Chapter 8: New Keynesian Monetary Economics*

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1 Introduction

For the past twenty years, the most common framework employed in monetary economics and monetary policy analysis has incorporated nominal wage and/or price rigidity into a dynamic, stochastic, general equilibrium (DSGE) framework that is based on optimizing behavior by the agents in the model. These modern DSGE models with nominal frictions are commonly labelled "new Keynesian" models because, like older versions of models in the Keynesian tradition, aggregate demand plays a central role in determining output in the short run and there is a presumption that some fluctuations both can be and should be dampened by countercyclical monetary and/or fiscal policy.[1] Early examples of models with these properties include those of Rotemberg and Woodford (1995), Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1997), and McCallum and Nelson (1999). Book length treatments of the new Keynesian model are provided by Woodford (2003a) and Galí (2008).

The first section of this chapter shows how a basic money-in-the-utility function (MIU) model, combined with the assumption of monopolistically competitive goods markets and price stickiness, can form the basis for a simple linear new Keynesian model.[2] The model is a consistent general

[1] Goodfriend and King (1997) proposed the name, "the new neoclassical synthesis," to emphasize the connection with the neoclassical, as opposed to Keynesian, traditions.

[2] See chapter 2 for a discussion of money-in-the-utility function models.

equilibrium model in which all agents face well-defined decision problems and behave optimally, given the environment in which they find themselves. To obtain a canonical new Keynesian model, three key modifications will be made to the MIU model of chapter 2. First, endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who show that, at least for the United States, there is little relationship between the capital stock and output at business-cycle frequencies. Endogenous capital stock dynamics play a key role in equilibrium business-cycle models in the real business-cycle tradition, but as Cogley and Nason (1995) show, the response of investment and the capital stock to productivity shocks actually contributes little to the dynamics implied by such models. For simplicity, then, the capital stock will be ignored.[3]

Second, the single final good in the MIU model is replaced by a continuum of differentiated goods produced by monopolistically competitive firms. These firms face constraints on their ability to adjust prices, thus introducing nominal price stickiness into the model. In the basic model, nominal wages will be allowed to fluctuate freely, although section 5.1 will explore the implications of assuming both prices and wages are sticky.

Third, monetary policy is represented by a rule for setting the nominal rate of interest. Most central banks today use a short-term nominal interest rate as their instrument for implementing monetary policy. The nominal quantity of money is then endogenously determined to achieve the desired nominal interest rate. Chapter 11 will take up the issues that arise when the use of an interest rate instrument is constrained by the zero lower bound on nominal rates.[4] Even absent the zero lower bound, there are also important issues involved in choosing between money supply policy procedures and interest rate procedures, and some of these will be discussed in chapter 12.

These three modifications yield a new Keynesian framework that is consistent with optimizing behavior by private agents and incorporates nominal rigidities, yet is simple enough to use for exploring a number of policy issues. It can be linked directly to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the workhorses for monetary policy analysis and is still common in most undergraduate texts. Once the basic framework has been developed, section 4 considers optimal policy, as well as a variety of policy rules and policy frameworks, including inflation targeting. Section 5 discusses the role of sticky wages in the new Keynesian model and the integration of modern theories of unemployment into the basic model.

[3] However, Dotsey and King (2006) and Christiano et al. (2005) emphasized the importance of variable capital utilization for understanding the behavior of inflation. New Keynesian models that are taken to the data incorporate investment and capital stock dynamics (e.g., Christiano et al. (2005), Smets and Wouter (2007), Altig et al. (2011)).

[4] It is perhaps better to speak of an effective lower bound on nominal interest rate as policy rates of the ECB, the Swedish Riksbank and the Danish central bank were all below zero by 2015.

2 The Basic Model

The model consists of households, firms, and a central bank. Households supply labor, purchase goods for consumption, and hold money and bonds, while firms hire labor and produce and sell differentiated products in monopolistically competitive goods markets. The basic model of monopolistic competition is drawn from Dixit and Stiglitz (1977). The model of price stickiness is taken from Calvo (1983).[5] Each firm sets the price of the good it produces, but not all firms reset their price in each period. Households and firms behave optimally; households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. The central bank, in contrast to households and firms, is initially assumed to follow a simple rule; optimal policy is explored in section 4.

2.1 Households

The preferences of the representative household are defined over a composite consumption good C_t, real money balances M_t/P_t, and the time devoted to market employment N_t. Households maximize the expected present discounted value of utility:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{1}$$

The composite consumption good consists of differentiated products produced by monopolistically competitive final goods producers (firms). There is a continuum of such firms of measure 1, and firm j produces good c_j. The composite consumption good that enters the household's utility function is defined as

$$C_t = \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \qquad \theta > 1. \tag{2}$$

The household's decision problem can be dealt with in two stages. First, regardless of the level of C_t the household decides on, it will always be optimal to purchase the combination of individual goods that minimizes the cost of achieving this level of the composite good. Second, given the cost of achieving any given level of C_t, the household chooses C_t, N_t, and M_t optimally.

Dealing first with the problem of minimizing the cost of buying C_t, the household's decision problem is to

$$\min_{c_{jt}} \int_0^1 p_{jt} c_{jt} dj$$

subject to

$$\left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \geq C_t, \tag{3}$$

[5] See section 7.2.4.

where p_{jt} is the price of good j. Letting ψ_t be the Lagrangian multiplier on the constraint, the first-order condition for good j is

$$p_{jt} - \psi_t \left[\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{1}{\theta-1}} c_{jt}^{-\frac{1}{\theta}} = 0.$$

Rearranging, $c_{jt} = (p_{jt}/\psi_t)^{-\theta} C_t$. From the definition of the composite level of consumption (2), this implies

$$C_t = \left[\int_0^1 \left[\left(\frac{p_{jt}}{\psi_t}\right)^{-\theta} C_t\right]^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} = \left(\frac{1}{\psi_t}\right)^{-\theta} \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{\theta}{\theta-1}} C_t.$$

Solving for ψ_t,

$$\psi_t = \left[\int_0^1 p_{jt}^{1-\theta} dj\right]^{\frac{1}{1-\theta}} \equiv P_t. \tag{4}$$

The Lagrangian multiplier is the appropriately aggregated price index for consumption as it gives the marginal cost of an additional unit of the consumption basket C_t. The demand for good j can then be written as

$$c_{jt} = \left(\frac{p_{jt}}{P_t}\right)^{-\theta} C_t. \tag{5}$$

The price elasticity of demand for good j is equal to θ. As $\theta \to \infty$, the individual goods become closer and closer substitutes, and, as a consequence, individual firms will have less market power.

Given the definition of the aggregate price index in (4), the budget constraint of the household is, in real terms,

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \left(\frac{W_t}{P_t}\right) N_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \left(\frac{B_{t-1}}{P_t}\right) + \Pi_t, \tag{6}$$

where M_t (B_t) is the household's nominal holdings of money (one-period bonds). Bonds pay a nominal rate of interest i_t. Real profits received from firms are equal to Π_t.

In the second stage of the household's decision problem, consumption, labor supply, money, and bond holdings are chosen to maximize (1) subject to (6). This leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium:

$$C_t^{-\sigma} = \beta(1 + i_t)\mathrm{E}_t \left(\frac{P_t}{P_{t+1}}\right) C_{t+1}^{-\sigma}; \tag{7}$$

$$\frac{\gamma\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-\sigma}}=\frac{i_t}{1+i_t}; \tag{8}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}=\frac{W_t}{P_t}. \tag{9}$$

These conditions represent the Euler condition for the optimal intertemporal allocation of consumption, the intratemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money, and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage.[6]

2.2 Firms

Firms maximize profits, subject to three constraints. The first is the production function summarizing the available technology. For simplicity, capital is ignored, so output is a constant returns to scale function solely of labor input N_{jt} and an aggregate productivity disturbance Z_t:

$$c_{jt}=Z_tN_{jt}, \qquad \mathrm{E}(Z_t)=1. \tag{10}$$

The second constraint on the firm is given by the demand curve (5) each firm faces. The third constraint is that each period some firms are not able to adjust their price. The specific model of price stickiness used is due to Calvo (1983). Each period, the firms that adjust their price are randomly selected, and a fraction $1-\omega$ of all firms adjust while the remaining ω fraction do not adjust. The parameter ω is a measure of the degree of nominal rigidity; a larger ω implies that fewer firms adjust each period and that the expected time between price changes is longer. Those firms that do adjust their price at time t do so to maximize the expected discounted value of current and future profits. Profits at some future date $t+s$ are affected by the choice of price at time t only if the firm has not received another opportunity to adjust between t and $t+s$. The probability of this is ω^s.[7]

Before analyzing the firm's pricing decision, consider its cost minimization problem, which involves minimizing W_tN_{jt} subject to producing $c_{jt}=Z_tN_{jt}$. This problem can be written, in real terms, as

$$\min_{N_t}\left(\frac{W_t}{P_t}\right)N_t+\varphi_t\left(c_{jt}-Z_tN_{jt}\right).$$

[6] See chapter 2 for further discussion of these first order conditions in a basic MIU model.

[7] In this formulation, the degree of nominal rigidity, as measured by ω, is constant, and the probability that a firm has adjusted its price is a function of time but not of the current state.

where φ_t is equal to the firm's real marginal cost.[8] The first-order condition implies

$$\varphi_t = \frac{W_t/P_t}{Z_t}; \tag{11}$$

to produce one extra unit of c_j the firm must hire $1/Z_t$ units of labor at a real cost of $(W_t/P_t)/Z_t$.

The firm's pricing decision problem then involves picking p_{jt} to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right) c_{jt+i} - \varphi_{t+i} c_{jt+i} \right],$$

where the discount factor $\Omega_{i,t+i}$ is given by $\beta^i (C_{t+i}/C_t)^{-\sigma}$. Using the demand curve (5) to eliminate c_{jt}, this objective function can be written as

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[\left(\frac{p_{jt}}{P_{t+i}} \right)^{1-\theta} - \varphi_{t+i} \left(\frac{p_{jt}}{P_{t+i}} \right)^{-\theta} \right] C_{t+i}.$$

While individual firms produce differentiated products, they all have the same production technology and face demand curves with constant and equal demand elasticities. In other words, they are essentially identical, except that they may have set their current price at different dates in the past. However, all firms adjusting in period t face the same problem, so all adjusting firms will set the same price. Let p_t^* be the optimal price chosen by all firms adjusting at time t. The first-order condition for the optimal choice of p_t^* is

$$\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \Omega_{i,t+i} \left[(1-\theta) \left(\frac{p_t^*}{P_{t+i}} \right) + \theta \varphi_{t+i} \right] \left(\frac{1}{p_t^*} \right) \left(\frac{p_t^*}{P_{t+i}} \right)^{-\theta} C_{t+i} = 0. \tag{12}$$

Using the definition of $\Delta_{i,t+i}$, (12) can be rearranged to yield

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \frac{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\theta}}{\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \left(\frac{P_{t+i}}{P_t} \right)^{\theta-1}}. \tag{13}$$

Consider the case of flexible prices so that all firms are able to adjust their prices every period ($\omega = 0$). When $\omega = 0$, (13) reduces to

$$\left(\frac{p_t^*}{P_t} \right) = \left(\frac{\theta}{\theta - 1} \right) \varphi_t = \mu \varphi_t. \tag{14}$$

Each firm sets its price p_t^* equal to a markup $\mu > 1$ over its nominal marginal cost $P_t \varphi_t$. This is

[8] The Lagrangian φ_t gives the effect on costs if the firm produces an additional unit of output.

the standard result in a model of monopolistic competition. Because price exceeds marginal cost, output will be inefficiently low. When prices are flexible, all firms charge the same price. In this case, $p_t^* = P_t$ and $\varphi_t = 1/\mu$. Using the definition of real marginal cost, this means $W_t/P_t = Z_t/\mu < Z_t$ in a flexible-price equilibrium. However, the real wage must also equal the marginal rate of substitution between leisure and consumption to be consistent with household optimization. This condition implies, from (9), that

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}} = \frac{W_t}{P_t} = \frac{Z_t}{\mu}. \tag{15}$$

With flexible prices, goods market clearing and the production function imply that $C_t = Y_t$ and $N_t = Y_t/Z_t$. Using these conditions in (15), and letting Y_t^f denote equilibrium output under flexible prices, Y_t^f is given by

$$Y_t^f = \left(\frac{1}{\chi\mu}\right)^{1/(\sigma+\eta)} Z_t^{(1+\eta)/(\sigma+\eta)}. \tag{16}$$

When prices are flexible, output is a function of the aggregate productivity shock, reflecting the fact that, in the absence of sticky prices, the new Keynesian model reduces to a real business cycle model.

When prices are sticky ($\omega > 0$), output can differ from the flexible-price equilibrium level. Because a firm will not adjust its price every period, it takes into account expected future marginal cost as well as current marginal cost whenever it has an opportunity to adjust its price. Equation (13) gives the optimal price to set, conditional on the current aggregate price level P_t. This aggregate price index, defined by (4), is an average of the price charged by the fraction $1-\omega$ of firms setting their price in period t and the average of the remaining fraction ω of all firms that do not change their price in period t. However, because the adjusting firms were selected randomly from among all firms, the average price of the non-adjusters is just the average price of all firms that prevailed in period $t-1$. Thus, from (4), the average price in period t satisfies

$$P_t^{1-\theta} = (1-\omega)(p_t^*)^{1-\theta} + \omega P_{t-1}^{1-\theta}. \tag{17}$$

Thus, (13) and (17) jointly describe the evolution of the price level.

2.3 Market clearing

In addition to affecting the evolution of the price level over time, price rigidity also affects the aggregate market clearing condition for goods. Let y_{jt} denote the output produced by firm j. Then for each good j, market clearing requires $y_{jt} = c_{jt}$, where c_{jt} is the demand for good j. Defining aggregate output as

$$Y_t = \left[\int y_{jt}^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}} \tag{18}$$

and using the definition of C_t from (2), aggregate goods market clearing implies

$$Y_t = C_t. \tag{19}$$

Because the production function for firm j is $y_{jt} = Z_t N_{jt}$, aggregate employment is, using (5),

$$N_t \equiv \int N_{jt} dj = \int \left(\frac{y_{jt}}{Z_t}\right) dj = \left(\frac{Y_t}{Z_t}\right) \int \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj = \left(\frac{C_t}{Z_t}\right) \Delta_t, \tag{20}$$

where

$$\Delta_t = \int \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} dj \geq 1 \tag{21}$$

is a measure of price dispersion across the individual firms. If all firms set the same price, $\Delta_t = 1$, than the total employment necessary to produce C_t is simply $C_t/Z_t = Y_t/Z_t$, as was assumed in deriving an expression for Y_t^f. With sticky prices, however, $\Delta_t \geq 1$, but this means that aggregate employment is $N_t = (C_t/Z_t)\,\Delta_t \geq (C_t/Z_t)$. Price dispersion means that more labor is required to produce a given overall consumption basket C_t than would be the case if all firms charged the same price. When firms are charging different prices, given that they all share the same technology, households purchase a combination of goods (more of the cheaper ones, less of the more expensive ones) that is socially inefficient. Because working generates disutility, price dispersion is costly in terms of the welfare of households. This inefficiency will be shown in section 4.1 to account for the costs of inflation variability in the new Keynesian model.[9]

[9] See problem 1 for an alternative derivation of the distortion generated by price dispersion.

It will be useful to note that (21) implies[10]

$$\Delta_t = (1-\omega)\left(\frac{p_t^*}{P_t}\right)^{-\theta} + \omega\left(\frac{P_t}{P_{t-1}}\right)^{\theta}\Delta_{t-1}. \tag{22}$$

3 A Linearized New Keynesian Model

Equations (7)-(9), (11), (13), (17)-(20), and (22) provide the equilibrium conditions characterizing private sector behavior for the basic new Keynesian model. They represent a system in C_t, N_t, M/P_t, Y_t, φ_t, P_t, p_t^*, W_t/P_t, Δ_t, and i_t that can be combined with a specification of monetary policy to determine the economy's equilibrium. These equations are nonlinear, but one reason for the popularity of the new Keynesian model is that it allows for a simple linear representation of private sector behavior in terms of an inflation adjustment equation, or Phillips curve, and an output and real interest rate relationship that corresponds to the IS curve of undergraduate macroeconomics. To derive this linearized version, the nonlinear equilibrium conditions of the model will be linearized around a steady state in which the inflation rate is zero. In what follows, let $\hat{x}_t$ denote the percentage deviation of a variable X_t around its steady state, and let the superscript f denote the flexible-price equilibrium.

3.1 The Linearized Phillips Curve

Equations (13) and (17) can be approximated around a zero average inflation, steady-state equilibrium to obtain an expression for aggregate inflation (see section 7.1 of the chapter appendix for details) of the form

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\hat{\varphi}_t \tag{23}$$

[10] A fraction $1-\omega$ of firms all set their price equal to p_t^*. Therefore

$$\Delta_t = \int \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} dj = (1-\omega)\left(\frac{p_t^*}{P_t}\right)^{-\theta} + \omega \int_{j\in NB}\left(\frac{p_{j,t}}{P_t}\right)^{-\theta} dj,$$

where the notation $j \in B$ indicates the second integral is over firms in the set of non-adjusting (NA) firms, of which there are a measure ω. Because for these firms, $p_{j,t} = p_{j,t-1}$ and these firms are a random sample of all firms,

$$\begin{aligned}\int_{j\in NB}\left(\frac{p_{j,t}}{P_t}\right)^{-\theta} dj &= \int_{j\in NB}\left(\frac{P_{t-1}}{P_t}\frac{p_{j,t-1}}{P_{t-1}}\right)^{-\theta} dj \\ &= \left(\frac{P_{t-1}}{P_t}\right)^{-\theta}\Delta_{t-1}.\end{aligned}$$

Thus,

$$\Delta_t = (1-\omega)\left(\frac{p_t^*}{P_t}\right) + \omega\left(\frac{P_t}{P_{t-1}}\right)^{\theta}\Delta_{t-1}.$$

.

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\beta\omega)}{\omega}$$

is an increasing function of the fraction of firms able to adjust each period and $\hat{\varphi}_t$ is real marginal cost, expressed as a percentage deviation around its steady-state value.[11]

Equation (23) is often referred to as the *new Keynesian Phillips curve*. Unlike more traditional Phillips curve equations, the new Keynesian Phillips curve implies that real marginal cost is the correct driving variable for the inflation process. It also implies that the inflation process is forward-looking, with current inflation a function of expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods. Solving (23) forward,

$$\pi_t = \tilde{\kappa} \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t \hat{\varphi}_{t+i},$$

which shows that inflation is a function of the present discounted value of current and future real marginal costs.

The new Keynesian Phillips curve also differs from traditional Phillips curves in having been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices). This derivation reveals how $\tilde{\kappa}$, the impact of real marginal cost on inflation, depends on the structural parameters β and ω. An increase in β means that the firm gives more weight to future expected profits. As a consequence, $\tilde{\kappa}$ declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in ω) reduces $\tilde{\kappa}$; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price and fewer firms adjust each period.

Equation (23) implies that inflation depends on real marginal cost and not directly on a measure of the gap between actual output and some measure of potential output or on a measure of unemployment relative to the natural rate, as is typical in traditional Phillips curves.[12] However, real marginal costs can be related to an output gap measure. The firm's real marginal cost is equal to the real wage it faces divided by the marginal product of labor (see 11). In a flexible price equilibrium, all firms set the same price, so (14) implies that real marginal cost will equal its steady-state value of $1/\mu$. Because nominal wages have been assumed to be completely flexible, the real wage must, according to (9), equal the marginal rate of substitution between leisure and

[11] Ascari (2004) shows that the behavior of inflation in the Calvo model can be significantly affected if steady-state inflation is not zero. See section 7.3.2.2.

[12] See Ravenna and Walsh (2008), Blanchard and Galí (2010) and Galí (2011) for models of labor market frictions that relate inflation to unemployment. Incorporating unemployment in the NK model is discussed in section 5.

consumption. Expressed in terms of percentage deviations around the steady state, (9) and (**??**) imply $\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{z}_t = \eta\hat{n}_t + \sigma\hat{c}_t - \hat{z}_t$. From the goods clearing condition, $C_t = Y_t$, so $\hat{c}_t = \hat{y}_t$. From (20), $N_t = Y_t\Delta_t/Z_t$. To first order, this becomes $\hat{n}_6 = \hat{y}_t - \hat{z}_t$.[13] Hence, the percentage deviation of real marginal cost around its steady-state value is

$$\begin{aligned}\hat{\varphi}_t &= \eta\left(\hat{y}_t - \hat{z}_t\right) + \sigma\hat{y}_t - \hat{z}_t \\ &= (\sigma+\eta)\left[\hat{y}_t - \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\right].\end{aligned}$$

To interpret the term involving $\hat{z}_t$, linearize (16) giving flexible-price output to obtain

$$\hat{y}_t^f = \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t. \tag{24}$$

Thus, (24) can be used to express real marginal cost as

$$\hat{\varphi}_t = (\sigma+\eta)\left(\hat{y}_t - \hat{y}_t^f\right). \tag{25}$$

Using this result, the inflation adjustment equation (23) becomes

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \tag{26}$$

where

$$\kappa = (\sigma+\eta)\tilde{\kappa} = \left[\frac{(1-\omega)(1-\beta\omega)}{\omega}\right](\sigma+\eta) \tag{27}$$

and $x_t \equiv \hat{y}_t - \hat{y}_t^f$ is the gap between actual output and flexible-price equilibrium output.

The proceeding has assumed that firms face constant returns to scale. If, instead, each firm's production function is $c_{jt} = Z_t N_{jt}^a$, where $0 < a \leq 1$, then the results must be modified slightly. When $a < 1$, firms with different production levels will face different marginal costs, and real marginal cost for firm j will equal

$$\varphi_{jt} = \frac{W_t/P_t}{aZ_t N_{jt}^{a-1}} = \frac{W_t/P_t}{ac_{jt}/N_{jt}}.$$

[13] When linearized, the last term becomes

$$\hat{\Delta}_t = -\theta\int\left(\hat{p}_{jt} - \hat{p}_t\right)dj,$$

but to a first order approximation, $\int \hat{p}_{jt}dj = \hat{p}_t$, so the devation of the price dispersion term around teh steady state is approximately (to first order) equal to zero.

Linearizing this expression for firm $j's$ real marginal cost and using the production function yields

$$\hat{\varphi}_{jt} = (\hat{w}_t - \hat{p}_t) - (\hat{c}_{jt} - \hat{n}_{jt}) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_{jt} - \left(\frac{1}{a}\right)\hat{z}_t. \tag{28}$$

Marginal cost for the individual firm can be related to average marginal cost, $\varphi_t = (W_t/P_t)/(aC_t/N_t)$, where

$$N_t = \int_0^1 N_{jt} dj = \int_0^1 \left(\frac{c_{jt}}{Z_t}\right)^{\frac{1}{a}} dj = \left(\frac{C_t}{Z_t}\right)^{\frac{1}{a}} \int_0^1 \left(\frac{p_{jt}}{P_t}\right)^{-\frac{\theta}{a}} dj.$$

When this last expression is linearized around a zero inflation steady state, the final term involving the dispersion of relative prices, turns out to be of second order, so one obtains

$$\hat{n}_t = \left(\frac{1}{a}\right)(\hat{c}_t - \hat{z}_t),$$

and

$$\hat{\varphi}_t = (\hat{w}_t - \hat{p}_t) - (\hat{c}_t - \hat{n}_t) = (\hat{w}_t - \hat{p}_t) - \left(\frac{a-1}{a}\right)\hat{c}_t - \left(\frac{1}{a}\right)\hat{z}_t. \tag{29}$$

Subtracting (29) from (28) and gives

$$\hat{\varphi}_{jt} - \hat{\varphi}_t = -\left(\frac{a-1}{a}\right)(\hat{c}_{jt} - \hat{c}_t)$$

Finally, employing the demand relationship (5) to express $\hat{c}_{jt} - \hat{c}_t$ in terms of relative prices,

$$\hat{\varphi}_{jt} = \hat{\varphi}_t - \left[\frac{\theta(1-a)}{a}\right](\hat{p}_{jt} - \hat{p}_t).$$

Firms with relatively high prices (and therefore low output) will have relatively low real marginal costs. In the case of constant returns to scale ($a = 1$), all firms face the same marginal cost. When $a < 1$, Sbordone (2002) and Gali et al. (2001) showed that the new Keynesian inflation adjustment equation becomes[14]

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\left[\frac{a}{a+\theta(1-a)}\right]\hat{\varphi}_t.$$

In addition, the labor market equilibrium condition under flexible prices becomes

$$\frac{W_t}{P_t} = \frac{aZ_t N_t^{a-1}}{\mu} = \frac{\chi N_t^{\eta}}{C_t^{-\sigma}},$$

[14] See the appendix, section 7.1, for further details on the derivation.

which implies flexible-price output is

$$\hat{y}_t^f = \left[\frac{1+\eta}{1+\eta+a(\sigma-1)}\right]\hat{z}_t.$$

When $a = 1$, this reduces to (24).

3.2 The Linearized IS Curve

Equation (26) relates output, in the form of the deviation around the level of output that would occur in the absence of nominal price rigidity, to inflation. It forms one of the two key components of the new Keynesian model. The other component is a linearized version of the household's Euler condition, (7). Because consumption is equal to output in this model (there is no government or investment as capital has been ignored), (7) can be approximated around the zero-inflation steady state as[15]

$$\hat{y}_t = \mathrm{E}_t\hat{y}_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t\pi_{t+1} - r), \tag{30}$$

where i_t is the deviation of the nominal interest rate and r is the steady-state real interest rate. Expressing this in terms of the output gap $x_t = \hat{y}_t - \hat{y}_t^f$,

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t\pi_{t+1} - r) + u_t, \tag{31}$$

where $u_t \equiv \mathrm{E}_t\hat{y}_{t+1}^f - \hat{y}_t^f$ depends only on the exogenous productivity disturbance (see 24). Combining (31) with (26) gives a simple two-equation, forward-looking, rational-expectations model for inflation and the output gap, once the behavior of the nominal rate of interest is specified.[16]

This two-equation model consists of the equilibrium conditions for a well-specified general equilibrium model. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. Equation (31) represents the demand side of the economy (an expectational, forward-looking IS curve), while the new Keynesian Phillips curve (26) corresponds to the supply side. In fact, both equations are derived from optimization problems, with (31) based on the Euler condition for the representative household's decision problem and (26) derived from the optimal pricing decisions of

[15] In the steady state with constant consumption, (7) implies $1 = \beta(1+r)$, where $r = (1+i)/(1+\pi)$ is the steady-state real interest rate. Hence, one can write (7) as

$$(1-\sigma\hat{c}_t) = \mathrm{E}_t\left[\frac{1+i_t}{(1+r)(1+\pi_{t+1})}\right](1-\sigma\hat{c}_{t+1}).$$

Following the approach used in chapter 2 (see the appendix to that chapter), and noting that $\hat{c}_t = \hat{y}_t$ yields (30). Previous editions defined $\hat{\imath}_t$ as $i_t - r$.

[16] With the nominal interest rate treated as the monetary policy instrument, (8) simply determines the real quantity of money in equilibrium.

individual firms.

There is a long tradition of using two equation, aggregate demand-aggregate supply (AD-AS) models in intermediate-level macroeconomic and monetary policy analysis. Models in the AD-AS tradition are often criticized as "starting from curves" rather than starting from the primitive tastes and technology from which behavioral relationships can be derived, given maximizing behavior and a market structure (Sargent (1982)). This criticism does not apply to (31) and (26). The parameters appearing in these two equations are explicit functions of the underlying structural parameters of the production and utility functions and the assumed process for price adjustment.[17] And (31) and (26) contain expectations of future variables; the absence of this type of forward-looking behavior is a critical shortcoming of older AD-AS frameworks. The importance of incorporating a role for future income was emphasized by Kerr and King (1996).

Equations (31) and (26) contain three variables: the output gap, inflation, and the nominal interest rate. The model can be closed by a monetary policy rule describing the central bank's behavior in setting the nominal interest rate.[18] Alternatively, if the central bank implements monetary policy by setting a path for the nominal supply of money, (26) and (31), together with the linearized version of (8), determine x_t, π_t, and i_t.[19]

3.3 Local Uniqueness of the Equilibrium

If a policy rule for the nominal interest rate is added to the model, this must be done with care to ensure that the policy rule does not render the system unstable or introduce multiple equilibria. For example, suppose monetary policy is represented by the following purely exogenous process for i_t:

$$i_t = r + v_t, \tag{32}$$

where v_t is a stationary, stochastic process. Combining (32) with (31) and (26), the resulting system of equations can be written as

$$\begin{bmatrix} 1 & \sigma^{-1} \\ 0 & \beta \end{bmatrix} \begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\kappa & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}.$$

Premultiplying both sides by the inverse of the matrix on the left produces

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix}, \tag{33}$$

[17] The process for price adjustment, however, has not been derived from the underlying structure of the economic environment.

[18] Important issues of price-level determinancy arise under interest-rate-setting policies, and these will be discussed in chapter 11.

[19] An alternative approach, discussed in section 6.4 specifies an objective function for the monetary authority and then derives the policymaker's decision rule for setting the nominal interest rate.

where

$$M = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & -\frac{1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Blanchard and Kahn (1980) showed that systems such as (33) have a unique, stationary solution if and only if the number of eigenvalues of M outside the unit circle is equal to the number of forward-looking variables, in this case, two (see). However, only the largest eigenvalue of this matrix is outside the unit circle, implying that multiple bounded equilibria exist and that the equilibrium is locally indeterminate. Stationary sunspot equilibria are possible.

This example illustrates that an exogenous policy rule – one that does not respond to the endogenous variables x and π – introduces the possibility of multiple equilibria. To understand why, consider what would happen if expected inflation were to rise. Because (32) does not allow for any endogenous feedback from this rise in expected inflation to the nominal interest rate, the real interest rate must fall. This decline in the real interest rate is expansionary, and the output gap increases. The rise in output increases actual inflation, according to (26). Thus, a change in expected inflation, even if due to factors unrelated to the fundamentals of inflation, can set off a self-fulfilling change in actual inflation.

This discussion suggests that a policy which raised the nominal interest rate when inflation rose, and raised $\hat{\imath}_t$ enough to increase the real interest rate so that the output gap fell, would be sufficient to ensure a unique equilibrium. For example, suppose the nominal interest rate responds to inflation according to the rule

$$i_t = r + \delta\pi_t + v_t. \tag{34}$$

Combining (34) with (26) and (31), i_t can be eliminated and the resulting system written as

$$\begin{bmatrix} \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = N \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \sigma^{-1} v_t - u_t \\ 0 \end{bmatrix} \tag{35}$$

where

$$N = \begin{bmatrix} 1 + \frac{\kappa}{\sigma\beta} & \frac{\beta\delta - 1}{\sigma\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} \end{bmatrix}.$$

Bullard and Mitra (2002) showed that a unique, stationary equilibrium exists as long as $\delta > 1$.[20] Setting $\delta > 1$ is referred to as satisfying the *Taylor principle*, because John Taylor was the first to stress the importance of interest-rate rules that called for responding more than one for one to changes in inflation.

Suppose that, instead of reacting solely to inflation, as in (34), the central bank responds to

[20] If the nominal interest rate is adjusted in response to expected future inflation (rather than current inflation), multiple solutions again become possible if i_t responds too strongly to $\mathrm{E}_t \pi_{t+1}$. See Clarida et al. (2000).

both inflation and the output gap according to

$$\hat{\imath}_t = r + \delta_\pi \pi_t + \delta_x x_t + v_t.$$

This type of policy rule is called a *Taylor rule* (Taylor (1993)), and variants of it have been shown to provide a reasonable empirical description of the policy behavior of many central banks (Clarida et al. (2000)).[21] With this policy rule, Bullard and Mitra (2002) showed that the condition necessary to ensure that the economy has a unique stationary equilibrium becomes

$$\kappa(\delta_\pi - 1) + (1 - \beta)\delta_x > 0. \tag{36}$$

Determinacy now depends on both the policy parameters δ_π and δ_x. A policy that failed to raise the nominal interest rate sufficiently when inflation rose would lead to a rise in aggregate demand and output. This rise in x could produce a rise in the real interest rate that served to contract spending if δ_x were large. Thus, a policy rule with $\delta_\pi < 1$ could still be consistent with a unique, stationary equilibrium. At a quarterly frequency, however, β is about 0.99, so δ_x would need to be vary large to offset a value of δ_π much below one.

The Taylor principle is an important policy lesson that has emerged from the new Keynesian model. It has been argued that the failure of central banks such as the Federal Reserve to respond sufficiently strongly to inflation during the 1970s provides an explanation for the rise in inflation experiences at the time (see Lubik and Schorfheide (2005)). Further, Orphanides (2001) argued that estimated Taylor rules for the Federal Reserve are sensitive to whether or not real-time data is used, and he found a much weaker response to inflation in the 1987-1999 period based on real-time data.[22] Because the Taylor principle is based on the mapping from policy response coefficients to eigenvalues in the state space representation of the model, one would expect the exact restrictions the policy responses must satisfy to ensure determinacy will depend on the specification of the model.

Two aspects of the model have been explored that lead to significant modifications of the Taylor principle. First, Hornstein and Wolman (2005), Ascari and Ropele (2007), and Kiley (2007) found that the Taylor rule can be insufficient to ensure determinacy when trend inflation is positive rather than zero as assumed when obtaining the standard linearized new Keynesian inflation equation. For example, Coibion and Gorodnichenko (2011b) showed, in a calibrated model, that the central bank's response to inflation in a rule such as (34) would need to be eight to ensure determinacy if steady-state inflation exceeded 6 percent. However, many models assume some form of indexation as

[21] Sometimes the term "Taylor rule" is reserved for the case in which $\delta_\pi = 1.5$ and $\delta_x = 0.5$ when inflation and the interest rate are expressed at annual rates. These are the values Taylor (1993) found matched the behavior of the federal funds rates rate during the Greenspan period.

[22] Other paper employing real-time data to estimate policy rules include Rudebusch (2006) for the U.S. and and Papell et al. (2008) for the U.S. and for Germany.

discussed in chapter 7, and for these models, the standard Taylor principle ($\delta_\pi > 1$) would continue to hold even in the face of a positive steady-state rate of inflation. In this context, it is important to note that (26) was obtained by linearizing around a zero-inflation steady state. If steady-state inflation is non-zero, then the linearized Calvo model takes a much more complex form, as shown by Ascari (2004).[23] For a survey on the implications of trend inflation, see Ascari and Sbordone (2013).

Second, the Taylor principle can be significantly affected when interest rates have direct effects on real marginal cost. Such an effect, usually referred to as the cost channel of monetary policy, is common in models in which firms need to finance wage payments, as in Christiano et al. (2005), Ravenna and Walsh (2006), or Christiano et al. (2014), or in which search frictions in the labor market introduce an intertemporal aspect to the firm's labor demand condition (Ravenna and Walsh (2008)). For example, Llosa and Tuesta (2009) for a model with a cost channel and Kurozumi and Zandweghe (2010) for a model with search and matching frictions in the labor market found that satisfying the standard Taylor principle of responding more than one-for-one to inflation may not ensure determinacy.

Note that if v_t and u_t are zero for all t, the solution to (35) would be $\pi_t = x_t = 0$ for all t. In this case, the parameter δ in the policy rule (34) could not be identified, yet the fact it exceeds one is necessary to ensure $\pi_t = x_t = 0$ is the unique equilibrium. As Cochrane (2011) emphasized, determinacy relies on assumptions about how the central bank would respond to movements of inflation out of equilibrium. Estimated Taylor rules may not reveal how policy would react in circumstances that are not observed. Cochrane also argued that determinacy requires the central bank to act in a manner that introduces an explosive root into the dynamic system; he characterized this as requiring the central bank to "blow up the world" to ensure determinacy.[24]

Finally, Benhabib et al. (2001) have emphasized that the equilibrium singled out when the Taylor Principle is satisfied is only locally unique. By introducing an explosive root, equilibria in which $\pi_t > 0$ are rule out as stationary solutions to (35) as they lead to explosive inflations. However, if $\pi_t < 0$, the explosive root leads to falling inflation and a falling nominal interest rate. But if the nominal interest rate is restricted to be non-negative, then it cannot cannot keep falling. Instead, once $i_t = 0$, the economy reaches a second stationary equilibrium with the nominal interest rate at zero. Thus, the standard equilibrium of (35) is locally but not globally unique. This issue will be discussed further in chapter 11 when the focus is on equilibria at the zero lower bound for nominal interest rates.

[23] Ascari and Ropele (2007) considered the implications of trend inflation for optimal monetary policy, while Lago Alves (2014) shows that the divine coincidence (that monetary policy can achieve a zero inflation and a zero output gap in the absence of cost shocks) no longer holds when trend inflation is non-zero. Cogley and Sbordone (2008) estimated a linearized Calvo model accounting for positive trend inflation.

[24] Recall that the basic model with $i_t = r + v_t$ had only one eigenvalue outside the unit circles, but two were needed to ensure a unique equilibrium.

3.4 The Monetary Transmission Mechanism

The model consisting of (26) and (31) assumes the impact of monetary policy on output and inflation operates through the real rate of interest. As long as the central bank is able to affect the real interest rate through its control of the nominal interest rate, monetary policy can affect real output. Changes in the real interest rate alter the optimal time path of consumption. An increase in the real rate of interest, for instance, leads households to attempt to postpone consumption. Current consumption falls relative to future consumption.[25] With sticky prices, the fall in current aggregate demand causes a fall in output.

Figure 1 illustrates the impact of a monetary policy shock (an increase in the nominal interest rate) in the model consisting of (26), (31), and the policy rule (34).[26] The parameter values used in constructing the figure are $\beta = 0.99$, $\sigma = \eta = 1$, $\delta = 1.5$, and $\omega = 0.8$. In addition, the policy shock v_t in the policy rule is assumed to follow an $AR(1)$ process given by $v_t = \rho_v v_{t-1} + \varepsilon_t$, with $\rho_v = 0.5$. The rise in the nominal rate causes inflation and the output gap to fall immediately. This reflects the forward-looking nature of both variables. In fact, all the persistence displayed by the responses arises from the serial correlation introduced into the process for the monetary shock v_t. If $\rho_v = 0$, all variables return to their steady-state values in the period after the shock.[27]

To emphasize the interest rate as the primary channel through which monetary influences affect output, it is convenient to express the output gap as a function of an *interest rate gap*, the gap between the current interest rate and the interest rate consistent with the flexible-price equilibrium. For example, let $r_t \equiv i_t - \mathrm{E}_t \pi_{t+1}$ be the real interest rate and write (31) as

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(r_t - \tilde{r}_t),$$

where $\tilde{r}_t \equiv r + \sigma u_t$. Woodford (2003a) has labeled $\tilde{r}_t$ the *Wicksellian real interest rate*. It is the interest rate consistent with output equaling the flexible-price equilibrium level. If $r_t = \tilde{r}_t$ for all t, then $x_t = 0$ and output is kept equal to the level that would arise in the absence of nominal rigidities. The interest rate gap $r_t - \tilde{r}_t$ then summarizes the effects on the actual equilibrium that are due to nominal rigidities.[28]

[25] Estrella and Fuhrer (2002) have noted that the forward-looking Euler equation implies counterfactual dynamics; (31) implies that $\mathrm{E}_t\hat{c}_{t+1} - \hat{c}_t = \sigma^{-1}(\hat{\imath}_t - \mathrm{E}_t\pi_{t+1})$, so that a rise in the real interest rate means that consumption must be expected to *increase* from t to $t+1$; $\hat{c}_t$ falls to ensure this is true.

[26] The programs used to obtain figures in this chapter are available at <http://people.ucsc.edu/~walshc/mtp4e/>.

[27] See Gali (2003) for a discussion of the monetary transmission mechanism incorporated in the basic new Keynesian model.

[28] Neiss and Nelson (2003) use a structural model to estimate the real interest-rate gap $r_t - \tilde{r}_t$ and find that it has value as a predictor of inflation.

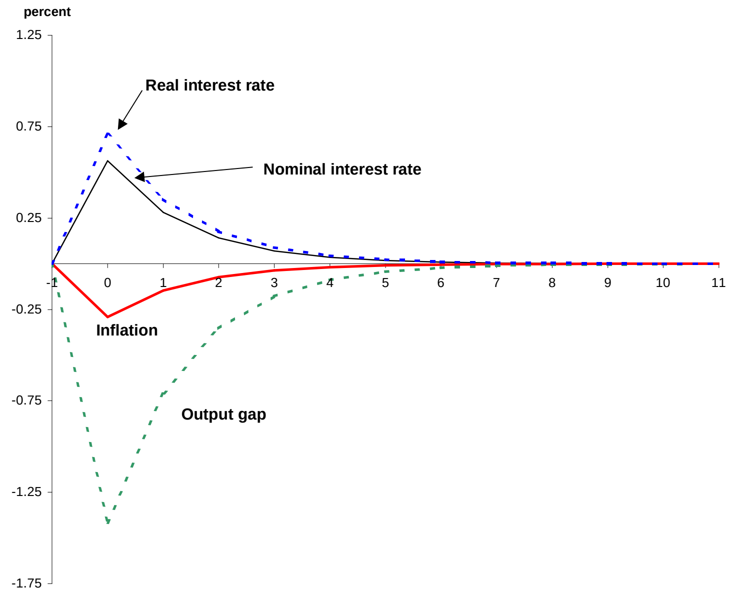

Figure 1: Output, Inflation, and Real Interest Rate Responses to a Policy Shock in the New Keynesian Model

The presence of expected future output in (31) implies that the future path of the one-period real interest rate matters for current demand. To see this, recursively solve (31) forward to yield

$$x_t = -\left(\frac{1}{\sigma}\right)\sum_{i=0}^{\infty} \mathrm{E}_t\left(r_{t+i} - \tilde{r}_{t+i}\right).$$

Changes in the one-period rate that are persistent will influence expectations of future interest rates. Therefore, persistent changes should have stronger effects on x_t than more temporary changes in real interest rates.

The basic interest rate transmission mechanism for monetary policy could be extended to include effects on investment spending if capital were reintroduced into the model (Christiano et al. (2005), Dotsey and King (2006)). Increases in the real interest rate would reduce the demand for capital and lead to a fall in investment spending. In the case of both investment and consumption, monetary policy effects are transmitted through interest rates.

In addition to these interest rate channels, monetary policy is often thought to affect the economy either indirectly through credit channels or directly through the quantity of money. Real money holdings represent part of household wealth; an increase in real balances should induce an increase in consumption spending through a wealth effect. This channel is often called the *Pigou effect* and was viewed as generating a channel through which price-level declines during a depression would eventually increase real balances and household wealth sufficiently to restore consumption spending. During the Keynesian/monetarist debates of the 1960s and early 1970s, some monetarists argued for a direct wealth effect that linked changes in the money supply directly to aggregate demand (Patinkin (1965)). The effect of money on aggregate demand operating through interest rates was viewed as a Keynesian interpretation of the transmission mechanism, whereas most monetarists argued that changes in monetary policy lead to substitution effects over a broader range of assets than Keynesians normally considered. Because wealth effects are likely to be small at business-cycle frequencies, most simple models used for policy analysis ignore them.[29]

Direct effects of the quantity of money are not present in this model as the quantity of money appears in neither (26) nor (31). The underlying model was derived from a MIU model, and the absence of money in (31) and (26) results from the assumption that the utility function is separable (see 1). If utility is not separable, then changes in the real quantity of money alter the marginal utility of consumption and/or leisure. This would affect the model specification in two ways. First, the real money stock would appear in the household's Euler condition and therefore in (31). Second, to replace real marginal cost with a measure of the output gap in (26), the real wage was equated to the marginal rate of substitution between leisure and consumption, and this would also involve real money balances if utility were nonseparable (see problem 7). Thus, the absence of money constitutes a special case. However, McCallum and Nelson (1999) and Woodford (2003a) have both argued that the effects arising with nonseparable utility are quite small, so that little is lost by assuming separability. Ireland (2004) finds little evidence for nonseparable preferences in a model estimated on U.S. data.

The quantity of money is not totally absent from the underlying model, because (8) must also hold in equilibrium. Linearizing this equation around the steady state yields[30]

$$\hat{m}_t - \hat{p}_t = \left(\frac{1}{bi^{ss}}\right)\left[\sigma\hat{y}_t - (i_t - i^{ss})\right]. \tag{37}$$

Given the nominal interest rate chosen by the monetary policy authority, this equation determines the nominal quantity of money. Alternatively, if the policymaker sets the nominal quantity of money, then (26), (31), and (37) must all be used to solve jointly for x_t, π_t, and i_t.

Chapter 10 discusses the role of credit channels in the monetary transmission process.

3.5 Adding Economic Disturbances

As the model consisting of (26) and (31) stands, there are no underlying nonpolicy disturbances that might generate movements in either the output gap or inflation other than the productivity

[29] For a recent analysis of the real balance effect, see Ireland (2001).
[30] See the appendix to chapter 2.

disturbance that affect the flexible-price output level. It is common, however, to include in these equations stochastic disturbances arising from other sources.

Remark 1 *Modify appendix welfare approximation to include ψ_t and χ_t preference shocks.*

Suppose the representative household's utility from consumption is subject to random shocks that alter the marginal utility of consumption and the marginal disutility of work. Specifically, let the utility function in (1) be modified to include stochastic taste shocks ψ_t and χ_t:

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{\left(\psi_{t+i} C_{t+i}\right)^{1-\sigma}}{1-\sigma} + \frac{\gamma}{1-b} \left(\frac{M_{t+i}}{P_{t+i}} \right)^{1-b} - \chi_t \frac{N_{t+i}^{1+\eta}}{1+\eta} \right]. \tag{38}$$

The Euler condition (7) becomes

$$\psi_t^{1-\sigma} C_t^{-\sigma} = \beta(1+i_t) \mathrm{E}_t \left(P_t / P_{t+1} \right) \left(\psi_{t+1}^{1-\sigma} C_{t+1}^{-\sigma} \right),$$

which, when linearized around the zero-inflation steady state, yields

$$\hat{c}_t = \mathrm{E}_t \hat{c}_{t+1} - \left(\frac{1}{\sigma} \right) \left(i_t - \mathrm{E}_t \pi_{t+1} - r \right) + \left(\frac{\sigma - 1}{\sigma} \right) \left(\mathrm{E}_t \hat{\psi}_{t+1} - \hat{\psi}_t \right). \tag{39}$$

If, in addition to consumption by households, the government purchases final output G_t, the goods market equilibrium condition becomes $Y_t = C_t + G_t$. When this is expressed in terms of percentage deviations around the steady state, one obtains

$$\hat{y}_t = \left(\frac{C}{Y} \right)^{ss} \hat{c}_t + \left(\frac{G}{Y} \right)^{ss} \hat{g}_t.$$

Using this equation to eliminate $\hat{c}_t$ from (39) and yields an expression for the output gap ($x_t \equiv \hat{y}_t - \hat{y}_t^f$),

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\tilde{\sigma}} \right) \left(i_t - \mathrm{E}_t \pi_{t+1} - r \right) + \xi_t, \tag{40}$$

where $\tilde{\sigma}^{-1} = \sigma^{-1} \left(C/Y \right)^{ss}$ and

$$\xi_t \equiv \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{C}{Y} \right)^{ss} \left(\mathrm{E}_t \hat{\psi}_{t+1} - \hat{\psi}_t \right) - \left(\frac{G}{Y} \right)^{ss} \left(\mathrm{E}_t \hat{g}_{t+1} - \hat{g}_t \right) + \left(\mathrm{E}_t \hat{y}_{t+1}^f - \hat{y}_t^f \right).$$

Equation (40) represents the Euler condition consistent with the representative household's intertemporal optimality condition linking consumption levels over time. It is also consistent with the resource constraint $Y_t = C_t + G_t$. The disturbance term arises from taste shocks that alter the marginal utility of consumption, shifts in government purchases, and shifts in the flexible-price equilibrium output.[31] In each case, it is expected changes in ψ, g, and $\hat{y}^f$ that matter. For example, an expected rise in government purchases implies that future consumption must fall. This reduces current consumption as forward-looking households respond immediately to the expected fall in future consumption.

Defining $r_t \equiv r - \tilde{\sigma}\xi_t$, (40) can be written in a convenient form as

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\tilde{\sigma}}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right). \tag{41}$$

Written this form shows that r_t is the equilibrium real interest rate consistent with a zero output gap. That is, if $x_t = 0$ for all t, then the actual real interest rate $i_t - \mathrm{E}_t\pi_{t+1}$ must equal r_t.

The source of a disturbance term in the inflation adjustment equation is both more critical for policy analysis and more controversial (section 4 takes up policy analysis). It is easy to see why exogenous shifts in (26) can have important implications for policy. Two commonly assumed objectives of monetary policy are to maintain a low and stable average rate of inflation and to stabilize output around full employment. These two objectives are often viewed as presenting central banks with a trade-off. A supply shock, such as an increase in oil prices, increases inflation and reduces output. To keep inflation from rising calls for contractionary policies that would exacerbate the decline in output; stabilizing output calls for expansionary policies that would worsen inflation. However, if the output objective is interpreted as meaning that output should be stabilized around its flexible-price equilibrium level, then (26) implies the central bank can always achieve a zero output gap (i.e., keep output at its flexible-price equilibrium level) and simultaneously keep inflation equal to zero. Solving (26) forward yields

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i}.$$

By keeping current and expected future output equal to the flexible-price equilibrium level, $\mathrm{E}_t x_{t+i} = 0$ for all i, and inflation remains equal to zero. Blanchard and Galí (2007) describe this as the "divine coincidence." This result holds even with the addition of a taste shock χ_t that affects the marginal rate of substitution between work and consumption and so affects the flexible-price output level (see end-of-chapter problem 3).

However, if an error term appears in the inflation adjustment equation so that

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{42}$$

[31] These three terms are not independent, as ψ_t and $\hat{g}_t$ will affect flexible-price output $\hat{y}_t^f$.

then

$$\pi_t = \kappa \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} + \sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i}.$$

As long as $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t e_{t+i} \neq 0$, maintaining $\sum_{i=0}^{\infty} \beta^i \mathrm{E}_t x_{t+i} = 0$ is not sufficient to ensure that inflation always remains equal to zero. A trade-off between stabilizing output and stabilizing inflation can arise. Disturbance terms in the inflation adjustment equation are often called *cost shocks* or *inflation shocks*. Because these shocks ultimately affect only the price level, they are also called *price shocks*.

Benigno and Woodford (2005) showed that a cost shock arises in the presence of stochastic variation in the gap between the welfare maximizing level of output and the flexible-price equilibrium level of output. In the model developed so far, only two distortions were present – one due to monopolistic competition and one due to nominal price stickiness. The first distortion implies the flexible-price output level is below the efficient output level even when prices are flexible. However, this "wedge", measured by the markup due to imperfect competition, is constant in the baseline model, so when the model is linearized, percent deviations of the flexible-price output and the efficient output around their respective steady-state values are equal. If there are time varying distortions such as would arise with stochastic variation in markups in product or labor markets or in distortionary taxes, then fluctuations in the two output concepts will differ. In this case, if x_t^w is the percent deviation of the welfare-maximizing output level around its steady state (the welfare gap),

$$x_t = x_t^w + \delta_t,$$

where δ_t represents these stochastic distortions. Because policymakers would be concerned with stabilizing fluctuations in x_t^w, the relevant constraint the policymaker will face is obtained by rewriting the Phillips curve (26) as

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t^w + \kappa \delta_t. \tag{43}$$

In this formulation, δ_t acts as a cost shock; stabilizing inflation in the face of non-zero realizations of δ_t cannot be achieved without creating volatility in the welfare gap x_t^w. One implication of (43) is that the variance of the cost shock will depend on κ^2. Thus, if the degree of price rigidity is high, implying that κ is small, cost shocks will also be less volatile (see Walsh (2005a)).

New Keynesian models, particularly those designed to be taken to the data, introduce a disturbance in the inflation equation by assuming that individual firms face random variation in the price elasticity of demand. That is, θ_t is assumed to be time varying (see 14). This modification leads to stochastic variation in markups, generating a wedge between flexible-price output and efficient output, and giving rise to cost shocks when the inflation equation is expressed in terms of the welfare gap as in (43).

4 Monetary Policy Analysis in New Keynesian Models

During the ten years after its first introduction, the new Keynesian model discussed in section 3 became the standard framework for monetary policy analysis. Clarida, Galí, and Gertler (1999), McCallum and Nelson (1999), Woodford (2003), and Svensson and Woodford (1999, 2005), among others, have popularized this simple model for use in monetary policy analysis. Galí (2002), and Galí and Gertler (2007) discusses some of the model's implications for monetary policy, while Galí (2008) provides an excellent treatment of the model and its implications for policy.

As seen in section 3, the basic new Keynesian model takes the form

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1} - r_t) \tag{44}$$

and

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{45}$$

where x_t is the output gap, defined as output relative to the equilibrium level of output under flexible prices, i_t is the nominal rate of interest, and π_t is the inflation rate. The demand disturbance r_t can arise from taste shocks to the preferences of the representative household, fluctuations in the flexible-price equilibrium output level, or shocks to government purchases of goods and services. The e_t shock is a cost shock that reflects exogenous stochastic variations in the markup. In this section, (44) and (45) are used to address issues of monetary policy design.

4.1 Policy Objectives

Given the economic environment that leads to (44) and (45), what are the appropriate objectives of the central bank? There is a long history in monetary policy analysis of assuming that the central bank is concerned with minimizing a quadratic loss function that depended on output and inflation. Models that make this assumption were discussed in chapter 6. While such an assumption is plausible, it is ultimately ad hoc. In the new Keynesian model, the description of the economy is based on an approximation to a fully specified general equilibrium model. One can therefore develop a policy objective function that can be interpreted as an approximation to the utility of the representative household. The general equilibrium foundations of (44) and (45) can then provide insights into the basic objectives central banks should pursue. Woodford (2003), building on the earlier work by Rotemberg and Woodford (1997), has provided the most detailed analysis of the link between a welfare criterion derived as an approximation to the utility of the representative agent and the types of quadratic loss functions common in the older literature.

Much of the literature that derives policy objectives based on the utility of the representative household follows Woodford (2003) in restricting attention to the case of a cashless economy, so real money balances do not appear in the utility function. Thus, assume the representative household seeks to maximize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta} \right], \tag{46}$$

where the consumption aggregate C_t is defined as in (2). Woodford demonstrates that deviations of the expected discounted utility of the representative agent around the level of steady-state utility can be approximated by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 \right] + t.i.p, \tag{47}$$

where $t.i.p.$ indicates terms independent of monetary policy. The detailed derivation of (47) and the values of Ω and λ are given in section 8.6.2 of the chapter appendix. In (47), x_t is the gap between output and the output level that would arise under flexible prices, and x^* is the gap between the steady-state efficient level of output (in the absence of the monopolistic distortions) and the actual steady-state level of output.

Equation (47) looks like the standard quadratic loss function employed in chapter 6 to represent the objectives of the monetary policy authority. There are, however, two critical differences. First, the output gap is measured relative to equilibrium output under flexible prices. In the traditional literature the output variable was more commonly interpreted as output relative to trend or output relative to the natural rate of output, which in turn was often defined as output in the absence of price surprises (see section 6.2.1).

A second difference between (47) and the quadratic loss function of chapter 6 arises from the reason inflation variability enters the loss function. When prices are sticky, and firms do not all adjust simultaneously, inflation results in an inefficient dispersion of relative prices and production among individual producers. Household respond to relative price dispersion by buying more of the relatively cheaper goods and less of the relatively more expensive goods. In turn, (20) showed that price dispersion means that more labor is required to produce an overall consumption basket C_t than would be the case if all firms charged the same price. Because working generates disutility, price dispersion is inefficient and reduces welfare. When each firm does not adjust its price every period, price dispersion is caused by inflation. These welfare costs can be eliminated under a zero inflation policy.

In Chapter 6, the efficiency distortion represented by $x^* > 0$ was used to motivate an overly ambitious output target in the central bank's objective function. The presence of x^* implies that a central bank acting under discretion to maximize (47) would produce a positive average inflation bias. However, with average rates of inflation in the major industrialized economies remaining low during the 1990s, many authors now simply assume that $x^* = 0$. In this case, the central bank is concerned with stabilizing the output gap x_t, and no average inflation bias arises.[32] If

[32] In addition, the inflation equation was derived by linearizing around a zero-inflation steady state. It would thus

tax subsidies can be used to offset the distortions associated with monopolistic competition, then one could assign fiscal policy the task of ensuring that $x^* = 0$. In this case, the central bank has no incentive to create inflationary expansions, and average inflation will be zero under discretion. Dixit and Lambertini (2003) showed that when both the monetary and fiscal authorities are acting optimally, the fiscal authority will use its tax instruments to set $x^* = 0$ and the central bank then ensures that inflation remains equal to zero.[33]

In the context of the linear-quadratic model, (47) represents a second order approximation to the welfare of the representative agent around the steady state. Expanding the period loss function,

$$\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 = \pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda \left(x^* \right)^2 .$$

Employing a first-order approximation for the structural equations will be adequate for evaluating the π_{t+i}^2 and x_{t+i}^2 terms, because any higher-order terms in the structural equations would become of order greater than two when squared. However, this is not the case for the $2\lambda x^* x_{t+i}$ term, which is linear in x_{t+i}. Hence, to approximate this correctly to the required degree of accuracy would require second-order approximations to the structural equations rather the linear approximations represented by (44) and (45). Thus, it is convenient to assume the fiscal authority employs an subsidy to undo the distortion arising from imperfect competition so that $x^* = 0$. In this case, the linear approximations to the structural equations will allow one to correctly evaluate the second-order approximation to welfare. See Benigno and Woodford (2005) for a discussion of optimal policy in the presence of a distorted steady state.

The parameter λ appearing in (47) plays a critical role in the evaluation of monetary policy as it governs the trade-off implied by the preferences of the representative household between volatility in inflation and volatility in real economic activity. The chapter appendix shows that

$$\lambda = \left[\frac{(1-\omega)(1-\omega\beta)}{\omega} \right] \left(\frac{\sigma + \eta}{\theta} \right) = \frac{\kappa}{\theta}, \tag{48}$$

where κ is defined in (27) and is the elasticity of inflation with respect to the output gap. Recall that ω is the fraction of firms that do not adjust price each period. An increase in ω represents an increase in the degree of price stickiness and reduces the weight placed on output gap volatility in the welfare function. With more rigid prices, inflation variability generates more relative price dispersion, leading to larger welfare losses. It therefore becomes more important to stabilize inflation. The welfare costs of inflation also depend on θ, the price elasticity of demand faced by individual firms. An increase in θ implies households respond more to changes in relative prices; thus, a given level of relative price dispersion generates larger distortions as households shift their expenditures from high-price to low-price firms. In this case, avoiding price dispersion by stabilizing inflation becomes

be inappropriate to use it to study situations in which average inflation is positive.

[33] See also Benigno and Woodford (2004) and Angeletos (2004).

more important, so λ falls.

4.2 Policy Trade-offs

The basic new Keynesian inflation adjustment equation given by (26) did not include a disturbance term, such as the e_t that was added to (45). The absence of e_t implies that there is no conflict between a policy designed to maintain inflation at zero and a policy designed to keep the output gap equal to zero. If $x_{t+i} = 0$ for all $i \geq 0$, then $\pi_{t+i} = 0$. In this case, a central bank that wants to maximize the expected utility of the representative household, assuming $x^* = 0$, will ensure that output is kept equal to the flexible-price equilibrium level of output. This also guarantees that inflation is equal to zero, thereby eliminating the costly dispersion of relative prices that arises with inflation. When firms do not need to adjust their prices, the fact that prices are sticky is no longer relevant. Thus, a key implication of the basic new Keynesian model is that price stability is the appropriate objective of monetary policy.[34]

The optimality of zero inflation conflicts with the Friedman rule for optimal inflation. Friedman (1969) concluded that the optimal inflation rate must be negative to make the nominal rate of interest zero (see chapter 4). The reason a different conclusion is reached here is the absence of any explicit role for money; (47) was derived from the utility function (46) in which money did not appear. In general, zero inflation still generates a monetary distortion. With zero inflation, the nominal rate of interest will be positive and the private opportunity cost of holding money will exceed the social cost of producing it. Khan et al. (2003) and Adao et al. (2003) considered models that integrate nominal rigidities and the Friedman distortion. Khan, King and Wolman introduced money into a sticky price model by assuming the presence of cash and credit goods, with money required to purchase cash goods. If prices are flexible, it is optimal to have a rate of deflation such that the nominal interest rate is zero. If prices are sticky, price stability would be optimal in the absence of the cash-in-advance constraint. With both sticky prices and the monetary inefficiency associated with a positive nominal interest rate, the optimal rate of inflation is less than zero but greater than the rate that yields a zero nominal interest rate. Khan, King, and Wolman conducted simulations in a calibrated version of their model and find that the relative price distortion dominates the Friedman monetary inefficiency. Thus, the optimal policy is close to the policy that maintains price stability.

In the baseline model with no monetary distortion and with $x^* = 0$, the optimality of price stability is a reflection of the presence of only one nominal rigidity. The welfare costs of a single nominal rigidity can be eliminated using the single instrument provided by monetary policy. Erceg et al. (2000) introduced nominal wage stickiness into the basic new Keynesian framework as a second nominal rigidity (see section 5.1). Nominal wage inflation with staggered adjustment of

[34] Notice that the conclusion that price stability is optimal is independent of the degree of nominal rigidity, a point made by Adao et al. (2004).

wages causes a distortions of relative wages and reduces welfare. Erceg, Henderson, and Levin showed that in this case the approximation to the welfare of the representative agent will include a term in wage inflation as well as the inflation and output gap terms appearing in (47). Nominal wage stability is desirable because it eliminates dispersion of hours worked across households. With two distortions – sticky prices and sticky wages – the single instrument of monetary policy cannot simultaneously offset both distortions. With sticky prices but flexible wages, the real wage can adjust efficiently in the face of productivity shocks and monetary policy should maintain price stability. With sticky wages and flexible prices, the real wage can still adjust efficiently to ensure that labor-market equilibrium is maintained in the face of productivity shocks, and monetary policy should maintain nominal wage stability. If both wages and prices are sticky, a policy that stabilizes either prices or wages will not allow the real wage to move so as to keep output equal to output with flexible prices and wages. Productivity shock will lead to movements in the output gap, and the monetary authority will be forced to trade-off stabilizing inflation, wage inflation, and the output gap. These issues are discussed further in section 5.1.

4.3 Optimal Commitment and Discretion

Suppose the central bank attempts to minimize a quadratic loss function such as (47), defined in terms of inflation and output relative to the flexible-price equilibrium.[35] Assume the steady-state gap between output and its efficient value is zero (i.e., $x^* = 0$). In this case, the central bank's loss function takes the form

$$L_t = \left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right). \tag{49}$$

Two alternative policy regimes can be considered. In a discretionary regime, the central bank behaves optimally in each period, taking as given the current state of the economy and private sector expectations. Given that the public knows the central bank optimizes each period, any promises the central bank makes about future inflation will not be credible – the public knows that whatever may have been promised in the past, the central bank will do what is optimal at the time it sets policy. The alternative regime is one of commitment. In a commitment regime, the central bank can make credible promises about what it will do in the future. By promising to take certain actions in the future, the central bank can influence the public's expectations about future inflation.

4.3.1 Commitment

A central bank able to precommit chooses a path for current and future inflation and the output gap to minimize the loss function (49) subject to the expectational IS curve (44) and the inflation

[35] Svensson (1999a) and Svensson (1999c) argued that there is widespread agreement among policymakers and academics that inflation stability and output gap stability are the appropriate objectives of monetary policy.

adjustment equation (45). Let θ_{t+i} and ψ_{t+i} denote the Lagrangian multipliers associated with the period $t+i$ constraints (44) and (45). The central bank's objective is to pick i_{t+i}, π_{t+i} and x_{t+i} to minimize

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \left(\frac{1}{2}\right) \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) + \theta_{t+i} \left[x_{t+i} - x_{t+i+1} + \sigma^{-1} \left(i_{t+i} - \pi_{t+i+1} - r_{t+i}\right)\right] \right.$$
$$\left. + \psi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - e_{t+i}\right) \right\}.$$

The first-order conditions for i_{t+i} take the form

$$\sigma^{-1} \mathrm{E}_t \theta_{t+i} = 0.$$

Hence, $\mathrm{E}_t \theta_{t+i} = 0$ for all $i \geq 0$. This result implies that (44) imposes no real constraint on the central bank as long as there are no restrictions on, or costs associated with, varying the nominal interest rate. Given the central bank's optimal choices for the output gap and inflation, (44) will simply determine the setting for i_t necessary to achieve the desired value of x_t. For that reason, it is often more convenient to treat x_t as if it were the central bank's policy instrument and drop (44) as an explicit constraint.

Setting $\mathrm{E}_t \theta_{t+i} = 0$, the remaining first-order conditions for π_{t+i} and x_{t+i} can be written as

$$\pi_t + \psi_t = 0, \tag{50}$$

$$\mathrm{E}_t \left(\pi_{t+i} + \psi_{t+i} - \psi_{t+i-1}\right) = 0, \qquad i \geq 1 \tag{51}$$

$$\mathrm{E}_t \left(\lambda x_{t+i} - \kappa \psi_{t+i}\right) = 0, \qquad i \geq 0. \tag{52}$$

Equations (50) and (51) reveal the dynamic inconsistency that characterizes the optimal commitment policy. At time t, the central bank sets $\pi_t = -\psi_t$ and promises to set $\pi_{t+1} = -\left(\psi_{t+1} - \psi_t\right)$ in the future. But when period $t+1$ arrives, a central bank that reoptimizes will again obtain $\pi_{t+1} = -\psi_{t+1}$ as its optimal setting for inflation. That is, the first-order condition (50) updated to $t+1$ will reappear.

An alternative definition of an optimal commitment policy requires that the central bank implement conditions (51) and (52) for all periods, including the current period. Woodford (1999) has labeled this the *timeless perspective* approach to precommitment.[36] One can think of such a policy as having been chosen in the distant past, and the current values of the inflation rate and output gap are the values chosen from that earlier perspective to satisfy the two conditions (51) and (52).

[36] See also Woodford (2000).

McCallum and Nelson (2004) provide further discussion of the timeless perspective and argue that this approach agrees with the one commonly used in many studies of precommitment policies.

Combining (51) and (52), under the timeless perspective optimal commitment policy inflation and the output gap satisfy

$$\pi_{t+i} = -\left(\frac{\lambda}{\kappa}\right)(x_{t+i} - x_{t+i-1}) \tag{53}$$

for all $i \geq 0$. Using this equation to eliminate inflation from (45) and rearranging, one obtains

$$\left(1 + \beta + \frac{\kappa^2}{\lambda}\right) x_t = \beta \mathrm{E}_t x_{t+1} + x_{t-1} - \frac{\kappa}{\lambda} e_t. \tag{54}$$

The solution to this expectational difference equation for x_t will be of the form $x_t = a_x x_{t-1} + b_x e_t$. To determine the coefficients a_x and b_x, note that if $e_t = \rho e_{t-1} + \varepsilon_t$, the proposed solution implies $\mathrm{E}_t x_{t+1} = a_x x_t + b_x \rho e_t = a_x^2 x_{t-1} + (a_x + \rho) b_x e_t$. Substituting this into (54) and equating coefficients, the parameter a_x is the solution less than 1 of the quadratic equation

$$\beta a_x^2 - \left(1 + \beta + \frac{\kappa^2}{\lambda}\right) a_x + 1 = 0$$

and b_x is given by

$$b_x = -\left\{\frac{\kappa}{\lambda\left[1 + \beta\left(1 - \rho - a_x\right)\right] + \kappa^2}\right\}.$$

From (53), equilibrium inflation under the timeless perspective policy is

$$\pi_t = \left(\frac{\lambda}{\kappa}\right)(1 - a_x) x_{t-1} + \left[\frac{\lambda}{\lambda\left[1 + \beta\left(1 - \rho - a_x\right)\right] + \kappa^2}\right] e_t. \tag{55}$$

Woodford (1999) stressed that, even if $\rho = 0$, so that there is no natural source of persistence in the model itself, $a_x > 0$ and the precommitment policy introduces inertia into the output gap and inflation processes. Because the central bank responds to the lagged output gap (see 53), past movements in the gap continue to affect current inflation. This commitment to inertia implies that the central bank's actions at date t allow it to influence expected future inflation. Doing so leads to a better trade-off between gap and inflation variability than would arise if policy did not react to the lagged gap. Equation (45) implies that the inflation impact of a positive cost shock, for example, can be stabilized at a lower output cost if the central bank can induce a fall in expected future inflation. Such a fall in expected inflation is achieved when the central bank follows (53).

A condition for policy such as (53) that is derived from the central bank's first-order conditions and only involves variables that appear in the objective function (in this case, inflation and the output gap), is generally called a *targeting rule or criterion* (e.g., Svensson and Woodford (2005)). It represents a relationship among the targeted variables that the central bank should maintain, because doing so is consistent with the first-order conditions from its policy problem.

Because the timeless perspective commitment policy is not the solution to the policy problem under optimal commitment (it ignores the different form of the first-order condition (50) in the initial period), the policy rule given by (53) may be dominated by other policy rules. For instance, it may be dominated by the optimal discretion policy (see next section). Under the timeless perspective, inflation as given by (53) is the same function each period of the current and lagged output gap; the policy displays the property of continuation in the sense that the policy implemented in any period continues the plan it was optimal to commit to in an earlier period. Blake (2002), Damjanovic et al. (2008) and Jensen and McCallum (2010) considered *optimal* continuation policies that require the policy instrument, in this case x_t, to be a time-invariant function, as under the timeless perspective, but rather than ignore the first period as is done under the timeless perspective, they focused on the optimal unconditional continuation policy to which the central bank should commit. This policy minimizes the unconditional expectation of the objective function, so that the Lagrangian for the policy problem becomes

$$\begin{aligned}\tilde{\mathrm{E}}\mathcal{L} &= \tilde{\mathrm{E}}\left\{\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i\left[\frac{1}{2}\left(\pi_{t+i}^2+\lambda x_{t+i}^2\right)\right.\right.\\ &\qquad \left.\left.+\psi_{t+i}\left(\pi_{t+i}-\beta\mathrm{E}_t\pi_{t+i+1}-\kappa x_{t+i}-e_{t+i}\right)\right]\right\},\end{aligned}$$

where $\tilde{\mathrm{E}}$ denotes the unconditional expectations operator. Because

$$\tilde{\mathrm{E}}\left(\mathrm{E}_t\psi_{t+i}\pi_{t+i+1}\right)=\tilde{\mathrm{E}}\left(\psi_{t-1}\pi_t\right),$$

the unconditional Lagrangian can be expressed as

$$\tilde{\mathrm{E}}\mathcal{L}=\left(\frac{1}{1-\beta}\right)\tilde{\mathrm{E}}\left\{\left[\frac{1}{2}\left(\pi_t^2+\lambda x_t^2\right)+\psi_t\pi_t-\beta\psi_{t-1}\pi_t-\kappa\psi_t x_t-\psi_t e_t\right]\right\}.$$

The first-order conditions then become

$$\pi_t+\psi_t-\beta\psi_{t-1}=0 \tag{56}$$

and

$$\lambda x_t-\kappa\psi_t=0.$$

Combining these to eliminate the Lagrangian multiplier yields the optimal unconditional continuation policy:

$$\pi_t=-\left(\frac{\lambda}{\kappa}\right)\left(x_{t+i}-\beta x_{t+i-1}\right). \tag{57}$$

Comparing this to (53) shows that rather than give full weight to past output gaps, the optimal unconditional continuation policy discounts the past slightly (at a quarterly frequency, $\beta \approx 0.99$).

Notice that neither (54) nor (55) involve the aggregate productivity shock that affect the economy's flexible-price equilibrium output. By definition, actual output is $\hat{y}_t = \hat{y}_t^f + x_t$. Thus, under the optimal commitment policy, monetary policy prevents a positive productivity shock from affecting the output gap, allowing output to move as it would if prices were flexible. The response to a positive productivity shock involves an increase firms' labor demand at the initial real wage. The efficient response requires a rise in the real wage to ensure labor supply and demand balance. The real wage is free to adjust appropriately because only prices have been assumed to be sticky; the nominal wage is free to adjust to the real wage and output to adjust as they would if prices had been flexible.

4.3.2 Discretion

When the central bank operates with discretion, it acts each period to minimize the loss function (49) subject to the inflation adjustment equation (45). Because the decisions of the central bank at date t do not bind it at any future dates, the central bank is unable to affect the private sector's expectations about future inflation. Thus, the decision problem of the central bank becomes the single-period problem of minimizing $\pi_t^2 + \lambda x_t^2$ subject to the inflation adjustment equation (45).

The first-order condition for this problem is

$$\kappa\pi_t + \lambda x_t = 0. \tag{58}$$

Equation (58) is the optimal targeting rule under discretion. Notice that by combining (50) with (52) evaluated at time t, one obtains (58); thus, the central bank's first-order condition relating inflation and the output gap at time t is the same under discretion or under the fully optimal precommitment policy (but not under the timeless perspective policy). The differences appear in subsequent periods. For $t+1$, under discretion $\kappa\pi_{t+1} + \lambda x_{t+1} = 0$, whereas under commitment (from 51 and 52), $\kappa\pi_{t+1} + \lambda(x_{t+1} - x_t) = 0$.

The equilibrium expressions for inflation and the output gap under discretion can be obtained by using (58) to eliminate inflation from the inflation adjustment equation. This yields

$$\left(1 + \frac{\kappa^2}{\lambda}\right) x_t = \beta \mathrm{E}_t x_{t+1} - \left(\frac{\kappa}{\lambda}\right) e_t. \tag{59}$$

Guessing a solution of the form $x_t = \delta e_t$, so that $\mathrm{E}_t x_{t+1} = \delta\rho e_t$, one obtains

$$\delta = -\left[\frac{\kappa}{\lambda(1-\beta\rho) + \kappa^2}\right].$$

Equation (58) implies that equilibrium inflation under optimal discretion is

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right) x_t = \left[\frac{\lambda}{\lambda(1-\beta\rho)+\kappa^2}\right] e_t. \tag{60}$$

According to (60) the unconditional expected value of inflation is zero; there is no average inflation bias under discretion. However, when forward-looking expectations play a role, as in (45), discretion will lead to what is known as a *stabilization bias* in that the response of inflation to a cost shock under discretion differs from the response under commitment. This can be seen by comparing (60) to (55).[37]

4.3.3 Discretion versus Commitment

The impact of a cost shock on inflation and the output gap under the timeless perspective optimal precommitment policy and optimal discretionary policy can be obtained by calibrating the model and numerically solving for the equilibrium under the alternative policies. Four unknown parameters appear in the model: β, κ, λ and ρ. The discount factor, β, is set equal to 0.99, appropriate for interpreting the time interval as one quarter. A weight on output fluctuations of $\lambda = 0.25$ is used. This value is also used by Jensen (2002) and McCallum and Nelson (2004) and is the value used by Debortoli et al. (2015) to represent the Fed's dual mandate of price stability and maximum sustainable employment.[38] The parameter κ captures both the impact of a change in real marginal cost on inflation and the comovement of real marginal cost and the output gap and is set equal to 0.05. McCallum and Nelson (2004) reported that empirical evidence is consistent with a value of κ in the range [0.01, 0.05]. Roberts (1995) reports higher values; his estimate of the coefficient on the output gap is about 0.3 when inflation is measured at an annual rate, so this translates into a value for κ of 0.075 for inflation at quarterly rates. Jensen (2002) used a baseline value of $\kappa = 0.1$, while Walsh (2003b) used 0.05.

The solid lines in figures 2 and 3 show the response of the output gap and inflation to a transitory, one standard deviation cost push shock under the optimal precommitment policy. Despite the fact that the shock itself has no persistence, the output gap displays strong positive serial correlation. By keeping output below potential (a negative output gap) for several periods into the future after a positive cost shock, the central bank is able to lower expectations of future inflation. A fall in $\mathrm{E}_t\pi_{t+1}$ at the time of the positive inflation shock improves the trade-off between inflation and output gap stabilization faced by the central bank.

Outcomes under optimal discretion are shown by the dashed lines in the figures. There is no

[37] In models containing an endogenous state variable such as the stock of capital or government debt, issues of determinacy discussed earlier with respest to instrument rules can also arise under optimal discretion. See Blake and Kirsanova (2012) and Dennis and Kirsanova (2013).

[38] If (49) is intrepreted as an approximation to the welfare of the representative agent, the implied value of λ would be much smaller.

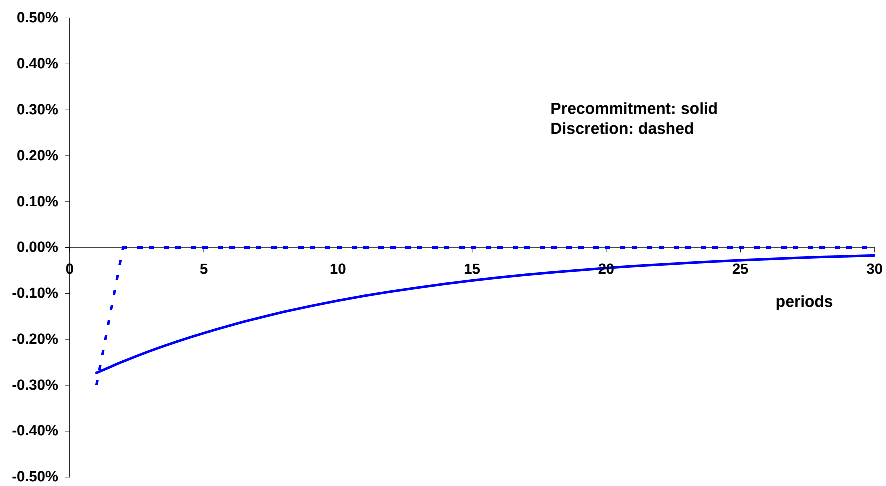

Figure 2: Output Gap Response to a Cost Shock: Timeless Precommitment and Pure Discretion

inertia under discretion; both the output gap and inflation return to their steady-state values in the period after the shock occurs. The difference in the stabilization response under commitment and discretion is the stabilization bias due to discretion. The intuition behind the suboptimality of discretion can be see by considering the inflation adjustment equation given by (45). Under discretion, the central bank's only tool for offsetting the effects on inflation of a cost shock is the output gap. In the face of a positive realization of e_t, x_t must fall to help stabilize inflation. Under commitment, however, the central bank has two instruments; it can affect both x_t and $\mathrm{E}_t\pi_{t+1}$. By creating expectations of a deflation at $t+1$, the reduction in the output gap does not need to be as large. Of course, under commitment a promise of future deflation must be honored, so actually inflation falls below the baseline beginning in period $t+1$ (see figure 3). Consistent with producing a deflation, the output gap remains negative for several periods.[39]

The analysis so far has focused on the goal variables, inflation and the output gap. Using (44), the associated behavior of the interest rate can be derived. For example, under optimal discretion,

[39] While it is not obvious from the figures, the unconditional expectation of $\pi_t^2 + \lambda x_t^2$ is $0.9901\sigma_e^2$ under discretion and $0.9134\sigma_e^2$ under commitment, using the same calibration as in the figures. This represents a 7.74% improvement under commitment.

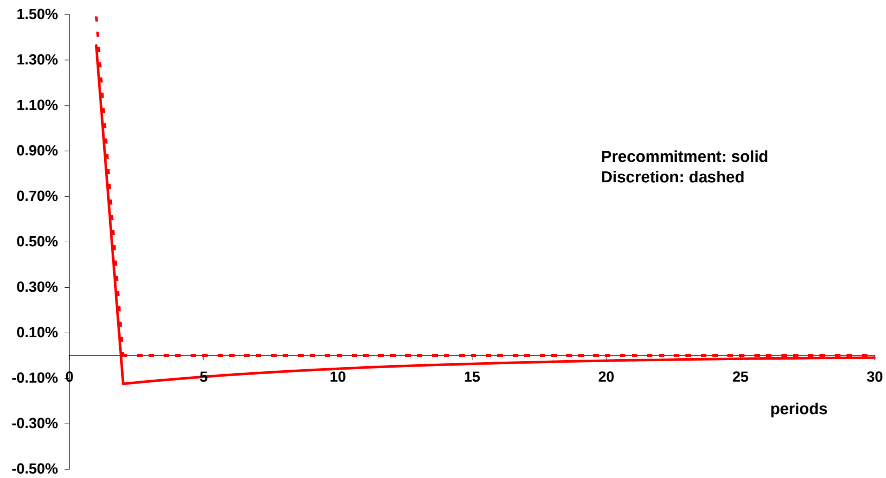

Figure 3: Response of Inflation to a Cost Shock: Timeless Precommitment and Pure Discretion

the output gap is given by

$$x_t = -\left[\frac{\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t,$$

while inflation is given by (60). Using these to evaluate $\mathrm{E}_t x_{t+1}$ and $\mathrm{E}_t \pi_{t+1}$ and then solving for i_t from (44) yields

$$\begin{aligned} i_t &= r_t + \mathrm{E}_t\pi_{t+1} + \sigma\left(\mathrm{E}_t x_{t+1} - x_t\right) \\ &= r_t + \left[\frac{\lambda\rho + (1-\rho)\sigma\kappa}{\lambda(1-\beta\rho)+\kappa^2}\right]e_t. \end{aligned} \tag{61}$$

Equation (61) is the reduced-form solution for the nominal rate of interest. The nominal interest rate is adjusted to offset completely the impact of the demand disturbance r_t on the output gap. As a result, it affects neither inflation nor the output gap. Section 3.3 illustrated how a policy that commits to a rule that calls for responding to the exogenous shocks renders the new Keynesian model's equilibrium indeterminate. Thus, it is important to recognize that (61) describes the *equilibrium* behavior of the nominal interest rate under optimal discretion; (61) is not an instrument rule (see Svensson and Woodford (2005)).

4.4 Commitment to a Rule

In the Barro-Gordon model popular in the 1980s and 1990s and examined in chapter 6, optimal commitment was interpreted as commitment to a policy that was a (linear) function of the state variables. In the present model consisting of (44) and (45), the only state variable is the current realization of the cost shock e_t. Suppose then that the central bank can commit to a rule of the form[40]

$$x_t = b_x e_t. \tag{62}$$

What is the optimal value of b_x? With x_t given by (62), inflation satisfies

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t,$$

and the solution to this expectational difference equation is[41]

$$\pi_t = b_\pi e_t, \qquad b_\pi = \frac{1+\kappa b_x}{1-\beta\rho}. \tag{63}$$

Using (62) and (63), the loss function can be written as

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right) = \left(\frac{1}{2}\right) \sum_{i=0}^{\infty} \beta^i \left[\left(\frac{1+\kappa b_x}{1-\beta\rho}\right)^2 + \lambda b_x^2\right] e_t^2.$$

This is minimized when

$$b_x = -\left[\frac{\kappa}{\lambda(1-\beta\rho)^2 + \kappa^2}\right].$$

Using this solution for b_x in (63), equilibrium inflation is given by

$$\pi_t = \left(\frac{1+\kappa b_x}{1-\beta\rho}\right) e_t = \left[\frac{\lambda(1-\beta\rho)}{\lambda(1-\beta\rho)^2 + \kappa^2}\right] e_t. \tag{64}$$

Comparing the solution for inflation under optimal discretion, given by (60), and the solution under commitment to a simple rule, given by (64), note that they are identical if the cost shock is serially uncorrelated ($\rho = 0$). If $0 < \rho < 1$, there is a stabilization bias under discretion relative to the case of committing to a simple rule.

Clarida et al. (1999) argued that this stabilization bias provides a rationale for appointing a

[40] This commitment does not raise the same uniqueness of equilibrium problem that would arise under a commitment to an instrument rule of the form $i_t = r_t + b_i e_t$. See problem 2

[41] To verify this is the solution, note that

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa b_x e_t + e_t = \beta b_\pi \rho e_t + \kappa b_x e_t + e_t \\ &= \left[\beta b_\pi \rho + \kappa b_x + 1\right] e_t, \end{aligned}$$

so that $b_\pi = \beta b_\pi \rho + \kappa b_x + 1 = (\kappa b_x + 1)/(1-\beta\rho)$.

Rogoff-conservative central banker – a central bank who puts more weight on inflation objectives that is reflected in the social loss function – when $\rho > 0$, even though in the present context there is no average inflation bias.[42] A Rogoff-conservative central banker places a weight $\hat{\lambda} < \lambda$ on gap fluctuations (see section 6.3.2). In a discretionary environment with such a central banker, (60) implies inflation will equal

$$\pi_t = \left[\frac{\hat{\lambda}}{\hat{\lambda}(1-\beta\rho)+\kappa^2} \right] e_t.$$

Comparing this with (64) reveals that if a central banker is appointed for whom $\hat{\lambda} = \lambda(1-\beta\rho) < \lambda$, the discretionary solution will coincide with the outcome under commitment to the optimal simple rule. Such a central banker stabilizes inflation more under discretion than would be the case if the relative weight placed on output gap and inflation stability were equal to the weight in the social loss function, λ. Because the public knows inflation will respond less to a cost shock, future expected inflation rises less in the face of a positive e_t shock. As a consequence, current inflation can be stabilized with a smaller fall in the output gap. The inflation-output trade-off is improved.

Recall, however, that the notion of commitment used here is actually suboptimal. As seen earlier, fully optimal commitment leads to inertial behavior in that future inflation depends not on the output gap but on the change in the gap.

4.5 Endogenous Persistence

The empirical research on inflation (see section 7.3.2) has generally found that when lagged inflation is added to (45), its coefficient is statistically and economically significant. If lagged inflation affects current inflation, then even under discretion the central bank faces a dynamic optimization problem; decisions that affect current inflation also affect future inflation, and this intertemporal link must be taken into account by the central bank when setting current policy. Svensson (1999b) and Vestin (2006) illustrated how the linear-quadratic structure of the problem allows one to solve for the optimal discretionary policy in the face of endogenous persistence.

To analyze the effects introduced when inflation depends on both expected future inflation and lagged inflation, suppose (45) is replaced by

$$\pi_t = (1-\phi)\beta \mathrm{E}_t \pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t. \tag{65}$$

The coefficient ϕ measures the degree of backward-looking behavior exhibited by inflation.[43] If the

[42] Rogoff (1985) proposed appointing a conservative central banker as a way to solve the average infltion bias that can arise under discretionary policies, an issue discussed in chapter 6. There is no average inflation bias in the present model because we have assumed that $x^* = 0$, ensuring that the central bank's loss function depends on output only through the gap between actual output and flexible-price equilibrium output.

[43] Gali and Gertler (1999), Woodford (2003a), and Christiano et al. (2005) developed inflation-adjustment equations in which lagged inflation appears by assuming that some fraction of firms do not reset their prices optimally (see

central bank's objective is to minimize the loss function given by (49), the policy problem under discretion can be written in terms of the value function defined by

$$\begin{aligned} V(\pi_{t-1}, e_t) &= \min_{\pi_t, x_t} \left\{ \left(\frac{1}{2}\right) \left(\pi_t^2 + \lambda x_t^2\right) + \beta \mathrm{E}_t V(\pi_t, e_{t+1}) \right. \\ &\quad \left. + \psi_t \left[\pi_t - (1-\phi)\beta \mathrm{E}_t \pi_{t+1} - \phi \pi_{t-1} - \kappa x_t - e_t\right] \right\}. \end{aligned} \tag{66}$$

The value function depends on π_{t-1} because lagged inflation is an endogenous state variable.

Because the objective function is quadratic and the constraints are linear, the value function will be quadratic, and one can hypothesize that it takes the form

$$V(\pi_{t-1}, e_t) = a_0 + a_1 e_t + \frac{1}{2} a_2 e_t^2 + a_3 e_t \pi_{t-1} + a_4 \pi_{t-1} + \frac{1}{2} a_5 \pi_{t-1}^2. \tag{67}$$

As Vestin demonstrated, this guess is only needed to evaluate $\mathrm{E}_t V_\pi(\pi_t, e_{t+1})$, and $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_3 \mathrm{E}_t e_{t+1} + a_4 + a_5 \pi_t$. If one assumes the cost shock is serially uncorrelated, $\mathrm{E}_t e_{t+1} = 0$ and, as a consequence, the only unknown coefficients in (67) that will play a role are a_4 and a_5.

The solution for inflation will take the form

$$\pi_t = b_1 e_t + b_2 \pi_{t-1}. \tag{68}$$

Using this proposed solution, one obtains $\mathrm{E}_t \pi_{t+1} = b_2 \pi_t$. This expression for expected future inflation can be substituted into (65) to yield

$$\pi_t = \frac{\kappa x_t + \phi \pi_{t-1} + e_t}{1 - (1-\phi)\beta b_2}, \tag{69}$$

which implies $\partial \pi_t / \partial x_t = \kappa / \left[1 - (1-\phi)\beta b_2\right]$.

Collecting these results, the first-order condition for the optimal choice of x_t by a central bank whose decision problem is given by (66) is

$$\left[\frac{\kappa}{1 - (1-\phi)\beta b_2}\right] \left[\pi_t + \beta \mathrm{E}_t V_\pi(\pi_t, e_{t+1})\right] + \lambda x_t = 0. \tag{70}$$

Using (69) to eliminate x_t from (70) and recalling that $\mathrm{E}_t V_\pi(\pi_t, e_{t+1}) = a_4 + a_5 \pi_t$, one obtains

$$\pi_t = \left[\frac{\Psi}{\kappa^2(1 + \beta a_5) + \lambda \Psi^2}\right] \left[\lambda \phi \pi_{t-1} + \lambda e_t - \left(\frac{\beta \kappa^2}{\Psi}\right) a_4\right], \tag{71}$$

where $\Psi \equiv 1 - (1-\phi)\beta b_2$.

section 7.3). See also Eichenbaum and Fisher (2007). Lagged inflation also appears when firms index prices to past inflation.

From the envelope theorem and (70),

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= a_3 e_t + a_4 + a_5 \pi_{t-1} \\ &= \left[\frac{\phi}{1-(1-\phi)\beta b_2}\right] \left[\pi_t + \mathrm{E}_t V_\pi(\pi_t, e_{t+1})\right] = -\left(\frac{\lambda\phi}{\kappa}\right) x_t. \end{aligned}$$

Again using (69) to eliminate x_t,

$$\begin{aligned} V_\pi(\pi_{t-1}, e_t) &= -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{\Psi\pi_t - \phi\pi_{t-1} - e_t}{\kappa}\right] \\ &= -\left(\frac{\lambda\phi}{\kappa}\right)\left[\frac{(\Psi b_2 - \phi)\,\pi_{t-1} + (\Psi b_1 - 1)\, e_t}{\kappa}\right]. \end{aligned} \tag{72}$$

However, (67) implies that $V_\pi(\pi_{t-1}, e_t) = a_3 e_t + a_4 + a_5 \pi_{t-1}$. Comparing this with (72) reveals that $a_4 = 0$,

$$a_3 = \lambda\phi\left(\frac{1 - \Psi b_1}{\kappa^2}\right),$$

and

$$a_5 = \lambda\phi\left(\frac{\phi - \Psi b_2}{\kappa^2}\right).$$

Finally, substitute these results into (71) to obtain

$$\pi_t = \left[\frac{\Psi}{\kappa^2 + \beta\lambda\phi\,(\phi - \Psi b_2) + \lambda\Psi^2}\right]\left[\lambda\phi\pi_{t-1} + \lambda e_t\right].$$

Equating coefficients with (68),

$$b_1 = \left[\frac{\lambda\Psi}{\kappa^2 + \beta\lambda\phi\,(\phi - \Psi b_2) + \lambda\Psi^2}\right]$$

and

$$b_2 = \left[\frac{\lambda\Psi\phi}{\kappa^2 + \beta\lambda\phi\,(\phi - \Psi b_2) + \lambda\Psi^2}\right]. \tag{73}$$

Because Ψ also depends on the unknown parameter b_2, (73) does not yield a convenient analytic solution. To gain insights into the effects of backward-looking aspects of inflation, it is useful to employ numerical techniques. This is done to generate figure 4, which shows the response of the output gap and inflation under optimal discretion when $\phi = 0.5$. Also shown for comparison are the responses under the optimal commitment policy. Both the output gap and inflation display more persistence than when $\phi = 0$ (see figures 2 and 3), and inflation returns to zero more slowly under discretion.

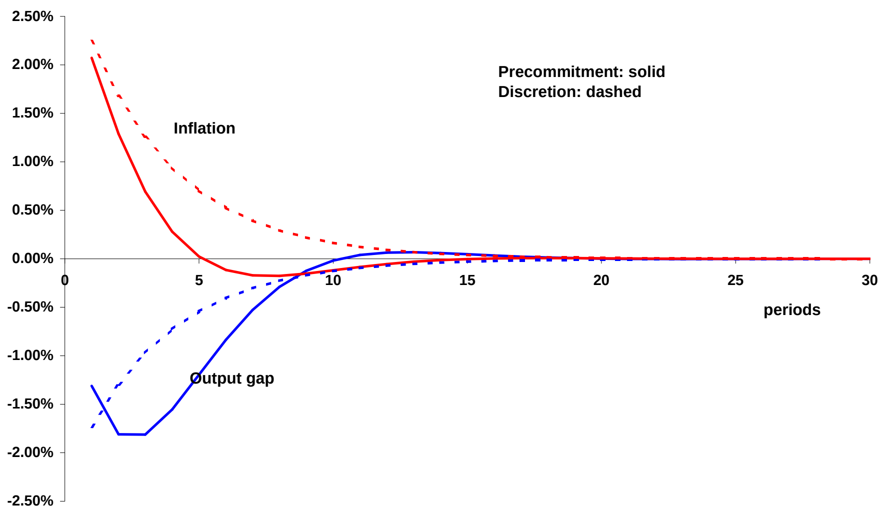

Figure 4: Responses to a Cost Shock with Endogenous Persistence ($\phi = 0.5$)

It is insightful to consider explicitly the first-order conditions for the optimal policy problem under commitment when lagged inflation affects current inflation. Adopting the timeless perspective, maximizing (49) subject to (65) leads to the following first-order conditions:

$$\pi_t = (1 - \phi)\beta \mathrm{E}_t \pi_{t+1} + \phi \pi_{t-1} + \kappa x_t + e_t$$

$$\pi_t + \psi_t - (1 - \phi)\psi_{t-1} - \beta\phi \mathrm{E}_t \psi_{t+1} = 0$$

$$\lambda x_t - \kappa \psi_t = 0,$$

where ψ_t is the Lagrangian multiplier associated with (65). Eliminating this multiplier, the optimal targeting criterion becomes

$$\pi_t = -\left(\frac{\lambda}{\kappa}\right)\left[x_t - (1 - \phi)x_{t-1} - \beta\phi \mathrm{E}_t x_{t+1}\right]. \tag{74}$$

As noted earlier, the presence of forward-looking expectations in the new Keynesian Phillips curve led optimal policy to be backward-looking by introducing inertia through the appearance of x_{t-1} in the optimal targeting rule. The presence of lagged inflation in the inflation adjustment equation when $\phi > 0$ leads policy to be forward-looking through the role of $\mathrm{E}_t x_{t+1}$ in the targeting rule. This illustrates a key aspect of policy design; when policy affects the economy with a lag, policymakers must be forward-looking.

4.6 Targeting Regimes and Instrument Rules

The analysis of optimal policy contained in section 8.5 specified an objective function for the central bank. The central bank was assumed to behave optimally, given its objective function and the constraints imposed on its choices by the structure of the economy. A policy regime in which the central bank is assigned an objective is commonly described as a *targeting regime*. A targeting regime is defined by 1) the variables in the central bank's loss function (the objectives) and 2) the weights assigned to these objectives, with policy implemented under discretion to minimize the expected discounted value of the loss function.[44] Targeting rules were also discussed in chapter 6.3.5, in the context of solving the inflation bias that can arise under discretion.

The most widely analyzed targeting regime is inflation targeting.[45] In 1990, New Zealand became the first country to adopted formal targets for inflation, while now almost 30 countries are formal inflation targeters (see Roger (2010) and Rose (2014)). Experiences with inflation targeting are analyzed by Ammer and Freeman (1995), , Bernanke B. S. and Posen (1998), Mishkin and Schmidt-Hebbel (2002), Mishkin and Schmidt-Hebbel (2007), Amato and Gerlach (2002), and the papers in Lowe (1997) and Leiderman and Svensson (1995). Some of the lessons from inflation targeting are discussed in Walsh (2009) and Walsh (2011).

This section also briefly discusses instrument rules. These constitute an alternative approach to policy that assumes the central bank can commit to a simple feedback rule for its policy instrument. The best known of such rules is the Taylor rule (Taylor (1993)).[46]

4.6.1 Inflation Targeting

The announcement of a formal target for inflation is a key component of any inflation targeting regimes, and this is often accompanied by publication of the central bank's inflation forecasts. An inflation targeting regime can be viewed as the assignment to the central bank of an objective

[44] This definition of a targeting regime is consistent with that of Svensson (1999c), who states, "By a targeting rule, I mean, at the most general level, the assignment of a particular loss function to be minimized" (p. 617). An alternative interpretation of a targeting regime is that it is a rule for adjusting the policy instrument in the face of deviations between the current (or expected) value of the targeted variable and its target level (see, for example, McCallum (1990) and the references he cites). Jensen (2002) and Rudebusch (2002) illustrate these two alternative interpretations of targeting.

[45] Early contributions to the literature on inflation targeting include (Bernanke and Mishkin (1997), Svensson (1997a), Svensson (1997b), Svensson (1999a), Svensson (1999c), and Svensson and Woodford (2005).

[46] Walsh (2015) compares a regime such as inflation targeting in which the central bank is assigned a goal (e.g., achieve 2 percent inflation) to a regime in which the central bank is assigned an instrument rule (e.g., follow the Taylor rule).

function of the form

$$L_t^{IT} = \left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T\right)^2 + \lambda_{IT} x_{t+i}^2 \right], \tag{75}$$

where π^T is the target inflation rate and λ_{IT} is the weight assigned to achieving the output gap objective relative to the inflation objective. λ_{IT} may differ from the weight placed on output gap stabilization in the social loss function (49). As long as $\lambda_{IT} > 0$, specifying inflation targeting in terms of the loss function (75) assumes that the central bank is concerned with output stabilization as well as inflation stabilization. An inflation targeting regime in which $\lambda_{IT} > 0$ is described as a flexible inflation targeting regime.[47]

In the policy problems analyzed so far, the central bank's choice of its instrument i_t allows it to affect both output and inflation immediately. This absence of any lag between the time a policy action is taken and the time it affects output and inflation is unrealistic. If policy decisions taken in period t only affect future output and inflation, then the central bank must rely on forecasts of future output and inflation when making its policy choices. In analyzing the case of such policy lags, Svensson (1997a) and Svensson and Woodford (2005) emphasize the role of inflation-forecast targeting. To illustrate the role of forecasts in the policy process, suppose the central bank must set i_t prior to observing any time t information. This assumption implies that the central bank cannot respond to time-t shocks contemporaneously; information about shocks occurring in period t will affect the central bank's choice of i_{t+1} and, as a consequence, x_{t+1} and π_{t+1} can be affected. Assume that the demand shock in (44) is serially uncorrelated. The central bank's objective is to choose i_t to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_{t-1} \sum_{i=0}^{\infty} \beta^i \left[\left(\pi_{t+i} - \pi^T\right)^2 + \lambda_{IT} x_{t+i}^2 \right],$$

where the subscript on the expectations operator is now $t-1$ to reflect the information available to the central bank when it sets policy. The choice of i_t is subject to the constraints represented by (44) and (45).[48] Taking expectations based on the central bank's information, these two equations can be written as

$$\mathrm{E}_{t-1} x_t = \mathrm{E}_{t-1} x_{t+1} - \left(\frac{1}{\sigma}\right) \left(i_t - \mathrm{E}_{t-1}\pi_{t+1} - \mathrm{E}_{t-1} r_t\right) \tag{76}$$

[47] This is the terminology used in section 6.3.5.

[48] Because (45) was obtained by linearizing around a zero-inflation steady state, one should set $\pi^T = 0$ for consistency. A common assumption in empirical models is that the firms who are not optimally adjusting their price index their price to the central bank's target for inflation. In this case, (45) would be replaced with

$$\pi_t - \pi^T = \beta \mathrm{E}_t \left(\pi_{t+1} - \pi^T\right) + \kappa x_t + e_t.$$

and

$$\mathrm{E}_{t-1}\pi_t = \beta \mathrm{E}_{t-1}\pi_{t+1} + \kappa \mathrm{E}_{t-1}x_t + \rho e_{t-1}, \tag{77}$$

where the cost shock follows an $AR(1)$ process: $e_t = \rho e_{t-1} + \varepsilon_t$. Under discretion, the first-order condition for the central bank's choice of i_t implies that

$$\mathrm{E}_{t-1}\left[\kappa\left(\pi_t - \pi^T\right) + \lambda x_t\right] = 0. \tag{78}$$

Rearranging this first-order condition yields

$$\mathrm{E}_{t-1}x_t = -\left(\frac{\kappa}{\lambda}\right)\mathrm{E}_{t-1}\left(\pi_t - \pi^T\right).$$

Thus, if the central bank forecasts that period-t inflation will exceed its target rate of inflation, it should adjust policy to ensure that the forecast of the output gap is negative.

Svensson (1997a) and Svensson and Woodford (2005) provided detailed discussions of inflation forecast targeting, focusing on the implications for the determinacy of equilibrium under different specifications of the policy decision process. The possibility of multiple equilibria becomes particularly relevant if the central bank bases its own forecasts on private sector forecasts which are in turn based on expectations about the central bank's actions.

4.6.2 Other Targeting Regimes

Inflation targeting is just one example of a policy targeting regime. A number of alternative targeting regimes have been analyzed in the literature. These include price level targeting (Dittmar et al. (1999), Svensson (1999b), Vestin (2006), Dib et al. (2013), Kryvtsov et al. (2008), Cateau et al. (2009) and Billi (2015)), nominal income growth targeting (Jensen (2002)), hybrid price level-inflation targeting (Batini and Yates (2001)), average inflation targeting (Nessen and Vestin (2005)), and regimes based on the change in the output gap or its quasi-difference (Jensen and McCallum (2002), Walsh (2003b)). In each case, it is assumed that, given the assigned loss function, the central bank chooses policy under discretion. The optimal values for the parameters in the assigned loss function, for example, the value of λ_{IT} in (75), are chosen to minimize the unconditional expectation of the social loss function (49).

The importance of forward-looking expectations in affecting policy choice is well illustrated by work on price-level targeting. The traditional view argued that attempts to stabilize the price level, as opposed to the inflation rate, would generate undesirable levels of output variability. A positive cost shock that raised the price level would require a deflation to bring the price level back on target, and this deflation would be costly. However, as figure 3 shows, an optimal commitment policy that focuses on output and inflation stability also induces a deflation after a positive cost shock. By reducing $\mathrm{E}_t\pi_{t+1}$, such a policy achieves a better trade-off between inflation variability and output variability. The deflation generated under a discretionary policy concerned with output and price-level stability might actually come closer to the commitment policy outcomes than discretionary inflation targeting would. Using a basic new Keynesian model, Vestin (2006) showed that this intuition is correct. In fact, when inflation is given by (45) and the cost shock is serially uncorrelated, price level targeting can replicate the timeless precommitment solution exactly if the central bank is assigned the loss function $p_t^2 + \lambda_{PL} x_t^2$, where λ_{PL} differs appropriately from the weight λ in the social loss function.

Jensen (2002) showed that a nominal income growth targeting regime can also dominate inflation targeting. Walsh (2003b) added lagged inflation to the inflation adjustment equation and showed that the advantages of price-level targeting over inflation targeting decline as the weight on lagged inflation increases. Walsh analyzed discretionary outcomes when the central bank targets inflation and the change in the output gap (a *speed limit* policy). Introducing the change in the gap induces inertial behavior similar to that obtained under precommitment. For empirically relevant values of the weight on lagged inflation (ϕ in the range 0.3 to 0.7), speed limit policies dominate price-level targeting, inflation targeting, and nominal income growth targeting. For ϕ below 0.3, price-level targeting does best. Svensson and Woodford (2005) considered interest-rate-smoothing objectives as a means of introducing into discretionary policy the inertia that is optimal under commitment.

4.6.3 Instrument Rules

The approach to policy analysis adopted in the preceding sections starts with a specification of the central bank's objective function and then derives the optimal setting for the policy instrument. An alternative approach specifies an instrument rule directly. The best known of such instrument rules is the Taylor rule (Taylor (1993)). Taylor showed that the behavior of the federal funds interest rate in the United States from the mid-1980s through 1992 (when Taylor was writing) could be fairly well matched by a simple rule of the form

$$i_t = r^* + \pi_t + 0.5 x_t + 0.5 \left(\pi_t - \pi^T \right),$$

where π^T was the target level of average inflation (Taylor assumed it to be 2%) and r^* was the equilibrium real rate of interest (Taylor assumed this too was equal to 2%). The Taylor rule for general coefficients is often written

$$i_t = r^* + \pi^T + \alpha_x x_t + \alpha_\pi \left(\pi_t - \pi^T \right). \tag{79}$$

The nominal interest rate deviates from the level consistent with the economy's equilibrium real rate and the target inflation rate if the output gap is nonzero or if inflation deviates from target. A positive output gap leads to a rise in the nominal rate, as does a deviation of actual inflation above target. With Taylor's original coefficients, $\alpha_\pi = 1.5$, so that the nominal rate is changed more than one-for-one with deviations of inflation from target. Thus, the rule satisfies the Taylor principle (see section 8.3.3); a greater than one-for-one reaction of i_t ensures that the economy has a unique stationary rational expectations equilibrium. Lansing and Trehan (2003) explored conditions under which the Taylor rule emerges as the fully optimal instrument rule under discretionary policy.

A large literature has estimated Taylor rules, or similar simple rules, for a variety of countries and time periods. For example, Clarida et al. (1998) did so for the central banks of Germany, France, Italy, Japan, the UK and the U.S. In their specification, however, actual inflation is replaced by expected future inflation so that the central bank is assumed to be forward-looking in setting policy. Estimates for the United States under different Federal Reserve chairmen are reported by Judd and Rudebusch (1997). In general, the basic Taylor rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching the actual behavior of the policy interest rate. However, Orphanides (2000) found that when estimated using the data on the output gap and inflation actually available at the time policy actions were taken (i.e., using real-time data), the Taylor rule does much more poorly in matching the U.S. funds rate. Clarida et al. (1998) found the Fed moved the funds rate less than one for one during the period 1960-1979, thereby violating the Taylor principle, thereby failing to ensure a determinant equilibrium. Coibion and Gorodnichenko (2011a) show that when average inflation is positive, assessing determinacy depends on the level of inflation and the policy responses to output, as well as the policy response to inflation. In a further example of the importance of using real-time data, however, Perez (2001) finds that when the Fed's reaction function is reestimated for this earlier period using real-time data, the coefficient on inflation is greater than 1. Lubik and Schorfheide (2004) estimated a complete dynamic stochastic general equilibrium (DSGE) new Keynesian model of the U.S. economy and found evidence that Federal Reserve policy has been consistent with determinacy since 1982. However, their estimates suggested policy was not consistent with determinacy prior to 1979. Questioning these results, Cochrane (2011) argued that the Taylor principle applies to beliefs about how the central bank would respond to off-the-equilibrium path behavior. Because such behavior is not observed, the relevant response coefficient is unidentified.

When a policy interest rate such as the federal funds rate in the United Sates is regressed on inflation and output gap variables, the lagged value of the interest rate normally enters with a statistically significant and large coefficient. The interpretation of this coefficient on the lagged interest rate has been the subject of debate. One interpretation is that it reflects inertial behavior of the sort seen in section 8.4.3 that would arise under an optimal precommitment policy and discussed by Woodford (2003b). It has also been interpreted to mean that central banks adjust gradually toward a desired interest rate level. For example, suppose that i_t^* is the central bank's desired value for its policy instrument, but it wants to avoid large changes in interest rates. Such an interest-smoothing objective might arise from a desire for financial market stability. If the central bank adjusts i_t gradually toward i_t^*, then the behavior of i_t may be captured by a partial adjustment model of the form

$$i_t = i_{t-1} + \theta\left(i_t^* - i_{t-1}\right) = (1-\theta)i_{t-1} + \theta i_t^*. \tag{80}$$

The estimated coefficient on i_{t-1} provides an estimate of $1-\theta$. Values close to 1 imply that θ is small; each period the central bank closes only a small fraction of the gap between its policy rate and its desired value.

The view that central banks adjust slowly has been criticized. Sack (2000) and Rudebusch (2002) argued that the presence of a lagged interest rate in estimated instrument rules is not evidence that the Fed acts gradually. Sack attributed the Fed's behavior to parameter uncertainty that leads the Fed to adjust the funds rate less aggressively than would be optimal in the absence of parameter uncertainty. Rudebusch argued that imperfect information about the degree of persistence in economic disturbances induces behavior by the Fed that appears to reflect gradual adjustment. He noted that if the Fed followed a rule such as (80), future changes in the funds rate would be predictable, but evidence from forward interest rates does not support the presence of predictable changes. Similarly, Lansing (2002) showed that the appearance of interest rate smoothing can arise if the Fed uses real-time data to update its estimate of trend output each period. When final data are used to estimate a policy instrument rule, the serial correlation present in the Fed's real-time errors in measuring trend output will be correlated with lagged interest rates, creating the illusion of interest rate-smoothing behavior by the Fed.

4.7 Model Uncertainty

Up to this point, the analysis has assumed that the central bank knows the true model of the economy with certainty. Fluctuations in output and inflation arose only from disturbances that took the form of additive errors. In this case, the linear-quadratic framework results in certainty equivalence holding; the central bank's actions depend on its expectations of future variables but not on the uncertainty associated with those expectations. When error terms enter multiplicatively, as occurs, for example, when the model's parameters are not known with certainty, equivalence will not hold. Brainard (1967) provided the classic analysis of multiplicative uncertainty. He showed that when there is uncertainty about the impact a policy instrument has on the economy, it will be optimal to respond more cautiously than would be the case in the absence of uncertainty.

Brainard's basic conclusion can be illustrated with a simple example. Suppose the inflation adjustment equation given by (45) is modified to take the following form:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa_t x_t + e_t, \tag{81}$$

where $\kappa_t = \bar{\kappa} + v_t$ and v_t is a white noise stochastic process. In this formulation, the central bank is uncertain about the true impact of the gap x_t on inflation. For example, the central bank may have an estimate of the coefficient on x_t in the inflation equation, but there is some uncertainty associated with this estimate. The central bank's best guess of this coefficient is $\bar{\kappa}$, while its actual realization is κ_t. The central bank must choose its policy before observing the actual realization of v_t.

To analyze the impact uncertainty about the coefficient has on optimal policy, assume that the central bank's loss function is

$$L = \frac{1}{2}\mathrm{E}_t\left(\pi_t^2 + \lambda x_t^2\right)$$

and assume that policy is conducted with discretion. In addition, assume that the cost shock e_t is serially uncorrelated.

Under discretion, the central bank takes $\mathrm{E}_t\pi_{t+1}$ as given, and the first-order condition for the optimal choice of x_t is

$$\mathrm{E}_t\left(\pi_t\kappa_t + \lambda x_t\right) = 0.$$

Because all stochastic disturbances have been assumed to be serially uncorrelated, expected inflation will be zero, so from (81), $\pi_t = \kappa_t x_t + e_t$. Using this to rewrite the first-order condition yields

$$\mathrm{E}_t\left[\left(\kappa_t x_t + e_t\right)\kappa_t + \lambda x_t\right] = \left(\bar{\kappa}^2 + \sigma_v^2\right)x_t + \bar{\kappa}e_t + \lambda x_t = 0.$$

Solving for x_t, one obtains

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t. \tag{82}$$

Equation (82) can be compared to the optimal discretionary response to the cost shock when there is no parameter uncertainty. In this case, $\sigma_v^2 = 0$ and

$$x_t = -\left(\frac{\bar{\kappa}}{\lambda + \bar{\kappa}^2}\right)e_t.$$

The presence of multiplicative parameter uncertainty ($\sigma_v^2 > 0$) reduces the impact of e_t on x_t. As uncertainty increases, it becomes optimal to respond less to e_t, that is, to behave more cautiously in setting policy.

Using (82) in the inflation adjustment equation (81),

$$\pi_t = \kappa_t x_t + e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}\left(\kappa_t - \bar{\kappa}\right)}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t = \left(\frac{\lambda + \sigma_v^2 - \bar{\kappa}v_t}{\lambda + \bar{\kappa}^2 + \sigma_v^2}\right)e_t.$$

Because the two disturbances v_t and e_t have been assumed to be uncorrelated, the unconditional variance of inflation is increasing in σ_v^2. In the presence of multiplicative uncertainty of the type modeled here, equilibrium output is stabilized more and inflation less in the face of cost shocks.

The reason for this result is straightforward. With a quadratic loss function, the additional inflation variability induced by the variance in κ_t is proportional to x_t. Reducing the variability of x_t helps to offset the impact of v_t on the variance of inflation. It is optimal to respond more cautiously, thereby reducing the variance of x_t but at the cost of greater inflation variability.

Brainard's basic result–multiplicative uncertainty leads to caution–is intuitively appealing, but it is not a general result. For example, Söderström (2002) examined a model in which there are lagged variables whose coefficients are subject to random shocks. He showed that in this case, optimal policy reacts more aggressively. For example, suppose current inflation depends on lagged inflation, but the impact of π_{t-1} on π_t is uncertain. The effect this coefficient uncertainty has on the variance of π_t depends on the variability of π_{t-1}. If the central bank fails to stabilize current inflation, it increases the variance of inflation in the following period. It can be optimal to respond more aggressively to stabilize inflation, thereby reducing the impact the coefficient uncertainty has on the unconditional variance of inflation.

Some studies have combined the notion of parameter uncertainty with models of learning to examine the implications for monetary policy (see Sargent (1999) and Evans and Honkapohja (2009)). Wieland (2000b) and Wieland (2000a) examined the trade-off between control and estimation that can arise under model uncertainty. A central bank may find it optimal to experiment, changing policy to generate observations that can help it learn about the true structure of the economy.

Another aspect of model uncertainty is measurement error or the inability to observe some relevant variables. For example, the flexible-price equilibrium level of output is needed to measure the gap variable x_t, but it is not directly observable. Svensson and Woodford (2003) and Svensson and Woodford (2004) provided a general treatment of optimal policy when the central bank's problem involves both an estimation problem (determining the true state of the economy such as the value of the output gap) and a control policy (setting the nominal interest rate to affect the output gap and inflation). In a linear-quadratic framework in which private agents and the central bank have the same information, these two problems can be dealt with separately[49] Svensson and Williams (2008) developed a general approach for dealing with a variety of sources of model and data uncertainty.

Finally, the approach adopted in section 4.1 derived welfare-based policy objectives from an approximation to the welfare of the representative agent. The nature of this approximation, however, will depend on the underlying model structure. For example, Steinsson (2003) showed that in the Gali and Gertler (1999) hybrid inflation model, in which lagged inflation appears in the inflation adjustment equation, the loss function also includes a term in the squared change in inflation. Woodford (2003a) found that if price adjustment is characterized by partial indexation to lagged

[49] As an example of the policy probles that arise when the true state of the economy is unobserable, Orphanides (2000) emphasized the role the productivity slowdown played during the 1970s in causing the Fed to overestimate potential output. See also Levin et al. (1999), Ehrmann and Smets (2003), Orphanides and Williams (2002), and Levin et al. (2006).

inflation so that the inflation adjustment equation involves $\pi_t - \gamma\pi_{t-1}$ and $\mathrm{E}_t\left(\pi_{t+1} - \gamma\pi_t\right)$ (see section 6.3.2), the period loss function includes $\left(\pi_t - \gamma\pi_{t-1}\right)^2$ rather than π_t^2. Thus, uncertainty about the underlying model will also translate into uncertainty about the appropriate objectives of monetary policy because policy objectives cannot be defined independently of the model that defines the costs of economic fluctuations (see Walsh (2005a)).

5 Labor market frictions and unemployment

In this section, the basic new Keynesian model is extended in two ways. First, sticky wages are introduced into the model. The resulting framework with sticky prices and sticky wages forms the core foundation of most empirical DSGE models, early examples of which include Christiano et al. (2005) and Smets and Wouters (2007). Second, the assumption of the basic model that all labor adjustment occurs through fluctuations in hours per worker is dropped, and instead adjustment in the number of workers employed is introduced. This change allows unemployment and variations in the fraction of the labor force that is employed to be incorporated. The model of unemployment is based on the search and matching framework of Mortensen and Pissarides (1994) and integrate modern theories of unemployment into a general equilibrium setting with nominal rigidities following Walsh (2003a) and Walsh (2005b).

5.1 Sticky Wages and Prices

The discussion so far has employed a basic new Keynesian model in which prices are sticky but wages have been assumed to be flexible. The underlying labor market in the model featured fluctuations in employment as output fluctuated, but the wage always adjusted to ensure households were able to work their desired number of hours. With prices sticky but wages flexible, a key relative price – the real wage – was able to adjust. It was for this reason that in the face of a productivity shock, actual output could move with the economy's flex-price output level, keeping the output gap equal to zero, while inflation was also kept at zero. For example, a positive productivity shock would increase the marginal product of labor; monetary policy could ensure aggregate demand rises in line with flex-price output, and the real wage would rise to maintain labor market equilibrium while prices could remain unchanged. If, however, both wages and prices are sticky, the real wage becomes sticky. Monetary policy would only be able to keep the output gap at zero if it allows inflation (or deflation) to achieve the required adjustment in the real wage.

Erceg et al. (2000) employed the Calvo specification to incorporate sticky wages *and* sticky prices into an optimizing framework.[50] The goods market side of their model is identical in structure to the one developed in section 3.2. However, Erceg, et. al. assumed that in the labor market individual

[50] Other models incorporating both wage and price stickiness include those of Ravenna (2000), Sbordone (2002) and Christiano et al. (2005). This is now standard in models being taken to the data.

households supply differentiated labor services. Firms combine these labor services to produce output. Output is given by a standard production function, $F(N_t)$, but the labor aggregate is a composite function of the individual types of labor services:

$$N_t = \left[\int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{\gamma}{\gamma-1}}, \qquad \gamma > 1, \tag{83}$$

where n_{jt} is the labor from household j that the firm employs. With this specification, households face a demand for their labor services that depends on the wage they set relative to the aggregate wage rate:

$$n_{jt} = \left(\frac{W_{jt}}{W_t} \right)^{-\gamma} N_t, \tag{84}$$

where $W_{j,t}$ is the nominal rate set by household j and W_t is the aggregate average nominal wage. Erceg, Henderson, and Levin assumed that a randomly drawn fraction of households optimally set their wage each period, just as the Calvo model of price stickiness assumes only a fraction of firms adjust their price each period.

The model of inflation adjustment based on the Calvo specification implies that inflation depends on real marginal cost. In terms of deviations from the flexible-price equilibrium, real marginal cost equals the gap between the real wage and the marginal product of labor (mpl). Similarly, wage inflation (when linearized around a zero inflation steady state) responds to a gap variable, but this time the appropriate gap depends on a comparison between the real wage and the household's marginal rate of substitution between leisure and consumption. With flexible wages, as in the earlier sections where only prices were assumed to be sticky, workers are always on their labor supply curves; nominal wages adjust to ensure the real wage equals the marginal rate of substitution between leisure and consumption (mrs). When nominal wages are also sticky, however, ω_t and mrs_t can differ. If $\omega_t < mrs_t$, workers will want to raise their nominal wage when the opportunity to adjust arises.[51] Letting π_t^w denote the rate of nominal wage inflation, Erceg, Henderson, and Levin showed that

$$\pi_t^w = \beta \mathrm{E}_t \pi_{t+1}^w + \kappa^w \left(mrs_t - \omega_t \right). \tag{85}$$

From the definition of the real wage,

$$\omega_t = \omega_{t-1} + \pi_t^w - \pi_t. \tag{86}$$

Equations (85) and (86), when combined with the new Keynesian Phillips curve in which inflation depends on $\omega_t - mpl_t$, constitute the inflation and wage adjustment block of an optimizing model with both wage and price rigidities.

[51] The variables mpl, mrs, and ω refer to the percent deviation of the marginal productivity of labor, the marginal rate of substitution between leisure and consumption and the real wage around their steady-state values, respectively.

5.1.1 Policy implications

The dispersion of relative wages that arises when not all workers can adjust wages every period will generate a welfare loss, just as a dispersion of relative prices in the goods market did. To see this, let N_t^s denote the total hours of work supplied by households, defined as

$$N_t^s \equiv \int_0^1 n_{jt} dj.$$

The demand for labor supplied by household j is given by (84), Thus,

$$N_t^s = \int_0^1 n_{jt} dj = \left[\int_0^1 \left(\frac{W_{jt}}{W_t} \right)^{-\gamma} dj \right] N_t = \Delta_{w,t} N_t \geq N_t,$$

where $\Delta_{w,t} \geq 1$ is a measure of relative wage dispersion (compare with 21). Output is $F\left(N_t\right) = F\left(\Delta_{w,t}^{-1} N_t^s\right) \leq F(N_t^s)$. The effective amount of labor, $\Delta_{w,t}^{-1} N_t^s$, is less then the total hours workers supply when relative wages differ across workers. Wage dispersion causes the hours of different labor types to be combined in production inefficiently.

Not surprisingly, wage inflation that generates a costly dispersion of relative wages will reduce welfare, just as price inflation did in the presence of sticky prices. Erceg, Henderson and Levin showed that the second-order approximation to the welfare of the representative household no longer is given by (47) but now depends on the volatility of price inflation, the output gap, *and* wage inflation. The welfare approximation takes the form

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda \left(x_{t+i} - x^* \right)^2 + \lambda_w \left(\pi_{t+i}^w \right)^2 \right] + t.i.p.$$

The parameter λ_w is increasing in the degree of wage rigidity, and, like λ, it is decreasing in the degree of price rigidity.

When wages are sticky, they adjust to the gap between the real wage and the marginal rate of substitution between leisure and consumption. When prices are sticky, they adjust to the gap between the marginal product of labor and the real wage. Gali et al. (2007) defined the *inefficiency gap* as the sum of these two gaps, the gap between the household's marginal rate of substitution between leisure and consumption (mrs_t) and the marginal product of labor (mpl_t). This inefficiency gap can be divided into its two parts, the wedge between the real wage and the marginal rate of substitution, which they labeled the *wage markup*, and the wedge between the real wage and the marginal product of labor, labeled the *price markup*. Based on U.S. data, they concluded that the wage markup accounts for most of the time-series variation in the inefficiency gap.

Levin et al. (2005) estimated a new Keynesian general equilibrium model with both price and wage stickiness. They found that the welfare costs of nominal rigidity is primarily generated by wage stickiness rather than by price stickiness. This finding is consistent with Christiano et al. (2005) who concluded that a model with flexible prices and sticky wages does better at fitting impulse responses estimated on U.S. data than a sticky price-flexible wage version of their model. Sbordone (2002) also suggested that nominal wage rigidity is more important empirically than price rigidity, while. Huang and Liu (2002) argued that wage stickiness is more important than price stickiness for generating output persistence. In contrast, Goodfriend and King (2001) argue that the long-term nature of employment relationships reduces the effects of nominal wage rigidity on real resource allocations. Models that incorporate the intertemporal nature of employment relationships based on search and matching models of unemployment were discussed in section 5.2.

The wage markup identified by Galí, Gertler, and Lopez-Salido could arise from fluctuations in markups in labor markets or from the presence of wage rigidities, both of which reflect welfare reducing inefficiencies. However, Chari et al. (2009) caution that this wedge could also reflect time variations in preferences which do not reflect any distortion or inefficiency. For example, suppose the marginal rate of substitution between leisure and consumption is given by $\chi_t N_t^{\eta}/C_t^{-\sigma}$ where χ_t is a taste shock. If μ_t^w equals a time varying markup due to imperfect competition in labor markets, then equilibrium with flexible wages will entail $\mu_t^w \chi_t N_t^{\eta}/C_t^{-\sigma} = W_t/P_t$. Expressing this condition in terms of percent deviations around the steady state yields

$$\omega_t - (\eta\hat{n}_t + \sigma\hat{c}_t) = \hat{\mu}_t^w + \hat{\chi}_t.$$

The labor wedge depends on the shock to the markup and the shock to preferences. The left side of this equation depends on observable variables (the real wage, employment and consumption), so conditional on estimates of η and σ, one can obtain a measure of the labor wedge as $\omega_t - (\eta\hat{n}_t + \sigma\hat{c}_t)$. However this measure alone does not allow one to infer whether fluctuations in $\omega_t - (\eta\hat{n}_t + \sigma\hat{c}_t)$ reflect distortionary shocks (the markup shocks) or nondistortionary shocks (the taste shocks). A decline in employment resulting from decreased market competition in labor markets (a positive markup shock) is welfare reducing; a fall in employment because households desire more leisure (a positive leisure taste shock) is not. For policy purposes, it is important to be able to identify which shock is affecting employment. Because the two shocks enter additively in the measured wedge $\omega_t - (\eta\hat{n}_t + \sigma\hat{c}_t)$, some additional identifying assumption is required, and various approaches have been explored. For example, Galí et al. (2012) used the unemployment rate as an additional observable in an estimated DSGE model in which movements in the unemployment rate only reflect $\hat{\mu}_t^w$ shocks, while in the baseline estimated DSGE model of Justiniano et al. (2013), low frequency movements in the labor wedge are attributed to taste shocks while much of the high frequency movement of wages is attributed to measurement error in the wage series. Thus, there is little role left for inefficient markup shocks to play. Sala et al. (2010) showed how assuming $\hat{\chi}_t$ follows an AR(1) process while $\hat{\mu}_t^w$ is white noise leads to very different estimates of the magnitude of inefficient fluctuations as compared to the case when $\hat{\chi}_t$ is assumed to be white noise while $\hat{\mu}_t^w$ is an AR(1) process.

5.2 Unemployment

The basic new Keynesian model adds imperfect competition and nominal rigidities to what is otherwise an equilibrium real business cycle model. In common with many real business cycle models, all labor adjustment occurs along the hours margin, with the measure of employment in the model defined as the fraction of time the representative household spends engaged in market work. When output in the model declines, hours per worker fall but all workers remain employed; there is no adjustment in the fraction of workers who are employment. Yet for an economy such as the U.S., most of the fluctuation in total hours over a business cycle results from movements in employment rather than in hours per employee. Log total hours is equal to the log of average hours per employee plus the log of employment, and over the 1960-2015 period, the standard deviation of log total hours is 1.87, that of log average hours is 0.50, and that of log employment is 1.57.[52] The correlation between the log of total hours and the log of employment is 0.97; the correlation between log total hours and log average hours per employee is 0.68. The 2008-2009 Great Recession in the U.S. was associated with a fall in total output in the nonfarm business sector of 7.60%, a fall in total labor hours of 7.22%, a fall in employment of 5.54% and a fall in average hours of 1.68%. Thus, most the labor adjustment occurs on the employment or extensive margin, and much less on the average hours or intensive margin. In this section, the new Keynesian model will be modified to incorporate fluctuations in the fraction of workers who have jobs and not just on the fluctuations of hours worker per employee. This modification allows unemployment to be introduced explicitly into the model. Extending the model to include unemployment also allows one to address such issues as the welfare effects of fluctuations in unemployment and the possible role that the unemployment rate, as distinct from the output gap, should play in the design of monetary policy.

The standard macroeconomic model of unemployment is provided by the search and matching Diamond-Mortensen-Pissarides (DMP) model (Mortensen and Pissarides (1994)). **Walsh (2003a)** and Walsh (2005b) were the first papers to integrate the DMP model into a new Keynesian model with sticky prices. Walsh assumed all labor adjustment occurred along the extensive margin, with hours per employee fixed and argued that in a model with habit persistence in consumption, inertia in the policy instrument rule, and search and matching frictions in the labor market, persistent effects of monetary policy shocks could be captured with a lower and more realistic degree of price stickiness than employed in standard new Keynesian models.[53] In a paper contemporaneous with Walsh (2005b), Trigari (2009) developed a similar model and estimates it using U.S. data. Other

[52] This is for the nonfarm business sector. All variables are HP filtered.

[53] Heer and Maußner (2010) find that this results depends on the assumption in Walsh (2005b) of a fixed capital stock.

contributions that add unemployment variation to a new Keynesian model include Blanchard and Galí (2007), Krause et al. (2008), Thomas (2008), Ravenna and Walsh (2008), Sala et al. (2008), Gertler and Trigari (2009), Blanchard and Galí (2010), Lechthaler and Snower (2010), Thomas (2011), Ravenna and Walsh (2011), Ravenna and Walsh (2012a), Ravenna and Walsh (2012b), Lago Alves (2012), Sala et al. (2012), and Galí (2013).

5.2.1 A sticky-price NK model with unemployment

To illustrate the implications of search and matching frictions in a monetary policy model, the discussion here will follow Ravenna and Walsh (2011). Given than most employment volatility occurs on the extensive margin, the model assumes hours per worker are fixed and all labor adjustment consists of changes in the fraction of the workforce that is employed. This reverses the standard new Keynesian specification in which hours per worker do all the varying. To adding a search and matching specification to capture labor market behavior, the basic sticky-price new Keynesian model will be modified in several ways. Because the model will contain two frictions (sticky prices and search frictions in the labor market), it will be convenient to follow Walsh (2003a) and introduce two types of firms, one with sticky prices and the other facing labor market frictions.[54]

Specifically, assume there is a measure one of retail firms who sell differentiated goods to households and whose prices are sticky. These retail firms do not employ labor but instead buy a homogeneous intermediate good that they use to produce their final output. Price adjustment by retail firms follows the Calvo model, so price inflation of the consumption goods purchased by households will depend on expected future inflation and the real marginal cost of retail firms. Their real marginal cost is simply P_t^I/P_t, where P_t^I is the price of intermediate inputs and P_t is the consumer price index. The other type of firms hire labor and produce the intermediate good. This good is sold in a competitive market to the retail firms, and the price of the intermediate good is flexible.

Rather than assuming households equate the marginal rate of substitution between leisure and consumption to the real wage, workers are assumed either to be employed, in which case they work a fixed number of hours, or unemployed and searching for a new job. Employment will be an endogenous state variable, and a new equation is added to keep track of its evolution. Employment will decrease if the flow of workers from employment to unemployment exceeds the flow of unemployed workers into jobs. For simplicity, the flow of workers who separate from jobs and become unemployed is taken to occur at a constant rate s per period.[55] Assume in period t a fraction q_t^u of the unemployed job seekers find jobs. If employment is denoted by e_t and the number

[54] The decision problem of a firm simultaneously facing both price adjustment frictions and labor market search frictions are analyzed in Thomas (2008), Thomas (2011), and Lago Alves (2012).

[55] For NK search and matching models with endogenous separations, see Walsh (2005b) or Ravenna and Walsh (2012a).

of job seekers in period t is denoted by u_t,

$$u_t = 1 - (1 - s)\, e_{t-1}, \tag{87}$$

and employment is

$$e_t = (1 - s)\, e_{t-1} + q_t^u u_t. \tag{88}$$

With this particular specification, u_t is predetermined and equals the fraction of the labor force that is seeking jobs during period t.[56]

The key innovation of Mortensen and Pissarides (1994) was to model the process by which unemployment workers and vacant positions at firms lead to actual employment matches. Let v_t denote the number of job vacancies. Then the number of new job matches m_t is given by

$$m_t = m\left(u_t, v_t\right), \tag{89}$$

where m is increasing in both u and v. With random search, the job finding rate for an unemployed worker is m_t/u_t and the job filling rate for a firm with a vacancy is m_t/v_t. It is common (and consistent with empirical evidence), to assume the matching function m displays constant returns to scale, and a Cobb-Douglas function form is often assumed. In this case, one can write

$$m_t = m_0 u_t^a v_t^{1-a} = m_0 \theta_t^{1-a} u_t,\ 0 < a < 1,$$

where $\theta_t \equiv v_t/u_t$ is a measure of labor market tightness. The rate at which unemployed find jobs, $m_t/u_t = q_t^u = m_0\theta_t^{1-a}$ is an increasing function of labor market tightness. The job filling rate is $m_t/v_t = m_0\theta_t^{1-a}u_t/v_t = m_0\theta_t^{-a}$ is a decreasing in labor market tightness.

Since a new variable, job vacancies, has been added. a theory of job creation is needed. Assume a firm faces a cost k per period to post a job vacancy.[57] If there is free entry to job posting, firms will create job openings until the expected return net of the cost k is driven to zero:

$$\left(\frac{m_t}{v_t}\right) V_t^J = m_0\theta_t^{-a} V_t^J = k, \tag{90}$$

where $m_0\theta_t^{-a}$ is the number of hires the firm expects to make if it posts a job opening for a period and V_t^J is the value of having a job filled.[58] Assuming a constant returns to scale production

[56] The choice of timing allows workers who find jobs in period t to produce within the period. With hours per employee fixed, this ensures in a sticky price new Keynesian model that in response to an aggregate demand shock, output can expand within the same period.

[57] Petrosky-Nadeau and Zhang (2013a) and Petrosky-Nadeau and Zhang (2013b) discuss the case of non-fixed costs of posting job vacancies.

[58] If V_t^V is the value to the firm of having a job opening posted, then

$$V_t^V = -k + m_0\theta_t^{-a} V_t^J + \left(1 - m_0\theta_t^{-a}\right) \mathrm{E}_t V_{t+1}^V.$$

technology with labor the only variable input, output at firm j is $y_{j,t} = Z_t N_{j,t}$. The value of a worker to the firm, expressed in terms of final consumption goods, is therefore

$$V_t^J = \left(\frac{P_t^I}{P_t}\right) Z_t - \omega_t + (1-s)\,\beta \mathrm{E}_t \Omega_{t,t+1} V_{t+1}^J. \tag{91}$$

To understand this expression, define $\mu_t \equiv P_t/P_t^I$ as the price of retail goods relative to the intermediate good (the retail price markup). The net profit from the hire is the value of output produced net of the real real wage, $\left(P_t^I/P_t\right) Z_t - \omega_t = \mu_t^{-1} Z_t - \omega_t$. Because with probability $1-s$ the worker does not separate, the current value of the worker to the firm also includes the expected future value of the worker, V_{t+1}^J. The term $\Omega_{t,t+1}$ is the stochastic factor for discounting time $t+1$ valuations back to period t.

Recall from (11) that in the basic new Keynesian model, real marginal cost with flexible prices (as is the case here in the intermediate goods sector), $Z_t/\mu_t = \omega_t$. From (90) and (91),

$$\frac{Z_t}{\mu_t} = \omega_t + \left(\frac{k}{m_0}\right)\left[\theta_t^a - (1-s)\,\beta \mathrm{E}_t \Omega_{t,t+1} \theta_{t+1}^a\right]. \tag{92}$$

The left side of this equation is the real marginal value of a worker; the right side is the cost of labor. It includes the wage plus the search cost of hiring the worker,[59] but it is reduced by the expected savings in search costs in $t+1$ from having a worker in place in time t. Importantly, given the current real wage, the firm's labor costs are increasing in current labor market tightness, as a rise in θ_t implies it takes longer to fill a job, but they are decreasing in expected future labor market tightness, as a rise in θ_{t+1} increasing the value to the firm of its existing workers.

To close the model, a specification of wage determination must be added. Because of the search frictions present in the labor market, as long as $V_t^J > 0$ the value to the firm of having a worker in place is greater than the alternative of having an unfilled vacancy. Similarly for a worker, being employed will be worth more to the worker than being unemployed.[60] Thus, there is a surplus to both parties to a job match. The surplus to the worker is the difference between having a job and not having one. Let V_t^E denote the value to the worker of having a job. It is given by the wage net of the disutility of working, plus the expected discounted value of still being employed (which occurs with probability $1-s$) and the value of not being employed. The latter, which occurs with probability s, consists of the expected value of finding a new job, $q_{t+1}^u V_{t+1}^E$ plus the expected value

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Profit maximizing firms will add job openings as long as $V_t^V > 0$ and will close job openings if $V_t^V < 0$. So in equilibrium, $V_t^V = 0$ for all t and

$$0 = -k + m_0 \theta_t^{-a} V_t^J.$$

Rearranging yields (90).

[59] If the probability of filling a vacancy is $m_0\theta_t^{-a}$, the excepted time it takes to hire a worker is $(1/m_0\theta_t^{-a})$ at a cost k per unit of time.

[60] We have essentially assumed this in assuming any unemployed worker actively searches for anotehr job.

of continuing to be unemployed, $\left(1-q^u_{t+1}\right)V^U_{t+1}$. Thus,[61]

$$V^E_t = \omega_t + \beta \mathrm{E}_t \Omega_{t,t+1}\left\{(1-s)\,V^E_{t+1} + s\left[q^u_{t+1}V^E_{t+1} + \left(1-q^u_{t+1}\right)V^U_{t+1}\right]\right\}.$$

The value of being unemployed arises from any unemployment benefit or home production produced when unemployment, ω^u_t, plus the expected gain if a new job is found:

$$V^U_t = \omega^u_t + \beta \mathrm{E}_t \Omega_{t,t+1}\left[q^u_{t+1}V^E_{t+1} + \left(1-q^u_{t+1}\right)V^U_{t+1}\right].$$

Noting that $q^u_{t+1} = m_0\theta^{1-a}_{t+1}$, the worker's surplus from being employed is therefore

$$V^E_t - V^U_t = \omega_t - \omega^u_t + (1-s)\,\beta \mathrm{E}_t \Omega_{t,t+1}\left(1-m_0\theta^{1-a}_{t+1}\right)\left(V^E_{t+1} - V^U_{t+1}\right). \tag{93}$$

Any wage such that $V^J_t \geq 0$ and $V^E_t - V^U_t \geq 0$ is compatible with the worker and firm each finding it individually rational to continue the employment match. To determine the wage, the standard approach in the literature has been to assume Nash bargaining with fixed bargaining weights.[62] Under Nash bargaining, both the worker and the firm have an incentive to maximize the joint surplus from the match and then to divide this maximized surplus with a fixed share α going to the worker and $1-\alpha$ going to the firm. Using (91) and (93), the joint surplus, $V^E_t - V^U_t + V^J_t$, is independent of the wage. The role of the wage is to ensure the appropriate division of the surplus between worker and firm. Thus, the wage ensures $V^E_t - V^U_t = a\left(V^E_t - V^U_t + V^J_t\right)$, or $(1-\alpha)\left(V^E_t - V^U_t\right) = \alpha V^J_t$. Using (91), (90), and (93),

$$\omega_t = \omega^u_t + \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\kappa}{m_0}\right)\left[\theta^a_t - (1-s)\,\beta \mathrm{E}_t \Omega_{t,t+1}\left(1-m_0\theta^{1-a}_{t+1}\right)\theta^a_{t+1}\right]. \tag{94}$$

The new Keynesian model with search and matching frictions in the labor market consists then of the standard household Euler condition, the model of price adjustment by retail firms, and the specification of the labor market. Equations (7), (13) with real marginal cost given by $1/\mu_t$, and (17) given the equilibrium conditions for consumption, optimal price setting by retail firms, and the definition of the aggregate price level as a function of the prices set by adjusting firms and the lagged price level. Goods clearing implies

$$C_t + kv_t = Y_t, \tag{95}$$

where Y_t is retail output which is used for consumption and the costs of posting vacancies. From

[61] For simplicity, this assumes there is no disutility from working; if there is, the wage should be interpreted as net of such costs.

[62] Petrosky-Nadeau and Zhang (2013b) and Christiano et al. (2013) adopt the alternating offer bargaining model of Hall and Milgrom (2008).

(20), output of the intermediate goods sector used to produce retail goods is

$$Y_t^I = Y_t \Delta_t, \tag{96}$$

where Δ_t is the measure of retail price dispersion given by (21). The aggregate production function in the intermediate sector is

$$Y_t^I = Z_t e_t, \tag{97}$$

and employment evolves according to

$$e_t = (1 - s)\, e_{t-1} + m_0 \theta_t^{1-a} \left[1 - (1 - s)\, e_{t-1}\right]. \tag{98}$$

This last equation is obtained from (87), (88) and the definition of q_t^u. From the definition of θ_t,

$$v_t = \frac{u_t}{\theta_t} = \frac{1 - (1 - s)\, e_t}{\theta_t}. \tag{99}$$

Collecting the equilibrium conditions, one has (7), (13), (17), (21), (92), (94), and (95)-(99). These eleven equations, plus the specification of monetary policy, determine the equilibrium values of consumption, the nominal interest rate, the optimal price chosen by adjusting firms, the retail price index, the measure of relative price dispersion, the retail price markup, the real wage, output in the retail sector, output in the intermediate goods sector, employment, vacancies, and labor market tightness.

Implications for monetary policy A number of authors have invested the role of monetary policy in models with nominal rigidities and search and matching frictions in the labor market. The focus in **Walsh (2003a)** and Walsh (2005b) was on the dynamic effects of labor market frictions. Faia (2008) considers monetary policy rules, and Kurozumi and Zandweghe (2010) study how search and matching frictions affect the conditions on policy required to ensure determinacy. Thomas (2008) , Faia (2009), and Ravenna and Walsh (2011) study optimal policy.[63]

In section 4.1, optimal policy in a basic NK model was studied using a second-order approximation to the welfare of the represented household. Ravenna and Walsh (2011) derive the second-order approximation to the welfare of the representative household in a NK model that includes a search and matching model of the labor market. They assume wages are flexible, set by Nash bargaining, but they allow the bargaining share to exhibit stochastic volatility. Shocks to the bargaining share operate like inefficeint markup shocks in a basic NK model. Their results illustrate directly the role that labor market variables play in a welfare-based policy objective. Assume inflation in the steady-state is zero and the steady-state output level is efficient, so that x^* in (47) is zero. As in

[63] See also Blanchard and Galí (2010).

the basic new Keynesian model, this requires a fiscal subsidy to offset the steady-state distortions arising from imperfect competition. It also requires that labor's share of the joint match surplus be equal to the elasticity of matches with respect to employment, or $\alpha = a$. This condition for efficiency is due to Hosios (1990). Ravenna and Walsh then show that the welfare approximation is given by[64]

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(V_{t+i} - \bar{V} \right) \approx -\Omega \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x \tilde{c}_{t+i}^2 + \lambda_\theta \tilde{\theta}_{t+i}^2 \right] + t.i.p., \tag{100}$$

where $\bar{V}$ is steady-state welfare, and $\tilde{c}_t$ and $\tilde{\theta}_t$ are the log gaps between consumption and labor market tightness and their efficient values, respectively. The first two terms, including the parameter λ_x on $\tilde{c}_t^2$, are identical to the squared inflation and output gap terms in (47) because in the basic model without capital, consumption and output are equal (to first order).[65] The new term, $\tilde{\theta}_t^2$ reflects inefficient labor market fluctuations. The weight on this term is

$$\lambda_\theta = \alpha \left(\frac{\lambda_x}{\sigma} \right) \frac{\kappa \bar{v}}{\bar{C}},$$

where $\kappa \bar{v} / \bar{C}$ equals steady state vacancy posting costs relative to consumption. Recall that output in this model is used for consumption or for job posting. If job posting costs are large relative to consumption, then it becomes more important to stabilize the labor market at its efficient level of tightness. For their baseline calibration, however, Ravenna and Walsh report that λ_θ is small reflecting both the finding in the basic NK model that λ_x is small, but also the assumption that vacancy posting costs are a small share of output. In fact, Blanchard and Galí (2010) assume such costs are small in deriving a welfare measure and so end up with only inflation and an output gap appearing in their policy objective function.

Ravenna and Walsh (2012a) find that when wages are set by Nash bargaining, even if the Hosios condition does not hold, the cost of labor search inefficiencies can be large, but the associated welfare cost is primarily a steady-state cost, so there is little scope for cyclical monetary policy to correct it. Price stability remains close to optimal.

5.2.2 Sticky wages in search and matching models

The model of the previous section took prices to be sticky but treated wages as being flexible. As Shimer (2005) demonstrated, the basic DMP model with flexible wages is unable to match the volatility of unemployment, implying too little volatility in unemployment and too much in wages

[64] Terms independent of policy are ignored.

[65] The value of λ_x in (47) depended on the inverse wage elasticity of labor supply, η, but in the model of this section, labor is supplied inelastically so $\eta = \infty$.

to be consistent with the macroeconomic evidence.[66] The standard response has been to modify the model by introducing wage stickiness as doing so increases the volatility of unemployment. Earlier examples include Gertler and Trigari (2009) who adopt a Calvo formulation in which a fraction of matches renegotiate wages each period.

In the search and matching approach to labor markets, it is the wage of newly hired workers that is relevant for the firm's job posting decisions, and the micro evidence suggest these wages are much more flexible than wages of existing workers.[67]. The Shimer puzzle has been studied primarily in models in which productivity shocks are the only source of aggregate fluctuations and both prices and wages are flexible. However, Andrés et al. (2006) shows that in a rich general equilibrium environment, price stickiness plays an important role in increasing the volatility of unemployment and vacancies closer to that observed in the data. Lago Alves (2012) shows that even if all wages are flexible, introducing a non-zero trend rate of inflation when prices adjust ala Calvo increases the volatility of unemployment sufficiently to solve the Shimer puzzle. In standard new Keynesian models, either trend inflation is assumed to be zero, or indexation not seen in the micro evidence is introduced to ensure trend inflation is neutral. As Alves shows, the assumption about indexation is critical in affecting unemployment dynamics.

The monetary policy implications of sticky wages in a search and matching framework are similar to those seen earlier. When combined with sticky prices, the presence of multiple sources of nominal frictions force the policymaker to make trade-offs in attempting to stabilize inflation, wage inflation, the output gap, and the labor tightness gap. In the extreme case of fixed nominal wages, labor market inefficiencies are large and volatile over the business cycle. However, Ravenna and Walsh (2012a) find that monetary policy is not an efficient instrument for correcting these distortions in labor markets as large and costly deviations from price stability would be required.

6 Summary

This chapter has reviewed the basic new Keynesian model that has come to dominate modern macroeconomics, particularly for addressing monetary policy issues. The basic model is a dynamic, stochastic general equilibrium model based on optimizing households, with firms operating in an environment of monopolistic competition and facing limited ability to adjust their prices. The staggered overlapping process of price adjustment apparent in the micro evidence (see chapter 6) is captured through the use of the Calvo mechanism. The details would differ slightly if an alternative model of price stickiness were employed, but the basic model structure would not change. This

[66] Shimer adopted standard values to calibrate the model. Hagedorn and Manovskii (2008) show a better match to the data is obtained if the value of the worker's outside option of unemployment is close to the value of employment. However, Costain and Reiter (2008) show that adopting the Hagedorn-Manovskii calibration implies the model is inconsistent with evidence of XXXXXXXXXX

[67] Kudlyak (2014) estimates the cyclicality of the user cost of labor. See also the references she cites.

structure consists of two basic parts. The first is an expectational IS curve derived from the Euler condition describing the first-order condition implied by intertemporal optimization on the part of the representative household. The second is a Phillips curve relationship linking inflation to an output gap measure. These two equilibrium relationships are then combined with a specification of monetary policy.

The model provides insights into the costs of inflation in generating an inefficient dispersion of relative prices. A model-consistent objective function for policy, derived as a second-order approximation to the welfare of the representative agent, calls for stabilizing inflation volatility and volatility in the gap between output and the output level that would arise under flexible prices.

The new Keynesian approach emphasizes the role of forward-looking expectations. The presence of forward-looking expectations implies that expectations about future policy actions play an important role, and a central bank that can influence these expectations, as assumed under a policy regime of commitment, can do better than one that sets policy in a discretionary manner.

7 Appendix

This appendix provides details on the derivation of the linear new Keynesian Phillips curve and on the approximation to the welfare of the representative household.

7.1 The New Keynesian Phillips Curve

In this section, (13) and (17) are used to obtain an expression for the deviations of the inflation rate around its steady-state level. The the steady state is assumed to involve a zero rate of inflation. Let $Q_t = p_t^*/P_t$ be the relative price chosen by all firms that adjust their price in period t. The steady-state value of Q_t is $Q = 1$; this is also the value Q_t equals when all firms are able to adjust every period. Dividing (17) by $P_t^{1-\theta}$, one obtains $1 = (1-\omega)Q_t^{1-\theta} + \omega\left(P_{t-1}/P_t\right)^{1-\theta}$. Expressed in terms of percentage deviations around the zero- inflation steady state, this becomes

$$0 = (1-\omega)\hat{q}_t - \omega\pi_t \Rightarrow \hat{q}_t = \left(\frac{\omega}{1-\omega}\right)\pi_t. \tag{101}$$

To obtain an approximation to (**??**), note that it can be written as

$$\left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}\right] Q_t = \mu\left[\mathrm{E}_t \sum_{i=0}^{\infty} \omega^i \beta^i C_{t+i}^{1-\sigma} \varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}\right]. \tag{102}$$

In the flexible-price equilibrium with zero inflation, $Q_t = \mu\varphi_t = 1$. The left side of (102) is approximated by

$$\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)+\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\hat{q}_t+C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+(\theta-1)\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right].$$

The right side is approximated by

$$\mu\left\{\left(\frac{C^{1-\sigma}}{1-\omega\beta}\right)\varphi+\varphi C^{1-\sigma}\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i}+(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+\theta\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right]\right\}.$$

Setting these two expressions equal and noting that $\mu\varphi=1$ yields

$$\begin{aligned}&\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t+\sum_{i=0}^{\infty}\omega^i\beta^i\left[(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+(\theta-1)\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right]\\ =\;&\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i}+(1-\sigma)\,\mathrm{E}_t\hat{c}_{t+i}+\theta\left(\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right)\right].\end{aligned}$$

Canceling the terms that appear on both sides of this equation leaves

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t=\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}-\hat{p}_t\right),$$

or

$$\left(\frac{1}{1-\omega\beta}\right)\hat{q}_t=\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}\right)-\left(\frac{1}{1-\omega\beta}\right)\hat{p}_t.$$

Multiplying by $1-\omega\beta$ and adding $\hat{p}_t$ to both sides yields

$$\hat{q}_t+\hat{p}_t=(1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{t+i}+\mathrm{E}_t\hat{p}_{t+i}\right).$$

The left side is the optimal nominal price $\hat{p}_t^*=\hat{q}_t+\hat{p}_t$, and this is set equal to the expected discounted value of future nominal marginal costs. This equation can be rewritten as $\hat{q}_t+\hat{p}_t=(1-\omega\beta)\left(\hat{\varphi}_t+\hat{p}_t\right)+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\hat{p}_{t+1}\right)$. Rearranging this expression yields

$$\begin{aligned}\hat{q}_t&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\hat{p}_{t+1}-\hat{p}_t\right)\\&=(1-\omega\beta)\hat{\varphi}_t+\omega\beta\left(\mathrm{E}_t\hat{q}_{t+1}+\mathrm{E}_t\pi_{t+1}\right).\end{aligned}$$

Now using (101) to eliminate $\hat{q}_t$, one obtains

$$\begin{aligned}\left(\frac{\omega}{1-\omega}\right)\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+1}\right] \\ &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}.\end{aligned}$$

Multiplying both sides by $(1-\omega)/\omega$ produces the forward-looking new Keynesian Phillips curve:

$$\pi_t = \tilde{\kappa}\hat{\varphi}_t + \beta\mathrm{E}_t\pi_{t+1},$$

where

$$\tilde{\kappa} = \frac{(1-\omega)(1-\omega\beta)}{\omega}.$$

When production is subject to diminishing returns to scale, firm specific marginal cost may differ from average marginal cost. Let $A = \theta(1-a)/a$. All firms adjusting at time t set their relative price such that

$$\begin{aligned}\hat{q}_t + \hat{p}_t &= (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left(\mathrm{E}_t\hat{\varphi}_{jt+i} + \mathrm{E}_t\hat{p}_{t+i}\right) \\ &= (1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+i} - A\left(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+i}\right) + \mathrm{E}_t\hat{p}_{t+i}\right].\end{aligned}$$

This equation can be rewritten as

$$\begin{aligned}\hat{q}_t + \hat{p}_t &= (1-\omega\beta)\left(\hat{\varphi}_t - A\hat{q}_t + \hat{p}_t\right) \\ &\quad \omega\beta(1-\omega\beta)\sum_{i=0}^{\infty}\omega^i\beta^i\left[\mathrm{E}_t\hat{\varphi}_{t+1+i} - A\left(\hat{q}_t + \hat{p}_t - \mathrm{E}_t\hat{p}_{t+1+i}\right) + \mathrm{E}_t\hat{p}_{t+1+i}\right].\end{aligned}$$

By rearranging this equation, and recalling that $\hat{q}_t = \omega\pi_t/(1-\omega)$, one obtains

$$\begin{aligned}\left(\frac{\omega}{1-\omega}\right)(1+A)\,\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left[\left(\frac{\omega}{1-\omega}\right)\mathrm{E}_t\pi_{t+1} + \mathrm{E}_t\pi_{t+1}\right] \\ &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(1+A)\left(\frac{1}{1-\omega}\right)\mathrm{E}_t\pi_{t+1}\end{aligned}$$

Multiplying both sides by $(1-\omega)/\omega(1+A)$ produces

$$\pi_t = \beta\mathrm{E}_t\pi_{t+1} + \left(\frac{\tilde{\kappa}}{1+A}\right)\hat{\varphi}_t$$

7.2 Approximating Utility

The details of the welfare approximation that lead to (47) are provided. In addition to the discussion provided in Woodford (2003a), see Chapter 4, Appendix A of Galí (2008).

To derive an approximation to the representative agent's utility, it is necessary to first introduce some additional notation. For any variable X_t, let $\bar{X}$ be its steady-state value, let X_t^* be its efficient level (if relevant), and let $\tilde{X}_t = X_t - \bar{X}$ be the deviation of X_t around the steady state. Let $\hat{x}_t = \log\left(X_t/\bar{X}\right)$ be the log deviation of X_t around its steady-state value. Using a second order Taylor approximation, the variables $\tilde{X}_t$ and $\hat{X}_t$ can be related as

$$\tilde{X}_t = X_t - \bar{X} = \bar{X}\left(\frac{X_t}{\bar{X}} - 1\right) \approx \bar{X}\left(\hat{x}_t + \frac{1}{2}\hat{x}_t^2\right). \tag{103}$$

Employing this notation, one can develop a second order approximation to the utility of the representative household.

Suppose

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} = \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}\right]. \tag{104}$$

The first term on the right of (46) is the utility from consumption. Start by approximating each term in the utility function

$$V_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}.$$

In general, if utility from consumption is $U(C_t)$, a second-order Taylor expansion around steady-state consumption $\bar{C}$ yields

$$\begin{aligned}
U(C_t) &\approx U(\bar{C}) + U_c(\bar{C})\left(C_t - \bar{C}\right) + \frac{1}{2}U_{cc}\left(\bar{C}\right)\left(C_t - \bar{C}\right)^2 \\
&= U(\bar{C}) + U_c(\bar{C})\tilde{C}_t + \frac{1}{2}U_{cc}\left(\bar{C}\right)\tilde{C}_t^2 \\
&= U(\bar{C}) + U_c(\bar{C})\bar{C}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \frac{1}{2}U_{cc}\left(\bar{C}\right)\bar{C}^2\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right)^2 \\
&= U(\bar{C}) + U_c(\bar{C})\bar{C}\left(\hat{c}_t + \frac{1}{2}\hat{c}_t^2\right) + \frac{1}{2}U_{cc}\left(\bar{C}\right)\bar{C}^2\left(\hat{c}_t^2 + \frac{1}{2}\hat{c}_t^3 + \frac{1}{4}\hat{c}_t^4\right) \\
&\approx U(\bar{C}) + U_c(\bar{C})\bar{C}\hat{c}_t + \frac{1}{2}U_c(\bar{C})\bar{C}\hat{c}_t^2 + \frac{1}{2}U_{cc}\left(\bar{C}\right)\bar{C}^2\hat{c}_t^2,
\end{aligned}$$

where U_c and U_{cc} denote the first and second derivatives of U and terms such as $\hat{c}_t^3$ and $\hat{c}_t^4$ have been ignored. When $U\left(C_t\right) = C_t^{1-\sigma}/\left(1-\sigma\right)$, the utility from consumption can then be approximated around the steady state as

$$\begin{aligned}
\frac{C_t^{1-\sigma}}{1-\sigma} &\approx \frac{\bar{C}^{1-\sigma}}{1-\sigma} + \bar{C}^{1-\sigma}\hat{c}_t + \frac{1}{2}\bar{C}^{1-\sigma}\hat{c}_t^2 - \frac{1}{2}\sigma\bar{C}^{1-\sigma}\hat{c}_t^2 \\
&\approx \frac{\bar{C}^{1-\sigma}}{1-\sigma} + \bar{C}^{1-\sigma}\left[\hat{c}_t + \frac{1}{2}(1-\sigma)\hat{c}_t^2\right],
\end{aligned} \tag{105}$$

where terms of order three or higher such as $\hat{c}_t^3$ and $\hat{c}_t^4$ have been ignored.

Next, one can analyze the term arising from the disutility of work. Expanding this around the steady state yields

$$\begin{aligned}
\chi\frac{N_t^{1+\eta}}{1+\eta} &\approx \chi\frac{\bar{N}^{1+\eta}}{1+\eta} + \chi\left[\bar{N}^{\eta}\tilde{N}_t + \frac{1}{2}\eta\bar{N}^{\eta-1}\tilde{N}_t^2\right] \\
&\approx \chi\frac{\bar{N}^{1+\eta}}{1+\eta} + \chi\bar{N}^{\eta}\left[\bar{N}\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2\right) + \frac{1}{2}\eta\bar{N}\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2\right)^2\right] \\
&= \chi\frac{\bar{N}^{1+\eta}}{1+\eta} + \chi\bar{N}^{1+\eta}\left[\hat{n}_t + \frac{1}{2}\hat{n}_t^2 + \frac{1}{2}\eta\left(\hat{n}_t^2 + \frac{1}{2}\hat{n}_t^3 + \frac{1}{4}\hat{n}_t^4\right)\right] \\
&\approx \chi\frac{\bar{N}^{1+\eta}}{1+\eta} + \chi\bar{N}^{1+\eta}\left[\hat{n}_t + \frac{1}{2}(1+\eta)\hat{n}_t^2\right].
\end{aligned} \tag{106}$$

Hence, the second order approximation of

$$V_t - \bar{V} \approx \bar{C}^{1-\sigma}\left[\hat{c}_t + \frac{1}{2}(1-\sigma)\hat{c}_t^2\right] - \chi\bar{N}^{1+\eta}\left[\hat{n}_t + \frac{1}{2}(1+\eta)\hat{n}_t^2\right]. \tag{107}$$

From the good market clearing condition, output is

$$Y_t = \int c_{jt}dj = Z_t\int N_{jt}dj = Z_tN_t.$$

But

$$Y_t = \int c_{jt}dj = C_t\int\left(\frac{p_{jt}}{P_t}\right)^{-\theta}dj = C_t\Delta_t,$$

where

$$\Delta_t = \int\left(\frac{p_{jt}}{P_t}\right)^{-\theta}dj$$

is a measure of price dispersion. From $Y_t = Z_tN_t$, $\ln Y_t = \ln Z_t + \ln N_t$ and so

$$\hat{y}_t = \hat{z}_t + \hat{n}_t, \tag{108}$$

while from $Y_t = \Delta_t C_t$, $\ln Y_t = \ln \Delta_t + \ln C_t$, so

$$\hat{y}_t = \hat{\Delta}_t + \hat{c}_t. \tag{109}$$

Combining these,

$$\hat{n}_t = \hat{y}_t - \hat{z}_t = \hat{c}_t + \hat{\Delta}_t - \hat{z}_t.$$

The next step is to obtain an expression for Δ_t. A second order approximation for Δ_t is obtaining by first noting that if $x_{jt} \equiv p_{jt}/P_t$,

$$\begin{aligned}
\left(\frac{p_{jt}}{P_t}\right)^{-\theta} &= x_{jt}^{-\theta} \approx 1 - \theta \bar{x}^{-\theta-1}\tilde{x}_{jt} + \frac{1}{2}\theta\left(1+\theta\right)\bar{x}^{-\theta-2}\tilde{x}_{jt}^2 \\
&= 1 - \theta\tilde{x}_{jt} + \frac{1}{2}\theta\left(1+\theta\right)\tilde{x}_{jt}^2 \\
&= 1 - \theta\left(\hat{x}_{jt} + \frac{1}{2}\hat{x}_{jt}^2\right) + \frac{1}{2}\theta\left(1+\theta\right)\hat{x}_{jt}^2 \\
&= 1 - \theta\hat{x}_{jt} + \frac{1}{2}\theta^2\hat{x}_{jt}^2.
\end{aligned}$$

Furthermore,

$$\begin{aligned}
\left(\frac{p_{jt}}{P_t}\right)^{1-\theta} &= x_{jt}^{1-\theta} \approx 1 + \left(1-\theta\right)\bar{x}^{-\theta}\tilde{x}_{jt} - \frac{1}{2}\theta\left(1-\theta\right)\bar{x}^{-\theta-1}\tilde{x}_{jt}^2 \\
&= 1 + \left(1-\theta\right)\tilde{x}_{jt} - \frac{1}{2}\theta\left(1-\theta\right)\tilde{x}_{jt}^2 \\
&= 1 + \left(1-\theta\right)\left(\hat{x}_{jt} + \frac{1}{2}\hat{x}_{jt}^2\right) - \frac{1}{2}\theta\left(1-\theta\right)\hat{x}_{jt}^2 \\
&= 1 + \left(1-\theta\right)\hat{x}_{jt} + \frac{1}{2}\left(1-\theta\right)^2\hat{x}_{jt}^2.
\end{aligned}$$

Integrating over j,

$$\int \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} dj = 1 + \left(1-\theta\right)\int \hat{x}_{jt}dj + \frac{1}{2}\left(1-\theta\right)^2\int \hat{x}_{jt}^2 dj. \tag{110}$$

But from the definition of

$$P_t = \left[\int p_{jt}^{1-\theta}dj\right]^{\frac{1}{1-\theta}}$$

it follows that

$$\int \left(\frac{p_{jt}}{P_t}\right)^{1-\theta} dj = 1,$$

so combined with (110) this implies

$$\int \hat{x}_{jt}dj = -\frac{1}{2}(1-\theta)\int \hat{x}_{jt}^2 dj.$$

Hence,

$$\begin{aligned}
\Delta_t &= \int \left(\frac{p_{jt}}{P_t}\right)^{-\theta} dj \approx 1-\theta\int \hat{x}_{jt}dj + \frac{1}{2}\theta^2\int \hat{x}_{jt}^2 dj \\
&= 1+\frac{1}{2}\theta(1-\theta)\int \hat{x}_{jt}^2 dj + \frac{1}{2}\theta^2\int \hat{x}_{jt}^2 dj \\
&= 1+\frac{1}{2}\theta\int \hat{x}_{jt}^2 dj. \\
&= 1+\frac{1}{2}\theta var_j(\ln p_{jt} - \ln P_t) \\
&= 1+\frac{1}{2}\theta var_j \hat{x}_{jt}.
\end{aligned}$$

Then

$$\hat{\Delta}_t \approx \frac{1}{2}\theta var_j(\ln p_{jt} - \ln P_t).$$

Notice that because $\bar{\Delta} = 1$ and $\hat{\Delta}_t = \frac{1}{2}\theta\int \hat{x}_{jt}^2 dj$, $\hat{\Delta}_t \approx 0$ to first order.

Using (108) and (109) in (107),

$$\begin{aligned}
V_t - \bar{V} &\approx \bar{C}^{1-\sigma}\left[\left(\hat{y}_t - \hat{\Delta}_t\right) + \frac{1}{2}(1-\sigma)\left(\hat{y}_t - \hat{\Delta}_t\right)^2\right] - \chi\bar{N}^{1+\eta}\left[(\hat{y}_t - \hat{z}_t) + \frac{1}{2}(1+\eta)(\hat{y}_t - \hat{z}_t)^2\right] \\
&\approx \bar{C}^{1-\sigma}\left[\hat{y}_t + \frac{1}{2}(1-\sigma)\hat{y}_t^2\right] - \bar{C}^{1-\sigma}\hat{\Delta}_t - \chi\bar{N}^{1+\eta}\left[\hat{y}_t - \hat{z}_t + \frac{1}{2}(1+\eta)\left(\hat{y}_t^2 - 2\hat{y}_t\hat{z}_t + \hat{z}_t^2\right)\right] \\
&= \left(\bar{C}^{1-\sigma} - \chi\bar{N}^{1+\eta}\right)\hat{y}_t + \frac{1}{2}\left[(1-\sigma)\bar{C}^{1-\sigma} - (1+\eta)\chi\bar{N}^{1+\eta}\right]\hat{y}_t^2 - \bar{C}^{1-\sigma}\hat{\Delta}_t \\
&\quad + (1+\eta)\chi\bar{N}^{1+\eta}\hat{z}_t\hat{y}_t + \chi\bar{N}^{1+\eta}\left[\hat{z}_t - \frac{1}{2}(1+\eta)\hat{z}_t^2\right],
\end{aligned}$$

because $\hat{y}_t\hat{\Delta}_t$ and $\hat{\Delta}_t^2$ are of order three and four respectively.

In the steady state, equilibrium in the labor market implies

$$\mu^w \frac{\chi N^\eta}{C^{-\sigma}} = \omega = \frac{1}{\mu},$$

where ω is the real wage, μ is the steady-state markup in the goods market and μ^w is the steady-state wage markup. In addition, goods market clearing and the aggregate production function implies $\bar{C} = \bar{Y} = \bar{N}$. Define $1 - \Phi \equiv 1/\mu\mu^w$. Then these results imply

$$\begin{aligned}
\bar{C}^{1-\sigma} - \chi \bar{N}^{1+\eta} &= \bar{C}^{1-\sigma}\left(1 - \frac{\chi N^{1+\eta}}{C^{1-\sigma}}\right) = \bar{C}^{1-\sigma}\left(1 - \frac{\bar{N}}{\bar{C}}\frac{\chi N^{\eta}}{C^{-\sigma}}\right) \\
&= \bar{C}^{1-\sigma}\left(1 - \frac{\chi N^{\eta}}{C^{-\sigma}}\right) = \bar{C}^{1-\sigma}\Phi
\end{aligned}$$

and

$$\chi \bar{N}^{1+\eta} = \bar{C}^{1-\sigma}\Phi.$$

This now allows the approximation to $V_t - \bar{V}$ to be written as

$$\begin{aligned}
V_t - \bar{V} &\approx \bar{C}^{1-\sigma}\Phi\hat{y}_t + \frac{1}{2}\bar{C}^{1-\sigma}\left[(1-\sigma) - (1+\eta)(1-\Phi)\right]\hat{y}_t^2 - \bar{C}^{1-\sigma}\hat{\Delta}_t \\
&\quad + (1+\eta)\bar{C}^{1-\sigma}(1-\Phi)\hat{z}_t\hat{y}_t + \bar{C}^{1-\sigma}(1-\Phi)\left[\hat{z}_t - \frac{1}{2}(1+\eta)\hat{z}_t^2\right].
\end{aligned}$$

The term Φ is a measure of the inefficiency generated from imperfect competition; if the steady-state markups were equal to one, $\Phi = 0$. Assume that Φ is small (of first order) so that terms such as $\Phi\hat{y}_t\hat{z}_t$ are third order. Then terms in the approximation of $V_t - \bar{V}$ that involve $\hat{y}_t$ can be written as

$$\begin{aligned}
\left\{ \begin{array}{c} \bar{C}^{1-\sigma}\Phi\hat{y}_t + \frac{1}{2}\bar{C}^{1-\sigma}\left[(1-\sigma) - (1+\eta)(1-\Phi)\right]\hat{y}_t^2 \\ +\bar{C}^{1-\sigma}(1+\eta)(1-\Phi)\hat{z}_t\hat{y}_t \end{array} \right\} &\approx \bar{C}^{1-\sigma}\Phi\hat{y}_t - \frac{1}{2}\bar{C}^{1-\sigma}(\sigma+\eta)\hat{y}_t^2 + \bar{C}^{1-\sigma}(1+\eta)\hat{z}_t\hat{y}_t \\
&= \bar{C}^{1-\sigma}\Phi\hat{y}_t - \frac{1}{2}\bar{C}^{1-\sigma}\left[(\sigma+\eta)\hat{y}_t^2 - 2(1+\eta)\hat{z}_t\hat{y}_t\right],
\end{aligned}$$

which can be written as

$$\bar{C}^{1-\sigma}\Phi\hat{y}_t - \frac{1}{2}\bar{C}^{1-\sigma}\left[(\sigma+\eta)\hat{y}_t^2 - 2(1+\eta)\hat{z}_t\hat{y}_t\right] = -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\hat{y}_t^2 - 2\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\hat{y}_t - 2\left(\frac{1}{\sigma+\eta}\right)\Phi\hat{y}_t\right].$$

Now subtracting and adding

$$\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\left(\frac{1+\eta}{\sigma+\eta}\right)^2\hat{z}_t^2 + \left(\frac{1}{\sigma+\eta}\right)^2\Phi^2 + 2\left(\frac{1}{\sigma+\eta}\right)\Phi\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\right],$$

$$
\begin{aligned}
-\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\hat{y}_t^2 - 2\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\hat{y}_t - 2\left(\frac{1}{\sigma+\eta}\right)\Phi\hat{y}_t\right] &= -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\hat{y}_t^2 - 2\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\hat{y}_t - 2\left(\frac{1}{\sigma+\eta}\right)\Phi\hat{y}_t \right. \\
&\quad -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\left(\frac{1+\eta}{\sigma+\eta}\right)^2\hat{z}_t^2 + \left(\frac{1}{\sigma+\eta}\right)^2\Phi^2 + 2\left(\frac{\quad}{\sigma}\right.\right. \\
&\quad +\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\left(\frac{1+\eta}{\sigma+\eta}\right)^2\hat{z}_t^2 + \left(\frac{1}{\sigma+\eta}\right)^2\Phi^2 + 2\left(\frac{\quad}{\sigma}\right.\right. \\
&= -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\begin{array}{r}\hat{y}_t^2 - 2\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t\hat{y}_t + \left(\frac{1+\eta}{\sigma+\eta}\right. \\ -2\left(\frac{1}{\sigma+\eta}\right)\Phi\hat{y}_t + 2\left(\frac{1}{\sigma+\eta}\right)\Phi\left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z} \end{array}\right. \\
&= -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left[\left(\hat{y}_t - \hat{y}_t^f\right)^2 - 2\left(\hat{y}_t - \hat{y}_t^f\right)x^* + \left(\hat{y}_t^f\right)^2 \right. \\
&= -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left(\hat{x}_t - x^*\right)^2 + t.i.p.
\end{aligned}
$$

where

$$\hat{y}_t^f \equiv \left(\frac{1+\eta}{\sigma+\eta}\right)\hat{z}_t$$

is the economy's flexible-price equilibrium output (expressed as a log deviation from steady state),

$$\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^f$$

is the output gap, and

$$x^* \equiv \left(\frac{1}{\sigma+\eta}\right)\Phi$$

is the steady-state gap between the economy's flexible-price output and efficient output.

These results imply

$$V_t - \bar{V} \approx -\frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma}\left(\hat{x}_t - x^*\right)^2 - \bar{C}^{1-\sigma}\hat{\Delta}_t + t.i.p..$$

is the economy's flexible-price equilibrium output (expressed as a log deviation from steady state). Thus, the second-order approximation to the discounted value of the welfare of the representative household is

$$\mathrm{E}_t\sum_{i=0}^{\infty}\beta^i V_{t+i} \equiv \left(\frac{1}{1-\beta}\right)\bar{V} - \mathrm{E}_t\bar{C}^{1-\sigma}\sum_{i=0}^{\infty}\beta^i\left[\frac{1}{2}(\sigma+\eta)\left(\hat{x}_{t+i} - x^*\right)^2 + \hat{\Delta}_t\right] + \mathrm{E}_t\sum_{i=0}^{\infty}\beta^i t.i.p.. \tag{111}$$

The last step is to relate the price dispersion term, $\hat{\Delta}_t$, to the average inflation rate across all firms. Earlier, it was shown that $\hat{\Delta}_t$ is related to the cross-sectional variance of prices across firms:

$$\hat{\Delta}_t \simeq \frac{1}{2}\theta var_j \left(\ln p_{jt} - \ln P_t\right).$$

Recall that the price-adjustment mechanism involves a randomly chosen fraction $1-\omega$ of all firms optimally adjusting price each period. Define $\bar{P}_t \equiv \mathrm{E}_j \log p_{jt}$. Then, because $var_j \bar{P}_{t-1} = 0$, one can write

$$\begin{aligned} var_j \left(\log p_{jt} - \bar{P}_{t-1}\right) &= \mathrm{E}_j \left(\log p_{jt} - \bar{P}_{t-1}\right)^2 - \left(\mathrm{E}_j \log p_{jt} - \bar{P}_{t-1}\right)^2 \\ &= \omega \mathrm{E}_j \left(\log p_{jt-1} - \bar{P}_{t-1}\right)^2 + (1-\omega)\left(\log p_t^* - \bar{P}_{t-1}\right)^2 \\ &\quad - \left(\bar{P}_t - \bar{P}_{t-1}\right)^2. \end{aligned}$$

where p_t^* is the price set at time t by the fraction $1-\omega$ of firms that reset their price. Given that $\bar{P}_t = (1-\omega)\log p_t^* + \omega \bar{P}_{t-1}$,

$$\log p_t^* - \bar{P}_{t-1} = \left(\frac{1}{1-\omega}\right)\left(\bar{P}_t - \bar{P}_{t-1}\right).$$

Using this result,

$$\begin{aligned} \hat{\Delta}_t &= \frac{1}{2}\theta \left[\omega \Delta_{t-1} + \left(\frac{\omega}{1-\omega}\right)\left(\bar{P}_t - \bar{P}_{t-1}\right)^2\right] \\ &\approx \frac{1}{2}\theta\omega\Delta_{t-1} + \frac{1}{2}\theta\left(\frac{\omega}{1-\omega}\right)\pi_t^2. \end{aligned}$$

This implies

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \hat{\Delta}_{t+i} = \frac{1}{2}\theta \left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right] \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \pi_{t+i}^2 + t.i.p.,$$

where the terms independent of policy also include the initial degree of price dispersion.

Combining this with (111) and ignoring terms independent of policy, the present discounted value of the utility of the representative household can be approximated by

$$\begin{aligned} \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} &\equiv \left(\frac{1}{1-\beta}\right)\bar{V} - \mathrm{E}_t \frac{1}{2}(\sigma+\eta)\bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i \left(\hat{x}_{t+i} - x^*\right)^2 - \mathrm{E}_t \bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i \hat{\Delta}_{t+i} \\ &= \left(\frac{1}{1-\beta}\right)\bar{V} - \mathrm{E}_t \frac{1}{2}\bar{C}^{1-\sigma} \sum_{i=0}^{\infty} \beta^i \left[(\sigma+\eta)\left(\hat{x}_{t+i} - x^*\right)^2 + \theta \left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\pi_{t+i}^2\right] \\ &= \left(\frac{1}{1-\beta}\right)\bar{V} - \mathrm{E}_t \frac{1}{2}\Omega \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda\left(\hat{x}_{t+i} - x^*\right)^2\right] \end{aligned}$$

where

$$\Omega = \frac{1}{2}\bar{C}^{1-\sigma}\left[\frac{\omega}{(1-\omega)(1-\omega\beta)}\right]\theta$$

and

$$\lambda = \left[\frac{(1-\omega)(1-\omega\beta)}{\omega}\right]\left(\frac{\sigma+\eta}{\theta}\right).$$

If fiscal tax and subsidy policies are used to offset the steady-state markups in the goods and labor markets, the steady-state output under flexible prices will be efficient. In this case, which corresponds to ensuring $\Phi = 0$, $x^* = 0$ and the welfare of the representative household is (again ignoring terms independent of policy) given by

$$\mathrm{E}_t \sum_{i=0}^{\infty} \beta^i V_{t+i} = \left(\frac{1}{1-\beta}\right)\bar{V} - \mathrm{E}_t \frac{1}{2}\Omega \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda \hat{x}_{t+i}^2\right).$$

8 Problems

Remark 2 *Add problems using dynare.*

1. NEW Suppose aggregate output is defined as

$$Y_t = \int c_{jt} dj.$$

   (a) Using the demand equation (5), show that goods market clearing implies $Y_t = \Delta_t C_t$, where Δ_t is the measure of price dispersion defined in (21).

   (b) If each firm faces the production function $c_{jt} = Z_t N_{jt}$, show that aggregate employment $N_t = \int N_{jt} dj$ is equal to Y_t / Z_t.

   (c) Use the results in parts (a) and (b) and the definition of C_t given by (2) to show

$$C_t = \Delta_t^{-1}(Z_t N_t) = Z_t \tilde{N}_t,$$

   where $\tilde{N}_t \equiv \Delta_t^{-1} N_t \leq N_t$. Explain how price dispersion reduces the effective amount of labor relative to actual employment N_t.

2. Consider a simple forward-looking model of the form

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}(i_t - \mathrm{E}_t \pi_{t+1}) + u_t,$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t.$$

Suppose policy reacts to the output gap:

$$i_t = \delta x_t.$$

Write this system in the form given by (33). Are there values of δ that ensure a unique stationary equilibrium? Are there values that do not?

3. Consider the model given by

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

Suppose policy sets the nominal interest rate according to a policy rule of the form

$$i_t = \phi_1 \mathrm{E}_t \pi_{t+1}$$

for the nominal rate of interest.

(a) Write this system in the form $\mathrm{E}_t z_{t+1} = M z_t + \eta_t$, where $z_t = [x_t,\ \pi_t]'$.

(b) For $\beta = 0.99$, $\kappa = 0.05$, and $\sigma = 1.5$, plot the absolute values of the two eigenvalues of M as a function of $\phi_1 > 0$.

(c) Are there values of ϕ_1 for which the economy does not have a unique stationary equilibrium?

4. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\chi_t N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, χ is a stochastic shock to "tastes," N is time spent working, Y output, and Z an aggregate productivity disturbance; σ and η are constants. The stochastic variable χ has a mean of 1.

(a) Derive the household's first-order condition for labor supply. Show how labor supply depends on the taste shock and explain how a positive realization of χ would affect labor supply.

(b) Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ for this economy.

(c) Does the taste shock affect the flexible-price equilibrium? If it does, explain how and why.

(d) The household's Euler condition for optimal consumption choice (expressed in terms of the output gap and in percent deviations around the steady-state) can be written as

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t^n\right).$$

How does r^n depend on the behavior of the flexible price equilibrium output? Does it depend on the taste shock χ? Explain intuitively whether a positive realization of χ raises, lowers, or leaves unchanged the flex-price equilibrium real interest rate.

5. NEW Assume the utility of the representative agent is

$$\frac{\psi_t C_t^{1-\sigma}}{1-\sigma} - \frac{\chi_t N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, ψ and χ are a stochastic shocks to "tastes," N is time spent working, Y output, and Z an aggregate productivity disturbance; σ and η are constants. The stochastic taste shocks have means of 1.

(a) Derive the household's first-order condition for labor supply. Show how labor supply depends on the taste shocks and explain how a positive realization of ψ affects labor supply.

(b) Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ for this economy. How is it affected by ψ?

(c) In the basic new Keynesian model, inflation depends on real marginal cost. Show that the linearized inflation equation (23) can still be written in the form given by (26) even with the introduction of taste shocks.

6. Suppose the economy is characterized by) is

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - E_t \pi_{t+1} - r_t^n\right)$$

and.

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.$$

What problems might arise if the central bank decides to set its interest rate instrument according to the rule $i_t = r_t^n$?

7. Suppose the economy is described by the basic new Keynesian model consisting of

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(i_t - \mathrm{E}_t \pi_{t+1}\right)$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t$$

$$i_t = \phi_\pi \pi_t + \phi_x x_t.$$

   (a) If $\phi_x = 0$, explain intuitively why $\phi_\pi > 1$ is needed to ensure that the equilibrium will be unique.

   (b) If both ϕ_π and ϕ_x are nonnegative, the condition given by (36) implies that the economy can still have unique, stable equilibrium even when

$$1 - \frac{(1-\beta)\phi_x}{\kappa} < \phi_\pi < 1.$$

   Explain intuitively why some values of $\phi_\pi < 1$ are still consistent with uniqueness when $\phi_x > 0$.

8. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{(1+\xi_t)N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t$. The notation is: C is consumption, ξ is a stochastic shock to "tastes," N is time spent working, Y is output, and $Z_t = (1 + z_t)$ is a stochastic aggregate productivity disturbance; σ and η are constants. Both ξ and z have zero means. Assume a standard model of monopolistic competition with Calvo pricing.

   (a) Assuming a zero steady-state rate of inflation, the inflation adjustment equation can be written as

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \mu_t,$$

   where μ_t is real marginal cost (expressed as a percent deviation around the steady-state). Derive an expression for μ_t in terms of an output gap.

   (b) Does the taste shock affect the output gap? Does it affect inflation? Explain.

9. Assume the utility of the representative agent is given by

$$\frac{C_t^{1-\sigma}\left(\frac{M_t}{P_t}\right)^{1-b}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}.$$

The aggregate production function is $Y_t = Z_t N_t^a$.

(a) Show that the household's first-order condition for labor supply takes the form

$$\eta \hat{n}_t + \sigma \hat{c}_t - \mu_t^w = \hat{w}_t - \hat{p}_t,$$

where $\mu_t^w = (1-b)(\hat{m}_t - \hat{p}_t)$.

(b) Derive an expression for the flexible-price equilibrium output $\hat{y}_t^f$ and the output gap $x_t = \hat{y}_t - \hat{y}_t^f$.

(c) Does money affect the flexible-price equilibrium? Does the nominal interest rate? Explain.

10. Suppose the economy is characterized by (44) and (45), and let the cost shock be given by $e_t = \rho e_{t-1} + \varepsilon_t$. The central bank's loss function is (49). Assume that the central bank can commit to a policy rule of the form $\pi_t = \gamma e_t$.

    (a) What is the optimal value of γ?

    (b) Find the expression for equilibrium output gap under this policy.

11. In section 4.4, the case of commitment to a rule of the form $x_t = b_x e_t$ was analyzed. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Suppose instead that the central bank commits to the rule $i_t = b_i e_t$ for some constant b_i. Does a unique, stationary, rational expectations equilibrium exist under such a commitment? Explain why the two cases differ.

12. Suppose the economy's inflation rate is described by the following equation (all variables expressed as percentage deviations around a zero-inflation steady state:

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t, \tag{112}$$

where x_t is the gap between output and the flexible price equilibrium output level, and e_t is a cost shock. Assume that

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where ε is a white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\frac{1}{2}\mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2 \right) \right].$$

(a) Derive the first-order conditions linking inflation and the output gap for the *fully* optimal commitment policy.

(b) Explain why the first-order conditions for time t differs from the first-order conditions for $t+i$ for $i > 0$.

(c) What is meant by a commitment policy that is optimal from a timeless perspective? (Explain in words.)

(d) What is the first-order condition linking inflation and the output gap that the central bank follows under an optimal commitment policy from a timeless perspective?

(e) Explain why, under commitment, the central bank promises a deflation in the period after a positive cost shock (assume the cost shock is serially uncorrelated).

13. Explain why inflation is costly in a new Keynesian model.

14. Suppose the economy is described by the following log-linearized system:

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)(i_t - \mathrm{E}_t \pi_{t+1}) + E_t\left(z_{t+1} - z_t\right) + u_t$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t,$$

where u_t is a demand shock, z_t is a productivity shock, and e_t is a cost shock. Assume that

$$u_t = \rho_u u_{t-1} + \xi_t$$

$$z_t = \rho_z z_{t-1} + \psi_t$$

$$e_t = \rho_e e_{t-1} + \varepsilon_t,$$

where ξ, ψ, and ε are white noise processes. The central bank sets the nominal interest rate i_t to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \left[\sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)\right].$$

(a) Derive the optimal time-consistent policy for the discretionary central banker. Write down the first-order conditions and the reduced-form solutions for x_t and π_t.

(b) Derive the interest-rate feedback rule implied by the optimal discretionary policy.

(c) Show that under the optimal policy, nominal interest rates are increased enough to raise the real interest rate in response to a rise in expected inflation.

(d) How will x_t and π_t move in response to a demand shock? To a productivity shock?

15. Suppose the central bank cares about inflation variability, output gap variability, and interest rate variability. The objective of the central bank is to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left[\pi_{t+i}^2 + \lambda_x x_{t+i}^2 + \lambda_i \left(i_{t+i} - i^*\right)^2\right].$$

The structure of the economy is given by

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + e_t$$

$$x_t = \mathrm{E}_t x_{t+1} - \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right),$$

where e and r are exogenous stochastic shocks. Let ψ_t denote the Lagrangian multiplier on the Phillips curve and let θ_t be the multiplier on the IS curve.

(a) Derive the first-order conditions for the optimal policy of the central bank under discretion. *Under discretion, the central bank takes expectations as given, so its decision problem can be written as*

$$\min_{\pi_t, x_t, i_t} \left(\frac{1}{2}\right)\left[\pi_t^2 + \lambda_x x_t^2 + \lambda_i \left(i_t - i^*\right)^2\right] + \psi_t \left[\pi_t - \beta \mathrm{E}_t \pi_{t+1} - \kappa x_t - e_t\right]$$
$$+\theta_t \left[x_t - \mathrm{E}_t x_{t+1} + \left(\frac{1}{\sigma}\right)\left(i_t - \mathrm{E}_t \pi_{t+1} - r_t\right)\right].$$

The first-order conditions are

$$\pi_t + \psi_t = 0$$

$$\lambda_x x_t - \kappa \psi_t + \theta_t = 0$$

and

$$\lambda_i \left(i_t - i^*\right) + \left(\frac{1}{\sigma}\right)\theta_t = 0.$$

(b) Show that θ is nonzero if $\lambda_i > 0$. Explain the economics behind this result. *From this last condition, if $\lambda_i = 0$, then $\theta_t = 0$ and the IS relationship does not impose a constraint on the central bank's policy choice. Because, when $\lambda_i = 0$, the central bank does not care about interest rate volatility, it can always adjust i_t to offset any shocks arising from the IS relationship, preventing these from affecting the things it does care about – inflation and the output gap.*

(c) Derive the first-order conditions for the fully optimal commitment policy. How do these differ from the conditions you found in (a)? *Under optimal commitment, the central* *bank's decision problem is*

$$\min \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{1}{2} \left[\pi_{t+i}^2 + \lambda x_{t+i}^2 + \lambda_i \left(i_{t+i} - i^* \right)^2 \right] \right.$$
$$+\psi_{t+i} \left(\pi_{t+i} - \beta \pi_{t+1+i} - \kappa x_{t+i} - e_{t+i} \right)$$
$$\left. +\theta_{t+i} \left[x_{t+i} - \mathrm{E}_t x_{t+1+i} + \left(\frac{1}{\sigma} \right) \left(i_{t+i} - \pi_{t+1+i} - r_{t+i} \right) \right] \right\}.$$

The first-order conditions are

$$\text{for } \pi_t\text{: } \pi_t + \psi_t = 0 \tag{113}$$

$$\text{for } \pi_{t+i}\text{: } \pi_{t+i} + \psi_{t+i} - \psi_{t+i-1} - \left(\frac{1}{\sigma \beta} \right) \theta_{t+i-1} = 0 \text{ for } i > 0 \tag{114}$$

$$\text{for } x_t\text{: } \lambda_x x_t - \kappa \psi_t + \theta_t = 0. \tag{115}$$

$$\text{for } x_{t+i}\text{: } \lambda_x x_{t+i} - \kappa \psi_{t+i} + \theta_{t+i} - \left(\frac{1}{\beta} \right) \theta_{t+i-1} = 0 \text{ for } i > 0. \tag{116}$$

$$\text{for } i_{t+i}\text{: } \lambda_i \left(i_{t+i} - i^* \right) + \left(\frac{1}{\sigma} \right) \theta_{t+i} = 0 \text{ for } i \geq 0. \tag{117}$$

*For $i = 0$, the conditions are the same as in part (a). For $i > 0$, they differ for the reasons explained in Problem 8.10 – see (**??**) - (**??**). Because $\lambda_i > 0$, the central bank cares about interest rate volatility, so the IS relationship becomes a constraint on policy choices ($\theta_t > 0$), and moving i_t to offset movements in r_t may lead to undesired interest rate volatility. Because the current output gap is affected by the expected future output gap through the IS relationship, the plan for x_{t+i}, $i > 0$, must take into account the effect this has on x_{t+i-1}, just as the choice of π_{t+i}, $i > 0$, affects π_{t+i-1}. This accounts for the θ_{t-1} term in (116). It is absent in (115) because x_t can no longer affect x_{t-1}.*

(d) Derive the first-order conditions for the optimal commitment policy from a timeless perspective. How do these differ from the conditions you found in (c)? *Under the timeless perspective, policy choices are such that (**??**), (116), and (117) are satisfied for all $i \geq 0$. That is, the special nature of the first period, when the effects on the past can be ignored, is not exploited and instead policy is set to satisfy*

$$\pi_{t+i} + \psi_t - \psi_{t+i-1} - \left(\frac{1}{\sigma \beta} \right) \theta_{t+i-1} = 0,$$

$$\lambda_x x_{t+i} - \kappa\psi_{t+i} + \theta_{t+i} - \left(\frac{1}{\beta}\right)\theta_{t+i-1} = 0,$$

and

$$\lambda_i\left(i_{t+i} - i^*\right) + \left(\frac{1}{\sigma}\right)\theta_{t+i} = 0$$

for all $i \geq 0$*. From the last of these,* $\theta_{t+i} = -\sigma\lambda_i\left(i_{t+i} - i^*\right)$*. Using this in the first-order condition for* x_{t+i}*,*

$$\lambda_x x_{t+i} - \kappa\psi_{t+i} - \sigma\lambda_i\left(i_{t+i} - i^*\right) + \left(\frac{1}{\beta}\right)\sigma\lambda_i\left(i_{t+i-1} - i^*\right) = 0$$

so

$$\psi_{t+i} = \left(\frac{1}{\kappa}\right)\left[\lambda_x x_{t+i} - \sigma\lambda_i\left(i_{t+i} - i^*\right)\right] + \left(\frac{1}{\kappa}\right)\left(\frac{1}{\beta}\right)\sigma\lambda_i\left(i_{t+i-1} - i^*\right).$$

Using this in the first-order condition for inflation yields the optimal targeting rule under the timeless perspective:

$$\begin{aligned}
\pi_{t+i} &= -\left(\psi_t - \psi_{t+i-1}\right) + \left(\frac{1}{\sigma\beta}\right)\theta_{t+i-1} \qquad (118)\\
&= -\left(\frac{\lambda_x}{\kappa}\right)\left(x_{t+i} - x_{t+i-1}\right) + \left(\frac{\sigma\lambda_i}{\kappa}\right)\left(i_{t+i} - i_{t+i-1}\right)\\
&\quad -\left(\frac{\sigma\lambda_i}{\beta\kappa}\right)\left(i_{t+i-1} - i_{t+i-2}\right) - \left(\frac{\lambda_i}{\beta}\right)\left(i_{t+i-1} - i^*\right)
\end{aligned}$$

If $\lambda_i = 0$ *as in the example from the text,* $\pi_{t+i} = -\left(\lambda_x/\kappa\right)\left(x_{t+i} - x_{t+i-1}\right)$*.*

(e) Eliminate any Lagrangian multipliers from the first-order conditions after adopting the timeless perspective. Write the result in the form of an interest rate rule. How many lagged values of the interest rate appear in the rule? Can you conclude anything about the size of the coefficient on i_{t-1}? *Reexpress (118) at time* t *with* i_t *on the left side:*

$$\begin{aligned}
i_t &= \left(1 + \frac{1}{\beta} + \frac{\kappa}{\sigma\beta}\right)i_{t-1} - \left(\frac{1}{\beta}\right)i_{t-2} + \left(\frac{\kappa}{\sigma\lambda_i}\right)\pi_t\\
&\quad + \left(\frac{\lambda_x}{\sigma\lambda_i}\right)\left(x_t - x_{t-1}\right) - \left(\frac{\kappa}{\sigma\beta}\right)i^*
\end{aligned}$$

The coefficient on i_{t-1} *is greater than* 1*, and the sum of the coefficients on* i_{t-1} *and* i_{t-2} *is larger than* 1*, so this looks like an explosive process. However, when combined with the equations for* x *and* π*, the model has a stationary equilibrium.*

16. Consider a basic new Keynesian model with Calvo adjustment of prices and flexible nominal wages.

(a) In this model, inflation volatility reduces the welfare of the representative agent. Explain why.

(b) In the absence of cost shocks, optimal policy would ensure inflation and the output gap both remain equal to zero. What does this imply for the behavior of output? Why can output fluctuate efficiently despite sticky prices?

(c) Suppose both prices and nominal wages are sticky (assume a Calvo model for wages). Will volatility in the rate of wage inflation be welfare reducing? Explain.

(d) Is zero inflation and a zero output gap still feasible? Explain.A key issue in the analysis

of policy trade offs is the source of the stochastic shocks in the model. Consider these two examples. 1) The utility function takes the form

$$\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta_t}}{1+\eta_t}$$

where η_t is stochastic. 2) There is a labor tax τ_t such that the after-tax wage is $(1-\tau_t)W_t$. Assume a standard model of monopolistic competition as in the lectures.

(e) Derive the condition for labor market equilibrium under flexible prices for each of the two cases.

(f) Linearize the conditions found in part (a) and, for each case, derive the flexible-price equilibrium output in terms of percent deviations from the steady state. Clearly state any assumptions you need to make on the η and τ processes or about other aspects of the model.

(g) Assume sticky prices ala Calvo. Express real marginal cost in terms of an output gap.

(h) Does either η_t or τ_t appear as a cost shock?

(i) Do you think either η_t or τ_t causes a wedge between the flexible-price output level and the efficient output level?

17. Suppose inflation adjustment is given by (65). The central bank's objective is to minimize

$$\left(\frac{1}{2}\right) \mathrm{E}_t \sum_{i=0}^{\infty} \beta^i \left(\pi_{t+i}^2 + \lambda x_{t+i}^2\right)$$

subject to (65). Use dynare to answer this question.

(a) Calculate the response of the output gap and inflation to a serially uncorrelated, positive cost shock for $\phi = 0$, 0.25, 0.5, 0.75, and 1 under the optimal discretionary policy.

(b) Now do the same for the optimal commitment policy.

(c) Discuss how the differences between commitment and discretion depend on ϕ, the weight on lagged inflation in the inflation adjustment equation.

18. Suppose

$$\pi_t - \gamma\pi_{t-1} = \beta\left(E_t\pi_{t+1} - \gamma\pi_t\right) + \kappa x_t + e_t$$

$$e_t = 0.25e_{t-1} + \varepsilon_t$$

and the period loss function is

$$L = \left(\pi_t - \gamma\pi_{t-1}\right)^2 + 0.25x_t^2.$$

(a) Analytically find the optimal targeting rule under discretion.

(b) Analytically find the optimal targeting rule under commitment (timeless perspective).

(c) Assume $\beta = 0.99$, $\kappa = 0.0603$, $\rho = 0.25$, and $\lambda = 0.25$. Set $\sigma_\varepsilon^2 = 1$. Under the targeting rules found in (a) and (b), plot the loss L as a function of $\gamma = [0\ 1]$.

19. Suppose the inflation equation contains lagged inflation:

$$\pi_t = (1-\phi)\beta E_t\pi_{t+1} + \phi\pi_{t-1} + \kappa x_t + e_t.$$

(a) Show that the optimal commitment policy from a timeless perspective is

$$\pi_t + (\lambda/\kappa)\left[x_t - (1-\phi)x_{t-1} - \beta\phi E_t x_{t+1}\right] = 0.$$

(b) Show that the unconditional optimal commitment policy takes the form

$$\pi_t + (\lambda/\kappa)\left[x_t - \beta(1-\phi)x_{t-1} - \phi E_t x_{t+1}\right] = 0.$$

20. The following model has been estimated by Linde (2005), though the values here are from Svensson and Williams (2008):

$$\pi_t = 0.4908E_t\pi_{t+1} + (1-0.4908)\pi_{t-1} + 0.0081y_t + \varepsilon_t^\pi$$

$$\begin{aligned} y_t \quad = \quad & 0.4408E_t y_{t+1} + (1-0.4408)\left[1.1778y_{t-1} + (1-1.1778)y_{t-2}\right] \\ & -0.0048\left(i_t - E_t\pi_{t+1}\right) + \varepsilon_t^y \end{aligned}$$

$$
\begin{aligned}
i_t &= (1 - 0.9557 + 0.0673)(1.3474\pi_t + 0.7948y_t) \\
&\quad +0.9557i_{t-1} - 0.0673i_{t-2} + \varepsilon_t^i
\end{aligned}
$$

with $\sigma_\pi = 0.5923$, $\sigma_y = 0.4126$, and $\sigma_i = 0.9918$.

(a) Write this system in the form $\mathrm{E}_t z_{t+1} = M z_t + \eta_t$ for appropriately defined vectors z and η.

(b) Plot the impulse response functions showing how inflation and the output gap response to each of the three shocks.

(c) How are the impulse responses affected if the coefficient on inflation in the policy rule is reduced from 1.3474 to 1.1?

21. Suppose the firm uses a labor aggregate N_t to produce output using the technology $Y_t = F(N_t)$, where $F' \geq 0$, $F'' \leq 0$. The labor aggregate is a composite function of the individual types of labor services and is given by

$$
N_t = \left[\int_0^1 n_{jt}^{\frac{\gamma-1}{\gamma}} dj\right]^{\frac{\gamma}{\gamma-1}}, \qquad \gamma > 1.
$$

where n_{jt} is the labor from household j that the firm employs. The real wage of labor type j is ω_{jt}. Show that if the firm takes wages as given, its optimal demand for labor type j, conditional on N_t, is given by (84).

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