1 The model

At period t, n_t individuals are born and endowed with some level of consumption of their parents when young that reduces the utility of their consumption when young (aspirations). These individuals, supply labor, earn wages, receive transfers, save and consume. At $t+1$, they are old, they consume the returns of their savings and also receive transfers. They suffer a negative effect of their consumption when young, that reduces their utility because they have acquired a consumption habit. Our modeling of both aspirations and habits follows Caballe and Moro (2014).

1.0.1 Households' problem

Individuals maximize

$$\max_{s_t,l_t} U_t = \max_{s_t,l_t} \left(\left(\log\left(\hat{c}_t^y\right) + \phi \log(1-l_t) \right) + \rho \log\left(\hat{c}_{t+1}^o\right) \right) \tag{1}$$

subject to

$$\hat{c}_t^y = c_t^y - \gamma_a c_{t-1}^y \tag{2}$$

and

$$\hat{c}_{t+1}^o = c_{t+1}^o - \gamma_h c_t^y \tag{3}$$

represent aspirations adjusted young consumption and consumption of the old adjusted by habit formation. Also,

$$c_t^y = w_t l_t - s_t + g_t, \tag{4}$$

$$c_{t+1}^o = (1+r_{t+1})s_t + g_{t+1}, \tag{5}$$

Consider first the case where individuals are myopic and do not anticipate the effect of their consumption when young on the utility of consumption when old. Thus consumption when young has a negative impact on second period's utility but does not influence decisions. Substituting (2)-(5) in (1) we obtain the first order condition for optimal saving

$$\frac{c_{t+1}^o - \gamma_h c_t^y}{c_t^y - \gamma_a c_{t-1}^y} = \rho\left(1+r_{t+1}\right) \tag{6}$$

and labour supply:

$$\frac{w_t}{c_t^y - \gamma_a c_{t-1}^y} = \frac{\phi}{1-l_t} \tag{7}$$

Together with the lifetime budget constraint:

$$c_t^y + \frac{c_{t+1}^o}{1+r_{t+1}} = w_t l_t + g_t + \frac{g_{t+1}}{1+r_{t+1}} \tag{8}$$

the first order conditions allow to characterize optimal consumption of an individual born at t when young and when old, and her labour supply for given factor prices and transfers and consumption of the old when they were young.

1.0.2 Comparative statics

Using (2), (3) and (4) and after some manipulation we obtain:

$$s_t = \frac{\rho\left(1+r_{t+1}\right)\left(w_t l_t + g_t - \gamma_a c_{t-1}^y\right) - g_{t+1} + \gamma_h c_t^y}{\left(1+\rho\right)\left(1+r_{t+1}\right)} \tag{9}$$

$$l_t = \frac{w_t + \phi\left(s_t - g_t + \gamma_a c_{t-1}^y\right)}{(1+\phi)w_t} \tag{10}$$

1.1 Firms' problem

A single good is produced in t out of physical capital K_t and efficient units of labor $A_t L_t : Y_t = K_t^{\alpha}\left(A_t L_t\right)^{1-\alpha}$. We can express production in terms of efficient labour, letting $\bar{y}_t = Y_t / A_t L_t$ and $\bar{k}_t = K_t / A_t L_t$, $\bar{y}_t = \bar{k}_t^{\alpha}$. Firms choose capital and efficient units of labour to

$$\max A_t L_t \left(\frac{K_t}{A_t L_t}\right)^{\alpha} - w_t A_t L_t - \left(\delta + r_t\right) K_t$$

Assume all capital to fully depreciate each period $\delta = 1$. Then

$$foc(K_t) : \alpha \bar{k}_t^{\alpha-1} = 1 + r_t$$

$$foc(A_t L_t) : \bar{k}_t^{a} - \alpha \bar{k}_t^{\alpha-1} \bar{k}_t = w_t$$

1.2 Equilibrium

Equilibrium requires the emptying of the markets for goods

$$K_{t+1} = n_t s_t$$

and labour

$$L_t = n_t l_t$$

The former, in efficiency units of labour

$$s_t = \left(1 + n_{t+1}\right) A_{t+1} \bar{k}_{t+1} l_{t+1}$$