

## LEARNING ONE

### Dynare Simulation

The objective is to analyse a version of the baseline New Keynesian model in which there are three shocks: interest rate shocks, cost-push shocks and aggregate demand shocks. The basic structure of the model is a dynamic IS curve, a Phillips curve, and a simple rule for monetary policy.

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + g_t \quad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + u_t \quad (2)$$

$$i_t = \phi_\pi \pi_t + v_t \quad (3)$$

where

$$k = \frac{(1 - \omega)(1 - \beta\omega)}{\alpha\omega}$$

The three shocks are assumed to be independent AR(1) processes, given by

$$\begin{pmatrix} v_{t+1} \\ u_{t+1} \\ g_{t+1} \end{pmatrix} = \begin{pmatrix} \rho_v & 0 & 0 \\ 0 & \rho_u & 0 \\ 0 & 0 & \rho_g \end{pmatrix} \begin{pmatrix} v_t \\ u_t \\ g_t \end{pmatrix} + \begin{pmatrix} \sigma_v & 0 & 0 \\ 0 & \sigma_u & 0 \\ 0 & 0 & \sigma_g \end{pmatrix} \begin{pmatrix} \epsilon_t^v \\ \epsilon_t^u \\ \epsilon_t^g \end{pmatrix}$$

#### Calibration

Parameter	Calibrated Value	Definition
$\beta$	0.99	Discount Rate
$\sigma$	1	Intertemporal Elasticity of Substitution
$\omega$	0.5	% of firms unable to change their price in each period
$\alpha$	3	$(1/\alpha)$ elasticity of wages with respect to output gap
$\phi_\pi$	1.5	Coef. of inflation in interest rate rule
$\rho_v$	0.5	Persistence of interest rate shock
$\rho_u$	0.8	Persistence of Supply shock (cost push)
$\rho_g$	0.3	Persistence of Demand shock
$\sigma_v$	1	Standard Deviation of interest rate shock
$\sigma_u$	0.5	Standard Deviation of Supply shock(cost push)
$\sigma_g$	1	Standard Deviation of Demand shock

1. Which are the backward-looking variables in the model? Which are the forward-looking variables?
2. Calculate via simulation the following stylised facts of this model economy: volatility, autocorrelation and correlations between output gap, inflation and interest rate.
3. Show the responses of interest rates, output gap and inflation to the three different shocks. Explain using the model, the intuition for the response of each type of shock.

- Use the forecast error variance decomposition to verify which of the three shocks contributes most to fluctuations in interest rates, the output gap and inflation.

### Welfare analysis

Assume that the welfare of agents in the economy can be measured by the following loss function:

$$\min \sum_{i=0}^{\infty} \beta^i [\pi^2 + y^2] \quad (4)$$

- Which type of shock is most detrimental to welfare in this economy?
- In the model, change the monetary policy rule as follows:

$$i_t = \phi_{\pi} \pi_t + \phi_y y_t + v_t \quad (5)$$

Where  $\phi_{\pi} = 1.5$  as before, and  $\phi_y = 0.5$ . Simulating the model again with this new policy rule, calculate the welfare function in (4) for this new specification. Which rule perform best? Should the central bank target income as well as inflation to improve the welfare of the consumers?

### LEARNING TWO

Consider a simple forward-looking model of the form:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) + u_t \quad (6)$$

$$\pi_t = \beta E_t \pi_{t+1} + k y_t + u_t \quad (7)$$

Suppose policy reacts to the output gap:

$$i_t = \vartheta y_t + v_t \quad (8)$$

Write the system in State-space form. Are there values of  $\vartheta$  that ensure a unique stationary equilibrium? Are there values that do not?