pi_{t+1}^p \frac{\lambda_t}{\beta \lambda_{t+1}} \tag{5}$$

$$\frac{\lambda_{t+1}}{\lambda_t} \beta \left[ r_{t+1}^k + (1-\delta) \right] = 1 \tag{6}$$

$$K_t = (1-\delta) K_{t-1} + I_t \tag{7}$$

$$W_t = C_t^\sigma h_t^\varphi = \frac{h_t^\varphi}{\lambda_t} \tag{8}$$

From the firms' problem, we obtain:

$$\frac{K_{t-1}}{h_t} = \frac{\alpha}{(1-\alpha)} \frac{w_t}{r_t^k} \tag{9}$$

$$mc_t = \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} \left( r_t^k \right)^\alpha w_t^{1-\alpha} \tag{10}$$

$$Y_t = K_{t-1}^\alpha h_t^{1-\alpha} \tag{11}$$

The following 4 equations define price dynamics:

$$1 = \xi_p \left( \frac{1}{\pi_t^p} \right)^{1-\varepsilon_p} + (1-\xi_p) \left( \frac{P_t^*}{P_t} \right)^{1-\varepsilon_p} \tag{12}$$

$$\frac{P_t^*}{P_t} = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{\psi_t^p}{\phi_t^p} \tag{13}$$

$$\psi_t^p = \lambda_t Y_t mc_t + \beta \xi_p E_t \pi_{t+1}^{\varepsilon_p} \psi_{t+1}^p \tag{14}$$

$$\phi_t^p = \lambda_t Y_t + \beta \xi_p E_t \pi_{t+1}^{\varepsilon_p - 1} \phi_{t+1}^p \tag{15}$$

Market clearing:

$$Y_t = C_t + I_t \tag{16}$$

Taylor rule:

$$\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\phi_R} \left[ \left( \frac{\pi_t^p}{\pi^p} \right)^{\phi_\pi} \left( \frac{Y_t}{Y} \right)^{\phi_y} \right]^{1-\phi_R} \tag{17}$$

where $\lambda_t$ is the Lagrange multiplier for the problem of households (real terms budget constraint), $\pi_t^p$ is gross price inflation, $w_t$ is the real wage and $r_t^k$ is th real price of capital.

# 3 Steady state

The steady state of the model is derived as follows:

$$\pi^p = 1$$

$$R = \frac{\pi^p}{\beta}$$

$$r^k = \frac{1}{\beta} - (1 - \delta)$$

$$\frac{I}{K} = \delta$$

$$mc = \frac{\varepsilon_p - 1}{\varepsilon_p}$$

$$\frac{Y}{K} = \frac{r^k}{\alpha} \frac{1}{mc}$$

$$\frac{I}{Y} = \frac{\frac{I}{K}}{\frac{Y}{K}}$$

$$\frac{C}{Y} = 1 - \frac{I}{Y}$$

$$P^* = P$$

$$w = \left[ mc \frac{\alpha^\alpha \left(1 - \alpha\right)^{1-\alpha}}{\left(r^k\right)^\alpha} \right]^{\frac{1}{1-\alpha}}$$

$$\frac{K}{h} = \frac{\alpha}{1 - \alpha} \frac{w}{r^k}$$

$$\frac{Y}{K} = \left(\frac{K}{h}\right)^{-(1-\alpha)}$$

$$\frac{C}{h} = \frac{Y}{K} \frac{K}{h} - \delta \frac{K}{h} = \left(\frac{K}{h}\right)^\alpha - \delta \frac{K}{h}$$

# 4 Log-linearized model

The set of log-linearized equations is the following:

$$-\sigma \hat{C}_t = \hat{\lambda}_t$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \hat{R}_t - \hat{\pi}^p_{t+1}$$

$$\hat{\lambda}_t = \hat{\lambda}_{t+1} + \left[1 - \beta\left(1-\delta\right)\right] \hat{r}^k_{t+1}$$

$$\hat{K}_t = \left(1-\delta\right) \hat{K}_{t-1} + \delta \hat{I}_t$$

$$\widehat{w}_t = \sigma \hat{C}_t + \varphi \hat{h}_t$$

$$\hat{K}_{t-1} - \hat{h}_t = \hat{w}_t - \hat{r}^k_t$$

$$\widehat{mc}_t = \alpha \hat{r}^k_t + \left(1-\alpha\right) \hat{w}_t$$

$$\hat{Y}_t = \alpha \hat{K}_{t-1} + \left(1-\alpha\right) \hat{h}_t$$

$$\hat{\pi}^p_t = \beta \hat{\pi}^p_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p} \widehat{mc}_t$$

$$\frac{C}{Y}\hat{C}_t + \frac{I}{Y}\hat{I}_t = \hat{Y}_t$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + \left(1-\phi_R\right)\left(\phi_\pi \hat{\pi}^p_t + \phi_y \hat{Y}_t\right)$$

# 5 Exercise

Suppose there are distortionary taxes on capital $\left(\tau^k_t\right)$. The budget constraint of households is modified as follows:

$$P_t C_t + P_t I_t + B_t = R_{t-1} B_{t-1} + W_t h_t + D_t + \left(1 - \tau^k_t\right) R^k_t K_{t-1} + \tau^k_t \delta P_t K_{t-1} - P_t T_t$$

Then also the government budget constraint is modified:

$$R_{t-1} B_{t-1} = B_t + P_t T_t + \tau^k_t \left(R^k_t - \delta P_t\right) K_{t-1}$$

Looking at Forni, Monteforte and Sessa (2009), suppose a fiscal feedback rule for $\tau^k_t$ such that:

$$\frac{\tau^k_t}{\tau^k} = \left(\frac{\tau^k_{t-1}}{\tau^k}\right)^{\rho_k} \left(\frac{b_{t-1}}{b}\right)^{\phi_{\tau,b}} \left(\frac{y_t}{y}\right)^{\phi_{\tau,y}} \varepsilon^k_t$$

where $b_t = B_t/P_t$ and $\varepsilon^k_t \sim i.i.d.N\left(0, \sigma^2_h\right)$ is an exogenous shock.

1. Looking at the literature, derive the model and calibrate parameters (the steady state of the model changes accordingly). Using the log-linearized model and Dynare, simulate a shock to the *capital tax rate*. Briefly comment the results.

2. Assume now that the law of motion of taxation is INDEPENDENT from $y$ and $b$. Repeat point 1.

You can refer to the papers assigned (not EVERY paper given is useful for your exercise in particular) or to other papers or references you can find.

When the exercise is completed, please send an email to bianca.barbaro@unimib.it attaching:

1. a *pdf* file with the solution of the exercise, highlighting in particular what is different from the standard model, and comments about the impulse response functions you get from the simulation of the model.

2. the *mod* file you used for the simulation with Dynare