$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\gamma}{2}\left[\frac{I_t}{I_{t-1}} - 1\right]^2 \tag{1}$$

How to log-linearize the following equilibrium condition:

$$Q_t\left[1 - S() - \frac{I_t}{I_{t-1}}S'()\right] + \beta\mathbb{E}_t\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 S'(\frac{I_{t+1}}{I_t}) = 0 \tag{2}$$

By assumption (ie. $S(1) = S'(1) = 0$), even by expliciting functional form and assuming $x_t = I_t/I_{t-1}$ and $x = 1$ at the steady state:

$$(1 + \hat{Q}_t)\left[1 - \frac{\gamma}{2}(x-1)^2(1 + 2\widehat{x_t - 1}) - \gamma x(x-1)(1 + \widehat{x_t - 1})\right]\ldots \tag{3}$$

The whole equation reduces to 0