

A RANK with a durable good

$$\begin{aligned}
\left. \begin{aligned}
\frac{1-\alpha}{C_t} &= \beta E_t \left\{ \frac{v_{C,t+1} + D_{C,t+1}}{v_{C,t}} \frac{1-\alpha}{C_{t+1}} \right\} \\
\frac{1-\alpha}{C_t} &= \beta E_t \left\{ \frac{v_{K,t+1} + D_{K,t+1}}{v_{K,t}} \frac{1-\alpha}{C_{t+1}} \right\} \\
\frac{1-\alpha}{C_t} &= \beta E_t \left\{ \frac{1+i_t}{1+\pi_{t+1}} \frac{1-\alpha}{C_{t+1}} \right\} \\
q_t \frac{1-\alpha}{C_t} &= \beta E_t \left\{ (1-\delta)(q_{t+1}) \frac{1-\alpha}{C_{t+1}} + \frac{\alpha}{K_{t+1}} \right\}
\end{aligned} \right\} \text{Household side} \\
\left. \begin{aligned}
C_t &= W_{r,t} N_t - Z_{t+1} + \frac{1+i_t-1}{1+\pi_t} B_t - v_{C,t}(\omega_{C,t+1} - \omega_{C,t}) + \omega_{C,t} D_{C,t} + \\
&\quad - q_t [K_{t+1} - (1-\delta)K_t] + \omega_{K,t} D_{K,t} - v_{K,t}(\omega_{K,t+1} - \omega_{K,t})
\end{aligned} \right\} \text{BC} \\
\nu(N_t)^\varphi &= \frac{1-\alpha}{C_t} W_{r,t} \left. \right\} \text{Labor supply} \\
\left. \begin{aligned}
\pi_{C,t}(1-\pi_{C,t}) &= \frac{\epsilon_C-1}{\psi} \left[\frac{\epsilon_C}{\epsilon_C-1} W_{r,t} - (1+\tau_C^S) \right] \\
D_{C,t} &= \left((1+\tau_C^S) Y_t - W_{r,t} N_{C,t} - \frac{\psi}{2} \pi_{C,t}^2 Y_t - \tau_C^S Y_t \right) \\
Y_t &= N_{C,t} \\
q_t &= \frac{\epsilon_K}{(\epsilon_K-1)(1+\tau_K^S)} W_{r,t} \\
q_t D_{K,t} &= q_t X_t^S - W_{r,t} N_{K,t} \\
X_t^S &= N_{K,t}
\end{aligned} \right\} \text{Firms side} \\
\left. \begin{aligned}
(1 - \frac{\psi}{2} \pi_t^2) Y_t &= C_t \\
X_t^D &= X_t^S \\
\omega_{C,t+1} &= \omega_{C,t} = 1 \\
\omega_{K,t+1} &= \omega_{K,t} = 1 \\
N_t &= N_{C,t} + N_{K,t}
\end{aligned} \right\} \text{Market equilibria} \\
\left. \begin{aligned}
B_{t+1} &= Z_{t+1} \\
Z_{t+1} &= 0
\end{aligned} \right\} \text{Money Balances} \\
1 + i_t &= \beta_t^{-1} \left(\frac{1+\pi_{C,t}}{1+\pi} \right)^\phi i_t^* \left. \right\} \text{Monetary policy} \\
K_{t+1} &= (1-\delta)K_t + X_t^S \left. \right\} \text{Durable good's evolution} \\
\log(i_t^*) &= i_0^* + \rho \log(i_{t-1}^*) + \epsilon_t \left. \right\} \text{AR(1) process for monetary shock}
\end{aligned}$$

Endogenous Variables: $\{C_t, K_{t+1}, Z_{t+1}, B_t, \omega_{C,t+1}, \omega_{K,t+1}, N_t, N_{C,t}, N_{K,t}, W_{r,t}, v_{C,t}, v_{K,t}, q_t, \pi_{C,t}, D_{C,t}, D_{K,t}, Y_t, i_t, X_t^D, X_t^S, i_t^*\}$.

Exogenous Variable: $\{\epsilon_t\}$, $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

B List of variables and parameters

C_t \equiv consumption by representative (rep) agent;

Z_{t+1} \equiv liquidity held at the end of period t by rep agent;

$\omega_{C,t+1}$ \equiv end of period t shares of firms producing ndg detained by rep agent (in fixed supply);

$\omega_{K,t+1}$ \equiv end of period t shares of firms producing dg detained by rep agent (in fixed supply);

N_t \equiv labor supply by rep agent;

$N_{C,t}$ \equiv labor demand by firms producing ndg;

$N_{K,t}$ \equiv labor demand by firms producing dg;

$W_{r,t}$ \equiv real wage in terms of price of ndg;

$v_{C,t}$ \equiv price of a single firm share producing ndg;

$v_{K,t}$ \equiv price of a single firm share producing dg;

q_t \equiv relative price of dg $\frac{P_{K,t}}{P_{C,t}}$;

$\pi_{C,t}$ \equiv net inflation for ndg $\frac{P_{C,t}-P_{C,t-1}}{P_{C,t-1}}$;

$D_{C,t}$ \equiv dividends by firms producing ndg;

$D_{K,t}$ \equiv dividends by firms producing dg;

Y_t \equiv total product of ndg;

i_t \equiv nominal interest rate;

X_t^D \equiv demand of new dg;

X_t^S \equiv supply of new dg;

i_t^* \equiv exogenous AR(1) process for the interest rate shock;

$\beta = 0.98$ \equiv Utility Discount factor;

$\phi = 1.5$ \equiv Monetary policy response to inflation parameter;

$\epsilon_C = 11$ \equiv Elasticity of substitution for non-durable good;

$\epsilon_k = 7.6667$ \equiv Elasticity of substitution for durable good;

$\psi = 125$ \equiv Cost of inflation;

$\tau_C^S = \frac{1}{\epsilon_C - 1}$ \equiv Subsidy rate and tax rate on production of non-durable good (forcing pricing at MC);

$\tau_K^S = \frac{1}{\epsilon_K - 1}$ \equiv Subsidy rate and tax rate on production of durable good (forcing pricing at MC);

$\sigma_\epsilon = 1$ \equiv Variance of shock to interest rate;

$\rho = 0.9$ \equiv AR(1) coefficient on last period monetary shock;

$\varphi = 1$ \equiv Inverse Frisch Elasticity;

$i_0^* = 0$ \equiv Null constant in AR(1) process for monetary shock;

$\alpha = 0.4896$ \equiv Weight of consumption of ndgs in consumption basket;

$\delta = 0$ \equiv Depreciation;

C Steady state values

The steady state is computed at zero inflation (for the price of the non-durable good). It is assumed that the representative agent works $\frac{1}{3}$ of his total time endowment. Variables with a bar on top denote steady state values.

$$\bar{\pi}_C = 0;$$

$$\bar{N} = \frac{1}{3};$$

$$\bar{i} = \frac{1}{\beta} - 1;$$

$$\bar{W}_r = \frac{\epsilon_C - 1}{\epsilon_C} (1 + \tau_C^S);$$

$$\bar{C} = \frac{3^\varphi(1-\alpha)}{\nu} \bar{W}_r;$$

$$\bar{Y} = \bar{C};$$

$$\bar{N}_C = \bar{Y};$$

$$\bar{D}_C = \bar{Y} - \bar{W}_r \bar{N}_C;$$

$$\omega_C = 1;$$

$$\omega_K = 1;$$

$$\bar{B} = 0;$$

$$\bar{Z} = 0;$$

$$\bar{N}_k = \frac{1}{3} - \bar{N}_C;$$

$$\bar{q} = \frac{\epsilon_K}{(\epsilon_K - 1)(1 + \tau_K^S)} \bar{W}_r;$$

$$\bar{v}_C = \frac{\bar{D}_C}{1 - \beta};$$

$$\bar{K} = \frac{\beta \alpha \bar{C}}{\bar{q}(1 - \alpha)[1 - \beta(1 - \delta)]};$$

$$\bar{X}^S = \delta \bar{K};$$

$$\bar{C}^D = \bar{X}^S;$$

$$\bar{D}_K = (\bar{q} - \bar{W}_r) \bar{N}_K \quad \bar{v}_K = \frac{\bar{D}_K}{1 - \beta};$$

Parameter ν is calibrated ex-post. Its value is as follows (expressed as a function of other parameters):

$$\nu = \frac{3^{\varphi+1} \epsilon_C \delta \beta \alpha (\epsilon_K - 1) (1 + \tau_K^S) + 3^{\varphi+1} \epsilon_K (1 - \alpha) (\epsilon_C - 1) (1 + \tau_C^S) [1 - \beta(1 - \delta)]}{\epsilon_C \epsilon_K [1 - \beta(1 - \delta)]}$$