Endowment BKK model

Complete markets

- Endowment economy
- Equivalence between CE and Pareto optimality: since $u(c)$ are concave, any optimum can be computed as the solution to the Planner's problem.
- 2 countries: home(h) and foreign(f)
- Same utility functions
- Initial assumption: equal size of countries

Competitive equilibrium

$$\max_{c_t^{i,j}} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) u(c_t^{i,j}(s^t)) \quad s.t.$$

$$c_t^{i,j}(s^t) + \sum_{s \in S} q(s') a_t^{i,j}(s') = y_t^{i,j}(s^t) + a_t^{i,j}(s^t) \quad \forall i = 1, 2 \quad j = h, f$$

Planner's problem

$$\max_{c_t^h, c_t^f} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t \left\{ \theta_1 u(c_t^h(s^t)) + \theta_2 u(c_t^f(s^t)) \right\}$$

s.t

$$Y\left(s^t\right) = c_t^h\left(s^t\right) + c_t^f\left(s^t\right)$$

Assume $u(c_t(s^t)) = \ln(c_t(s^t))$
FOC:

$$c_t^{1,h}(s^t) = \theta_1 Y(s^t)$$
$$c_t^{2,h}(s^t) = \theta_2 Y(s^t)$$

Under the assumption of equal size in each country : $\theta_i = \frac{1}{2}$.

$$c_t^h(s^t) = \frac{1}{2} Y(s^t)$$
$$c_t^f(s^t) = \frac{1}{2} Y(s^t)$$

System of equations in Dynare:

$$c_t^h = 0.5Y_t \tag{0.1}$$

$$c_t^f = 0.5Y_t \tag{0.2}$$

$$y_t^h + y_t^f = Y_t \tag{0.3}$$

$$\begin{bmatrix} y_t^h \\ y_t^f \end{bmatrix} = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \begin{bmatrix} y_t^h \\ y_t^f \end{bmatrix} + V \begin{bmatrix} \varepsilon_{t+1}^h \\ \varepsilon_{t+1}^f \end{bmatrix} \tag{0.4}$$