$$V(\mathrm{k_{t\text{-}1}}, \mathrm{b_t}) = \max_{\mathrm{c_t, n_t, x_t, k_t, b_{t+1}}} \left\{ u\left(\mathrm{c_{t,}} \mathrm{1\text{-}n_t}\right) + \beta \mathrm{EV}(\mathrm{k_t}, \mathrm{b_{t+1}}) \right\}$$

Subject to;

$$\mathrm{c_t + x_t + p_{t+1} b_{t+1} \leq w_t n_t + r_t k_{t\text{-}1} + b_t}$$

$$\mathrm{k_t} = (1-\delta)\mathrm{k_{t\text{-}1}} + \mathrm{x_t}$$

First order conditions;

$$\frac{\delta \mathrm{L}}{\delta \mathrm{c_t}} = \mathrm{u_{c_t}} + \lambda_{1,\mathrm{t}}(-1) = 0$$

$$\frac{\delta \mathrm{L}}{\delta \mathrm{n_t}} = \mathrm{u_{n_t}}(\text{-}1) + \lambda_{1,\mathrm{t}}(\mathrm{w_t}) = 0$$

$$\frac{\delta \mathrm{L}}{\delta \mathrm{x_t}} = \lambda_{1,\mathrm{t}}(-1) + \lambda_{2,\mathrm{t}} = 0$$

$$\frac{\delta \mathrm{L}}{\delta \mathrm{k_t}} = \beta \mathrm{EV_{k_t}} + \lambda_{2,\mathrm{t}}(-1) = 0$$

$$\frac{\delta \mathrm{L}}{\delta \mathrm{b_{t+1}}} = \beta \mathrm{EV_{b_{t+1}}} + \lambda_{1,\mathrm{t}}(\text{-}\mathrm{p_{t+1}}) = 0$$

$$\frac{\delta \mathrm{L}}{\delta \lambda_{1,\mathrm{t}}} = \mathrm{w_t n_t + r_t k_{t\text{-}1} + b_t \text{ - } c_t \text{ - } x_t \text{ - } p_{t+1} b_{t+1}} = 0$$

$$\frac{\delta \mathrm{L}}{\delta \lambda_{2,\mathrm{t}}} = (1-\delta)\mathrm{k_{t\text{-}1}} + \mathrm{x_t} \text{ - } \mathrm{k_t} = 0$$

Envelope theorem conditions;

$$\mathrm{V_{k_{t\text{-}1}}} = \lambda_{1,\mathrm{t}} \mathrm{r_t} + \lambda_{2,\mathrm{t}}(1-\delta)$$
$$\mathrm{V_{k_t}} = \lambda_{1,\mathrm{t+1}} \mathrm{r_{t+1}} + \lambda_{2,\mathrm{t+1}}(1-\delta)$$

$$\mathrm{V_{b_t}} = \lambda_{1,\mathrm{t}}$$
$$\mathrm{V_{b_{t+1}}} = \lambda_{1,\mathrm{t+1}}$$

Euler equations;

$$\mathrm{u_{n_t}} = \mathrm{u_{c_t}}(\mathrm{w_t})$$
$$\beta \mathrm{E}\left[\mathrm{u_{c_t+1}}(\mathrm{r_{t+1}} + (1-\delta))\right] = \mathrm{u_{c_t}}$$
$$\beta \mathrm{E}\left[\mathrm{u_{c_t+1}}\right] = \mathrm{p_{t+1} u_{c_t}}$$

Firm conditions;

$$y_t = z_t k_t^{\theta} n_t^{1\text{-}\theta}$$

$$w_t = (1\text{-}\theta) z_t k_t^{\theta} n_t^{\text{-}\theta}$$

$$r_t = \theta z_t k_t^{\theta\text{-}1} n_t^{1\text{-}\theta}$$

Equations for Dynare, for home and foreign country;

$$\frac{a}{1\text{-}n_t} = \frac{1}{c_{t,}}\left((1\text{-}\theta) z_t k_t^{\theta} n_t^{\text{-}\theta}\right)$$

$$\beta E\left[\frac{1}{c_{t+1,}}\left(\theta z_{t+1} k_{t+1}^{\theta\text{-}1} n_{t+1}^{1\text{-}\theta} + (1-\delta)\right)\right] = \frac{1}{c_{t,}}$$

$$(1\text{-}\theta) z_t k_t^{\theta} n_t^{\text{-}\theta} n_t + \theta z_t k_t^{\theta\text{-}1} n_t^{1\text{-}\theta} k_{t\text{-}1} + b_t \; = \; c_t \; + \; x_t \; + \; \beta E\left[\frac{c_{t,}}{c_{t+1,}}\right] b_{t+1}$$

$$k_t = (1-\delta) k_{t\text{-}1} + x_t$$

$$z_t = \rho z_{t\text{-}1} + \epsilon$$

$$z_t k_t^{\theta} n_t^{1\text{-}\theta} = c_{t,} + x_t$$

12 equations (6 home, 6 foreign) for 12 unknowns (c, n, k, b, x, z) for both countries.