Appendices: The Effects of Oil Price Shocks in a New-Keynesian Framework with Capital Accumulation

Verónica ACURIO VASCONEZ* Gaël GIRAUD[†] Florent Mc ISAAC[‡] Ngoc-Sang PHAM[§]

Centre d'Economie de la Sorbonne Paris School of Economics

March 16, 2015

These Appendices gather several technical issues related to the analysis of the DSGE model with energy and capital introduced in the main text.

Contents

${f Appen}$	dix A Model Derivations	2
A.1	Household	2
A.2	Final Good Firm	
A.3	Intermediate Goods Firms	
	A.3.1 Costs minimization	4
	A.3.2 Profit Maximization under Flexible Price	4
	A.3.3 Profit Maximization under Calvo Price setting	Ę
A.4	Equilibrium	7
A.5	Steady state	7
	Remark: Oil's Cost Share and Oil's Output elasticity	
Appen	dix B The Log-linear Model	ę
Appen	dix C Bayesian estimation procedure	10
C.1	Data treatment	1(
C.2	Identification Analysis	14
Appen	dix D Estimation results	19
	D.0.1 Restricted Estimation	20
D.1	Results	21

^{*}Email: veronica.acurio-vasconez@malix.univ-paris1.fr, Université Paris-1, Panthéon-Sorbonne, Maison des Sciences Economiques, 106-112 Boulevard de l'Hôpital, 75013 Paris

[†]CNRS, Paris School of Economics, Email: gael.giraud@parisschoolofeconomics.eu

[‡]Email: florent.mc-isaac@univ-paris1.fr, Université Paris-1.

[§]Email: pns.pham@gmail.com, Université Paris-1.

A Model Derivations

A.1 Household

The problem of the household is

$$\max_{\{C_t, L_t, B_t, K_{t+1}\}} \qquad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \big[u(C_t, L_t) \big], \quad 0 < \beta < 1,$$
 subject to :
$$P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} (K_{t+1} - (1 - \delta)K_t) + B_t$$

$$\leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t + T_t,$$

where the consumption flow is defined as:

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}, \tag{1}$$

with $x \in (0,1)$ being, at equilibrium, the share of oil in consumption, and $\Theta_x := x^{-x}(1-x)^{-(1-x)}$ and $C_{q,t} := \left(\int\limits_{[0,1]} C_{q,t} \frac{\epsilon-1}{\epsilon}(i)di\right)^{\frac{\epsilon}{\epsilon-1}}$ is a CES index of domestic goods. Note that, from (1), a fraction of imported oil is consumed by the household.

In order to ensure that this programme has a solution, we impose the following transversality condition (no Ponzi scheme):

$$\lim_{k \to \infty} \mathbb{E}_t \left(\frac{B_{t+k}}{\prod\limits_{s=0}^{t+k-1} (1+i_{s-1})} \right) \ge 0, \quad \forall t.$$

The optimal allocation of expenditures among different goods, domestic and foreign, yields:

$$P_{q,t}C_{q,t} = (1-x)P_{c,t}C_t$$

$$P_{e,t}C_{e,t} = xP_{c,t}C_t$$
CPI index:
$$P_{c,t} = P_{e,t}^x P_{q,t}^{1-x}$$

The Lagrangian associated with the maximization problem of the household has the following form

$$\mathcal{L}_{0} = \sum_{t=0}^{\infty} \beta^{t} \mathbb{E}_{0} \Big[u(C_{t}, L_{t}) + \lambda_{t} \big[P_{c,t} C_{t} + P_{k,t} I_{t} + B_{t} + T_{t} + (1 + i_{t-1}) B_{t-1} + W_{t} L_{t} + D_{t} + r_{t}^{k} P_{k,t} K_{t} \big] \Big]$$

Where λ_t is the Lagrange multiplier. The first order conditions are:

$$C_t: u_C(C_t, L_t) = \lambda_t P_{c,t}$$

$$L_t: u_L(C_t, L_t) = \lambda_t W_t$$

$$B_t: \lambda_t = \beta \mathbb{E}_t \Big[(1 + i_t) \lambda_{t+1} \Big]$$

$$K_{t+1}: \lambda_t P_{k,t} = \beta \mathbb{E}_t \Big[\lambda_{t+1} (r_{t+1}^k + 1 - \delta) P_{k,t+1} \Big].$$

Therefore, we have the following inter-temporal optimal conditions:

$$1 = \beta \mathbb{E}_{t} \left[(1+i_{t}) \frac{C_{t}}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \right]$$

$$1 = \beta \mathbb{E}_{t} \left[\frac{C_{t}}{C_{t+1}} \frac{P_{c,t}}{P_{c,t+1}} \frac{P_{k,t+1}}{P_{k,t}} (r_{t+1}^{k} + 1 - \delta) \right]$$

$$\frac{W_{t}}{P_{c,t}} = C_{t} L_{t}^{\phi}$$

One can define:

1. The stochastic discount factor from date t to date t+1

$$d_{t,t+1} := \frac{\beta u_C(C_{t+1}, L_{t+1})}{u_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+1}} =: \Delta_t^{t+1}, i.e, \quad \frac{1}{1+i_t} = \mathbb{E}_t(d_{t,t+1}).$$

2. The stochastic discount factor from date t to date t + k

$$d_{t,t+k} := \prod_{s=t}^{t+k-1} \Delta_s^{s+1}, \text{ then,} \quad d_{t,t+k} := \frac{\beta^k u_C(C_{t+k}, L_{t+k})}{u_C(C_t, L_t)} \frac{P_{c,t}}{P_{c,t+k}}.$$

A.2 Final Good Firm

A representative final good firm maximizes its profit without market power.

$$\max_{Q_t(\cdot)} P_{q,t} Q_t - \int_{[0,1]} P_{q,t}(i) Q_t(i) di$$
 subject to :
$$Q_t = \left(\int_{[0,1]} Q_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

The first order condition for $Q_t(i)$ is:

$$P_{q,t} \frac{\epsilon}{\epsilon - 1} \left(\int\limits_{[0,1]} Q_t(i)^{\frac{\epsilon - 1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1} - 1} \qquad \frac{\epsilon - 1}{\epsilon} Q_t(i)^{\frac{\epsilon - 1}{\epsilon} - 1} - P_{q,t}(i) = 0$$

$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon} Q_t$$

The price of the final good will therefore be:

$$P_{q,t} = \left(\int_{[0,1]} P_{q,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}.$$
 (2)

A.3 Intermediate Goods Firms

One can assume the following production function for the intermediate good firm i,

$$Q_t(i) := A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k}$$
$$\alpha_e, \alpha_\ell, \alpha_k \ge 0.$$

Intermediate goods firms solve a two-stage problem. Firstly, the costs minimization and, secondly, the profit maximization.

A.3.1 Costs minimization

One can derive the following Lagragian.

$$\mathcal{L}_{0} = P_{e,t}E_{t}(i) + W_{t}L_{t}(i) + r_{t}^{k}P_{k,t}K_{t}(i) - mc_{t}(i)\Big(A_{t}E_{t}(i)^{\alpha_{e}}L_{t}(i)^{\alpha_{\ell}}K_{t}(i)^{\alpha_{k}} - Q_{t}(i)\Big)$$

The first order conditions are:

 $E_t(i): P_{e,t} = mc_t(i)\alpha_e A E_t(i)^{\alpha_e - 1} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k}$

 $L_t(i): W_t = mc_t(i)\alpha_\ell A E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell - 1} K_t(i)^{\alpha_k}$

 $K_t(i): \qquad r_t^k P_{k,t} = mc_t(i)\alpha_k A E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k - 1}.$

Hence, the following relation must hold:

$$\frac{W_t L_t(i)}{\alpha_\ell} = \frac{r_t^k P_{k,t} K_t(i)}{\alpha_k} = \frac{P_{e,t} E_t(i)}{\alpha_e}.$$

On the other hand, we have

$$\begin{split} Q_t(i) &= A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k} \\ &= A_t \Big(\frac{\alpha_e m c_t(i) Q_t(i)}{P_{e,t}}\Big)^{\alpha_e} \Big(\frac{\alpha_\ell m c_t(i) Q_t(i)}{W_t}\Big)^{\alpha_\ell} \Big(\frac{\alpha_k m c_t(i) Q_t(i)}{r_t^k P_{k,t}}\Big)^{\alpha_k} \\ &= \frac{A_t \alpha_e^{\alpha_e} \alpha_\ell^{\alpha_\ell} \alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e} W_t^{\alpha_\ell} (r_t^k P_{k,t})^{\alpha_k}} \big[m c_t(i) Q_t(i)\big]^{\alpha}. \end{split}$$

Where $\alpha := \alpha_e + \alpha_k + \alpha_l$. Defining $F_t := \left(\frac{A_t \alpha_e^{\alpha_e} \alpha_\ell^{\alpha_\ell} \alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e} W_t^{\alpha_\ell} (r_t^k P_{k,t})^{\alpha_k}}\right)^{\frac{-1}{\alpha}}$.

Thus,

$$mc_t(i) = F_t Q_t(i)^{\frac{1}{\alpha} - 1}$$

And the cost function is:

$$cost(Q_t(i)) = \alpha F_t Q_t(i)^{\frac{1}{\alpha}}$$

A.3.2 Profit Maximization under Flexible Price

At each date t, firm i's profit maximization problem is

$$\max_{P_{q,t}(i)} P_{q,t}(i)Q_t(i) - cost(Q_t(i))$$
 subject to
$$Q_t(i) = \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon}Q_t.$$

Note that this problem does not depend on i. Consequently, its solution $P_{q,t}(i)$ does not depend on i, i.e. $P_{q,t}(i) = P_{q,t}^*$ for every i. Combining with (2), we have $P_{q,t}(i) = P_{q,t}$ for every i.

The first order condition for $P_{q,t}^*$ gives

$$P_{q,t}^* = \frac{\epsilon}{\epsilon - 1} m c_t^*,$$

where $mc_t^* := F_t Q_t^{\frac{1}{\alpha} - 1}$.

A.3.3 Profit Maximization under Calvo Price setting

In each period, the firm i has a probability θ to not reset its price. At each date t, firm i's profit maximization problem is

$$\begin{split} \max_{P_{q,t}(i)} \mathbb{E}_t \Big[\sum_{k=0}^{\infty} \theta^k d_{t,t+k} \big[P_{q,t}(i) Q_{t,t+k}(i) - cost(Q_{t,t+k}(i)) \big] \Big] \\ \text{subject to} \qquad Q_{t,t+k}(i) = \Big(\frac{P_{q,t}(i)}{P_{q,t+k}} \Big)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0. \end{split}$$

Note that this problem does not depend on i, hence its solution $P_{q,t}(i)$ does not either, we write $P_{q,t}(i) = P_{q,t}^o$. The first order condition for $P_{q,t}^o$ is:

$$\mathbb{E}_{t} \sum_{k=0}^{\infty} \theta^{k} d_{t,t+k} Q_{t,t+k}^{o} \Big[P_{q,t}^{o} - \mu^{p} m c_{t,t+k}^{o} \Big] = 0,$$

where $\mu^p := \frac{\epsilon}{\epsilon - 1}$, $mc_{t,t+k}^o := F_{t+k}(Q_{t,t+k}^o)^{\frac{1}{\alpha} - 1}$, and $Q_{t,t+k}^o := \left(\frac{P_{q,t}^o}{P_{q,t+k}}\right)^{-\epsilon} Q_{t+k}$ for every k > 0.

For computational purpose, one can re-write the first order condition recursively, such that:

$$A_t^o := \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o,$$

$$B_t^o := \mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o m c_{t,t+k}^o.$$

We have

$$\begin{array}{rcl} P_{q,t}^{o}A_{t}^{o} & = & \mu^{p}B_{t}^{o}, \\ A_{t}^{o} & := & Q_{t,t}^{o} + \theta \mathbb{E}_{t}d_{t,t+1}A_{t+1}^{o}, \\ B_{t}^{o} & := & Q_{t,t}^{o}mc_{t,t}^{o} + \theta \mathbb{E}_{t}d_{t,t+1}B_{t+1}^{o}. \end{array}$$

Denote

$$m_{t,t+k}^o := \frac{mc_{t,t+k}^o}{P_{q,t+k}}.$$

Then,

$$\mathbb{E}_{t} \sum_{k=0}^{\infty} \theta^{k} d_{t,t+k} Q_{t,t+k}^{o} \left[P_{q,t}^{o} - \mu^{p} m_{t,t+k}^{o} P_{q,t+k} \right] = 0,$$

The next three lemmas show the integration of the production function using Calvo price setting.

Lemma A.1. The aggregate production function:

$$\left(\int\limits_{[0,1]} \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{\frac{-\epsilon}{\alpha}} di\right)^{\alpha} Q_t = A_t E_t^{\alpha_e} L_t^{\alpha_\ell} K_t^{\alpha_k}.$$

Proof. One has

$$\begin{split} \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{-\epsilon} Q_t &= Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k} \\ &= A E_t(i)^{\alpha_e} \left(\frac{P_{e,t} E_t(i)}{W_t} \frac{\alpha_\ell}{\alpha_e}\right)^{\alpha_\ell} \frac{P_{e,t} E_t(i)}{r_t^k P_{k,t}} \frac{\alpha_k}{\alpha_e} \\ &= A_t E_t(i)^{\alpha} \left(\frac{P_{e,t}}{W_t} \frac{\alpha_\ell}{\alpha_e}\right)^{\alpha_\ell} \left(\frac{P_{e,t}}{r_t^k P_{k,t}} \frac{\alpha_k}{\alpha_e}\right)^{\alpha_k}. \end{split}$$

Hence we get

$$\left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{\frac{-\epsilon}{\alpha}}Q_t^{\frac{-\epsilon}{\alpha}} = E_t(i) \left[A \left(\frac{P_{e,t}}{W_t} \frac{\alpha_\ell}{\alpha_e}\right)^{\alpha_\ell} \left(\frac{P_{e,t}}{r_\star^k P_{k,t}} \frac{\alpha_k}{\alpha_e}\right)^{\alpha_k} \right]^{\frac{1}{\alpha}}.$$

By integrating out,

$$\left(\int\limits_{[0,1]} \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{\frac{-\epsilon}{\alpha}} di\right)^{\alpha} Q_{t} = E_{t}^{\alpha} A \left(\frac{P_{e,t}}{W_{t}} \frac{\alpha_{\ell}}{\alpha_{e}}\right)^{\alpha_{\ell}} \left(\frac{P_{e,t}}{r_{t}^{k} P_{k,t}} \frac{\alpha_{k}}{\alpha_{e}}\right)^{\alpha_{k}}.$$

Recall that

$$\frac{W_t L_t(i)}{\alpha_\ell} = \frac{r_t^k P_{k,t} K_t(i)}{\alpha_k} = \frac{P_{e,t} E_t(i)}{\alpha_e}.$$

By taking integral, we get

$$\frac{W_t L_t}{\alpha_\ell} = \frac{r_t^k P_{k,t} K_t}{\alpha_k} = \frac{P_{e,t} E_t}{\alpha_e}.$$

Combining with (3), the result obtains.

Lemma A.2. Under the Calvo price setting, the following "Aggregate Price Relationship" holds:

$$P_{q,t} = \left(\theta P_{q,t-1}^{1-\epsilon} + (1-\theta)(P_{q,t}^o)^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$

Proof. By definition we have

$$\begin{split} P_{q,t}^{1-\epsilon} &= \int\limits_{[0,1]} P_{q,t}(i)^{1-\epsilon} di \\ &= \int\limits_{\text{Firms that cannot change price}} P_{q,t}(i)^{1-\epsilon} di + \int\limits_{\text{Firms setting price optimally}} P_{q,t}(i)^{1-\epsilon} di \\ &= \int\limits_{[0,1]} \theta P_{q,t-1}(i)^{1-\epsilon} di + \int\limits_{[0,1]} (1-\theta) P_{q,t}(i)^{1-\epsilon} di \\ &= \theta P_{q,t-1}^{1-\epsilon} + (1-\theta) (P_{q,t}^o)^{1-\epsilon}. \end{split}$$

Define $v_t := \int\limits_{[0,1]} \left(\frac{P_{q,t}(i)}{P_{q,t}}\right)^{\frac{-\epsilon}{\alpha}} di.$

Lemma A.3. Under the Calvo price setting,

$$v_t = \theta v_{t-1} \Pi_{q,t}^{\frac{\epsilon}{\alpha}} + (1 - \theta) \left(\frac{P_{q,t}^o}{P_{q,t}} \right)^{\frac{-\epsilon}{\alpha}}$$

Proof. As Lemma A.2

A.4 Equilibrium

At equilibrium,

- (i) Each economic agent solves its maximization problem;
- (ii) All markets clear, i.e., the following equations hold:

Capital:
$$K_t = \int_{[0,1]} K_t(i)di$$
,
Labor: $L_t = \int_{[0,1]} L_t(i)di$,
Energy: $E_t = \int_{[0,1]} E_t(i)di$,

Resource constraint: $P_{c,t}C_t + P_{k,t}I_t + G_t = P_{q,t}Q_t - P_{e,t}E_t$.

(iii) And the government budget constraint is fulfilled:

$$(1+i_{t-1})B_{t-1} + G_t = B_t + T_t,$$

A.5 Steady state

$$\begin{array}{rcl} \text{Static problem of Household:} & C & = & \Theta_x C_e^x C_q^{1-x} \\ & \frac{P_c}{P_{q,t}} & = & S^x =: P_c^r \\ & C_q & = & (1-x)P_c^r C \\ & SC_e & = & xP_c^r C. \\ & 1 & = & \beta(r^k+1-\delta) \\ \text{Budget constraint:} & P_c^r C + \delta S_k K & = & W^r N + r^k S_k K + \Pi^r, \\ & \Pi^r & = & Q - SE - W^r L - r^k S_k K \\ \text{The FOC for the representative household is:} & W^r & = & P_c^r C L^\phi \\ & & \text{Production function:} & Q & = & AE^{\alpha_e} L^{\alpha_\ell} K^{\alpha_k} \\ & \text{The FOC for firms are:} & \frac{SE}{\alpha_e} & = & \frac{W^r L}{\alpha_\ell} = \frac{r^k S_k K}{\alpha_k} \\ & SE & = & \frac{\alpha_e(\epsilon-1)}{\epsilon} Q. \end{array}$$

And remember that, by assumption,

$$S = \frac{P_e}{P_q}$$
, where S is exogenous $S_k = \frac{P_k}{P_q}$, where S_k is exogenous

Where $W^r = W/P_q$. We have to find $(C, C_e, C_q, r^k, W, Q, E, L, K)$.

Solution: Without loss of generality, we normalize $S_k = S = 1$. Remember that $P_c^r C + \delta S_k K + G_r = Q - SE$, $E = \frac{\alpha_e(\epsilon - 1)}{\epsilon}Q$, and $\frac{E}{\alpha_E} = \frac{r^k K}{\alpha_k}$, so that

$$C = Q - SE - \delta S_k K - G$$
$$= Q \left(1 - \omega - \frac{\alpha_e(\epsilon - 1)}{\epsilon} - \frac{\alpha_e(\epsilon - 1)}{\epsilon} \frac{\delta \alpha_k}{\alpha_e r^k} \right)$$

Therefore, one can compute $\frac{Q}{C}$. The system of equations becomes

$$\begin{split} r^k &= \frac{1}{\beta} - 1 + \delta \\ C &= \left(1 - \omega - \frac{\alpha_e(\epsilon - 1)}{\epsilon} (1 + \delta \frac{\alpha_k}{\alpha_e r^k})\right) Q \\ C_q &= (1 - x)C \\ C_e &= xC \\ W^r &= CL^{\phi} \\ Q &= AE^{\alpha_e}L^{\alpha_\ell}K^{\alpha_k} \\ \frac{E}{\alpha_e} &= \frac{W^rL}{\alpha_\ell} \\ E &= \frac{\alpha_e(\epsilon - 1)}{\epsilon}Q \end{split}$$

By combining $W^r = CL^{\phi}$ with $\frac{E}{\alpha_e} = \frac{W^rL}{\alpha_\ell}$, we can compute the following quantities

$$L^{\phi+1} = \frac{(\epsilon - 1)\alpha_{\ell}}{\epsilon} \frac{Q}{C}$$

$$Q^{1-\alpha_{e}-\alpha_{k}} = AL^{\alpha_{\ell}} \left(\frac{\epsilon - 1}{\epsilon} \alpha_{\ell}\right)^{\alpha_{e}} \left(\frac{\epsilon - 1}{\epsilon} \frac{\alpha_{k}}{r^{k}}\right)^{\alpha_{k}}$$

$$E = \frac{\alpha_{e}(\epsilon - 1)}{\epsilon} Q$$

$$W^{r} = \frac{\alpha_{\ell}(\epsilon - 1)}{L\epsilon} Q.$$

A.6 Remark: Oil's Cost Share and Oil's Output elasticity

Let us define the oil's cost share as follows

Oil's cost share :=
$$\frac{P_e E}{P_y Y}$$

where Y is the GDP. Remember than in our case $P_yY = P_qQ - P_eE$. Then

Oil's cost share
$$\begin{split} &= \frac{P_e E}{P_q Q - P_e E} \\ &= \frac{\frac{P_e E}{P_q Q}}{1 - \frac{P_e E}{P_q Q}} \end{split}$$

at the steady state one has the following relationship

$$\frac{P_e E}{P_q Q} = \frac{\alpha_e}{\mu_p}$$

where μ_p is the price markup and α_e is the output elasticity. Then one has

B The Log-linear Model

The model is log-linearized using the following rules:

- All variables in non-capital letter stand for the log-deviation, e.g : $e_t = \log(E_t) \log(E)$, where variables without subscript stand for the steady state value.
- Exceptions for r_t^k , the log-deviation of the latter is \tilde{r}_t^k , I_t becomes \tilde{I}_t and mc_t becomes $\tilde{m}c_t$.
- All prices are in real value, in other words all prices $(P_{e,t}, P_{q,t}, P_{c,t} \& P_{k,t})$ are divided by the core CPI $(P_{q,t})$.
- One consequence of the previous item is that wages are expressed in real terms $(w_{r,t})$ and likewise for government spending $(g_{r,t})$.

$$\begin{array}{rcl} i_t & = & (1-\beta(1-\delta))\mathbb{E}_t[\tilde{r}_{t+1}^*] + \mathbb{E}_t[\pi_{k,t+1}] \\ c_t & = & \mathbb{E}_t[c_{t+1}] - (i_t - \mathbb{E}_t[\pi_{c,t+1}]) \\ w_{r,t} & = & c_t + \phi l_t + x s_{e,t} \\ i_t & = & \phi_\pi \pi_{q,t} + \phi_y y_t + \varepsilon_{i,t} \\ l_t + w_{r,t} & = & s_{e,t} + e_t \\ & = & \tilde{r}_t^k + s_{k,t} + k_t \\ & = & \tilde{r}_t^k + s_{k,t} + k_t \\ \hline \end{cases} \\ (S^xC)(c_t + x s_{e,t}) + (S_k I)(s_{k,t} + \tilde{I}_t) + G_r g_{r,t} = Qq_t - (SE)(s_{e,t} + e_t) \\ \delta \tilde{I}_t & = & k_{t+1} - (1-\delta)k_t \\ q_t & = & a_t + \alpha_t l_t + \alpha_e e_t + \alpha_k k_t \\ \pi_{q,t} - \beta \mathbb{E}_t[\pi_{q,t+1}] & = & \left(\frac{(1-\beta\theta)(1-\theta)\alpha}{\theta(\alpha+(1-\alpha)\epsilon)}\right) \tilde{m} c_{r,t} + \varepsilon_{p,t} \\ \tilde{m} c_{r,t} & = & \left[\frac{1-\alpha}{\alpha}q_t + F_t^r\right] \\ F_t^r & = & -\frac{1}{\alpha}(a_t - \alpha_e s_{e,t} - \alpha_t w_{r,t} - \alpha_k (r_t^k + s_{k,t})) \\ (S^xY)(y_t + x s_{e,t}) & = & Qq_t - (SE)(s_{e,t} + e_t) \\ \pi_{c,t} & = & \pi_{q,t} + x \Delta s_{e,t} \\ \pi_{k,t} & = & \pi_{q,t} + \Delta s_{k,t} \\ g_{r,t} & = & \rho_g g_{r,t-1} + \rho_{ag} e_{a,t} + e_{g,t} \\ s_{e,t} & = & \rho_s e s_{e,t-1} + e_{se,t} \\ s_{k,t} & = & \rho_s k s_{k,t-1} + e_{sk,t} \\ a_t & = & \rho_s k s_{k,t-1} + e_{sk,t} \\ \varepsilon_{t,t} & = & \rho_t \varepsilon_{t,t-1} + e_{t,t} \\ \varepsilon_{p,t} & = & \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1} \end{array}$$

- $\varepsilon_{i,t}$ stands for the exogenous part of the monetary policy.
- $\varepsilon_{p,t}$ stands for the price mark-up disturbance, it follows an ARMA(1,1), the inclusion of the MA part is designed to capture the high-frequency fluctuations in inflation.

This system has 20 variables and 20 equations. 14 endogenous variables, namely $(i_t, \tilde{r}_t^k, \pi_{k,t}, \pi_{c,t}, \pi_{q,t}, w_{r,t}, mc_{r,t}, c_t, l_t, y_t, e_t, k_t, \tilde{I}_t, q_t)$ and 6 exogenous disturbances $(a_t, s_{e,t}, s_{k,t}, g_{r,t}, \varepsilon_{i,t}, \varepsilon_{p,t})$.

C Bayesian estimation procedure

C.1 Data treatment

This section details the Bayesian estimation procedure of the DSGE model developed in the body of the paper.

We use six key macro-variables for our estimations. All series are quarterly. A description of the original series' sources is presented in Table 1 and the data is available upon request. The sample goes from 1984Q1 to 2007Q1.

Table 1: Original Sources

Serie	Description	Source
GDPC09	Real Gross Domestic Product, Chained Dollars (2009), Seasonally Adjusted, Annual Rate	Table 1.1.6 Bureau of Economic Analysis
GDPDEF	Implicit Price Deflators for Gross Domestic Product (2009), Seasonally Adjusted	Table 1.1.9. Bureau of Economic Analysis
PFI	Private Fixed Investment by Type, Seasonally Adjusted, Annual Rate	Table 5.3.5. Bureau of Economic Analysis
CE16OV	Civilian Employment, 16 and over, Seasonally Adjusted, Thousands	LNS12000000 Bureau of Labor Statistics
CE16OV Index	CE160V (2009)=1	
LNS10	Population level, civilian noninstitutional population, 16 and over, Seasonally Adjusted, Thousands	LNS10000000 Bureau of Labor Statistics
LNS10 Index	LNS10 (2009)=1	
PRS85006023	Nonfarm Business, All Persons, Average weekly hours worked Duration (2009), Seasonally Adjusted $$	PRS85006023 Bureau of Labor Statistics
FEDFUND	Federal funds effective rate, percent: Per Year, Average of Daily figures	Board of Governors of the Federal Reserve System
$O_{commercial}$	Total Petroleum Consumed by the Commercial Sector, Thousand barrels per day	Table 3.7a. U.S Energy Information Administration
$O_{industrial}$	Total Petroleum Consumed by the Industrial Sector, Thousand barrels per day $$	Table 3.7b. U.S Energy Information Administration
$O_{electrical}$	Total Petroleum Consumed by the Electrical Power Sector, Thousand barrels per day	Table 3.7c. U.S Energy Information Administration
$O_{transport}$	Total Petroleum Consumed by the Transport Sector, Thousand barrels per day	Table 3.7c. U.S Energy Information Administration
PSG	Passenger to freight, TBTu	Transportation Energy Intensity Indicators US Department of Energy

Our observable variables include: (i) real GDP, (ii) real Private Fixed Investment, (iii) hours worked, (iv) inflation (through the GDP price deflator), (v) the Federal Funds Rate and (vi) total oil use in production. The model is stationary, so we remove the trend of the first two series, that are trend stationary. The rest of the series are stationary, we do not remove their trends, but we take out their respective mean for the estimation period. A detailed explanation is presented on Table 2.

Table 2: Observable Variables

Observed Variable	Transformation
invobs	$detrend\left(ln\left(\frac{\frac{PFI}{GDPDEF}}{LNSIndex}\right)*100\right)$
yobs	$detrend\left(ln\left(\frac{GDPC09}{LNSIndex}\right)*100\right)$
labobs	$ln\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right)*100-mean\left(ln\left(\frac{PRS85006023*CE16OVIndex}{LNSIndex}\right)*100\right)$
infobs	$ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100-mean\left(ln\left(\frac{GDPDEF}{GDPDEF(-1)}\right)*100\right)$
iobs	$\left(ln\left(1 + \frac{FEDFUND}{400}\right) - mean\left(ln\left(1 + \frac{FEDFUND}{400}\right)\right)\right) * 100$
eobs	$ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100 - mean\left(ln\left(\frac{TotalSAOil}{LNSIndex}\right)*100\right)$

The total oil consumption of the production sector *TotalSAOil*, is constructed as follows:

$$TotalOil = O_{industrial} + O_{electrical} + O_{commercial} + (1 - PSG) * O_{transport},$$

where PSG is a measure of energy consumption in transport by households¹, computed as the ratio of the energy consumption of all passengers and the total energy consumption in transport (Total Energy consumption in transport=energy consumption of all passengers + total energy consumption of All Freight).

Then, seasonality is removed with X12-ARIMA software from the Census Bureau, implemented in the open-source GRETL software, from where we obtain the series TOTAL-SAOil.

Finally, we have to identify our observable variables to our model's variables. Note that we have few different types of prices in our model, among them: the domestic price, the CPI, which is equal to the GDP deflator by definition, and the price capital. Because we deflate the investment series by the GDP deflator (in the data treatment) and in our model the real series are deflated by the domestic price, we use the following observation equation for the investment:

$$invobs_t = \widehat{I}_t + s_{k,t} - xs_{e,t}$$

¹As for oil, it is the source of some 95% of transport fuels globally, and without oil-based transport none of the other energy forms (such as electricity) and other primary energy sources (such as coal, gas, biomass, wind, solar, hydro, and so on) can be delivered. In this specific sense oil remains the most critical of all energy sources, in particular in transports.

The other equations are:

$$ybos_t = y_t$$

$$labobs_t = l_t$$

$$eobs_t = e_t$$

$$infobs_t = \pi_{c,t}$$

$$iobs_t = i_t$$

C.2 Identification Analysis

In order to run an identification analysis, we need to specify starting values for all parameters to identify. We first initialize our parameters as in Table 3

Table 3: Starting values for the first identification

α_e	α_{ℓ}	α_k	ϕ	ϕ_{π}	ϕ_y	θ	$ ho_j$	σ_j
0.015	0.7	0.3	1.17	1.2	0.5	0.65	0.5	1

where $j \in \{a, se, sk, g, p, i\}$, so that ρ_j denotes all the autoregressive parameters in the model and σ_j , all the standard deviations.

The measure of identification strength developed by [Iskrev, 2010] and [Andrle, 2010] gives the following result

All parameters are identified in the model (rank of H).

All parameters are identified by J moments (rank of J).

Figure 1: Rank condition

Figure (2) refers to the identification and sensitivity methodologies with respect to the first and second moments proposed by [Iskrev, 2010] and [Andrle, 2010]. Firstly, all parameters are identified, this result confirms the necessary and sufficient conditions (printed in Figure (1)) discussed by [Iskrev, 2010] for local identifiability. Secondly, this setting allows us, nevertheless, to observe the lack of identifiability strength for the parameter α_e , here around 0.015. This last observation allows us to test whether this identification strength issue could be fixed using different initial values for the elasticities.

Figures (3) to (8) summarize the identification strength explained *supra* for the set of initial values in Table 4. Few observations of these graphs are worth making. Firstly, the higher α_e , the higher identification strength. Secondly, all parameters (except output elasticities) nearly keep the same ranking, in the sense that there is no shift greater than two positions. Thirdly, α_ℓ decreases in its identification strength, whereas α_k keeps the same ranking. Fourthly, one can note that in this experimentation, parameter θ , as opposed to the initial calibration, looses nearly all its identification strength. This explains why we estimate and compare the model with and without estimating θ .

Table 4: Set of starting values

Elasticity	(1)	(2)	(3)	(4)	(5)	(6)
α_e	0.1	0.2	0.3	0.4	0.5	0.6
$lpha_k$	0.3	0.3	0.2	0.2	0.2	0.1
α_ℓ	0.7	0.6	0.6	0.5	0.4	0.4

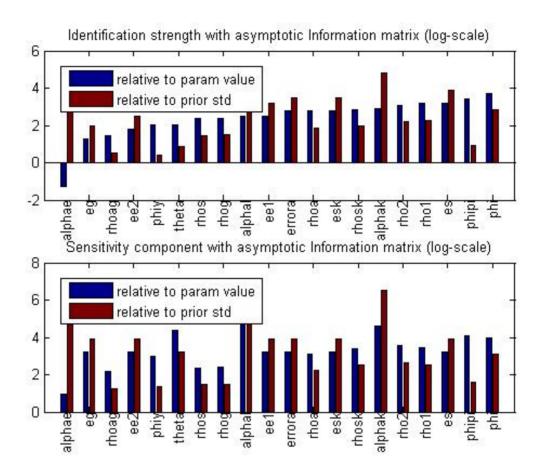


Figure 2: Identification Strength

.

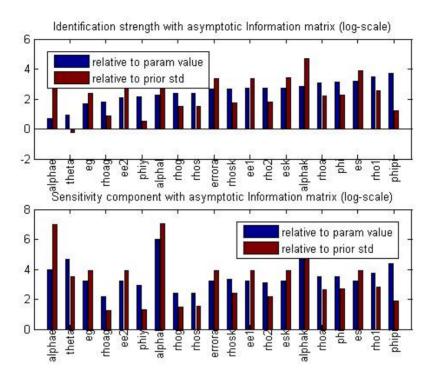


Figure 3: Identification Strength for (1)

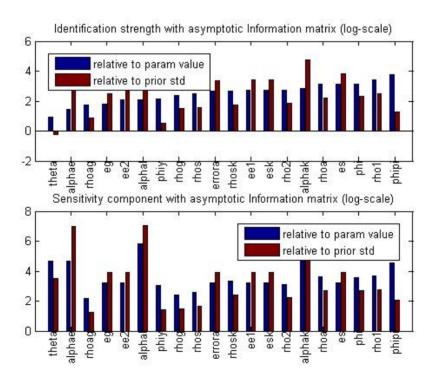


Figure 4: Identification Strength for (2)

.

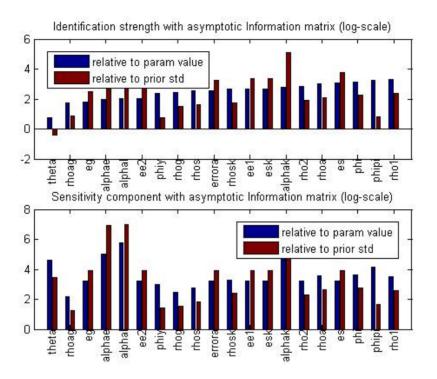


Figure 5: Identification Strength for (3)

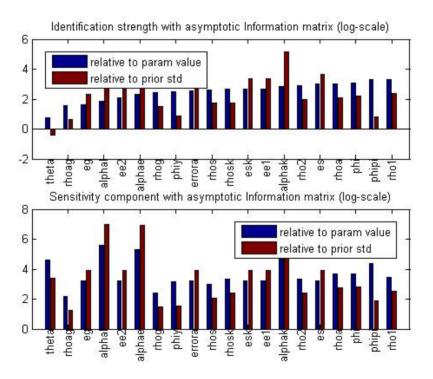


Figure 6: Identification Strength for (4)

.

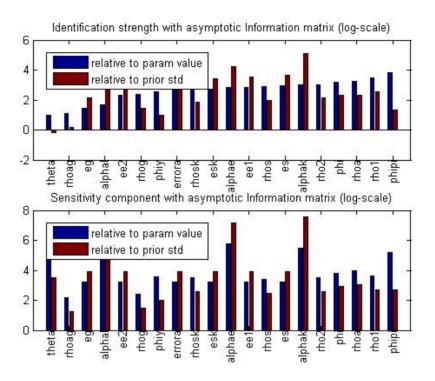


Figure 7: Identification Strength for (5)

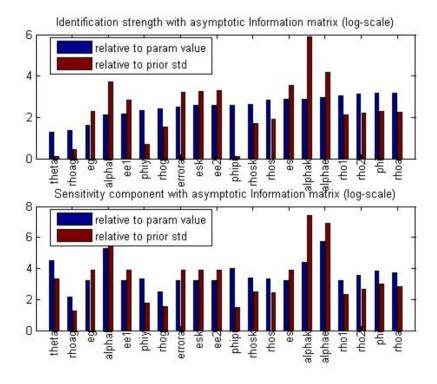


Figure 8: Identification Strength for (6)

D Estimation results

0.2

0.4

There are 26 parameters, including parameters that characterize the exogenous shocks. Of the 26 parameters, we fix 5 according to the literature. The discount factor β is calibrated at 0.99. The depreciation rate δ is calibrated at 0.025. We set the government spending output share ω to 0.18 and we calibrate ϵ to 8, that generate a steady state markup approximately equal to 1.14. These values are in line with empirical results. Finally, following [Blanchard and Galí, 2009], we calibrate the share of oil consumption for the households x, at 0.023. The calibration of these parameters are resumed in Table 5. Those parameters are calibrated due to their well-known lack of identification in macro-data. Note that for estimation proposes we add an ad-hoc shock for the New-Keynesian Phillips Curve $\varepsilon_{p,t}^2$, that can be interpreted as being a markup shock.

Table 5: Calibrated Parameters

β	δ	ω	x	ϵ
0.99	0.025	0.18	0.023	8

The priors distribution and priors' parameters of other variables remain unchanged except for the prior's parameters of α_k and α_ℓ that change along with α_e , as shown in Table 4.

 α_e prior value Log marg. density $\hat{\alpha}_e$ Sum of α_i θ estim. θ calib. θ estim. θ calib. θ calib. θ estim. θ calib. θ estim. 0.015 0.015 -567.16-570.930.11780.01171.36481.10770.3 0.5-567.65-589.990.0850.11771.3622 1.0952 0.5 0.2 -579.18-591.80 0.11380.0533 1.2002 1.0913 -586.98-592.990.60.10.12540.03561.12641.1188 0.1 0.6 -592.84-593.28 0.0820.1304 1.1168 1.0966 0.40.3-596.08 -596.51 0.1090 0.0625 1.0226 1.1023

0.0839

0.1055

1.1322

1.0915

-600.66

-596.92

Table 6: Summary of estimation results

Table 6 is ranking (ascending) with respect to the log-marginal density values. Several observations can emerge from this table. First, the first (best) three estimations when θ is estimated give us the sum of elasticities greater than the steady state markup $(\varepsilon/(\varepsilon-1) \approx 1.14)$. This gives rise to problematic economic interpretation due to the fact that one can show that if $\sum_{i \in \{e,l,k\}} \alpha_i > \varepsilon/(\varepsilon-1)$, the steady state value of firm's profit is negative. This is not surprising, since the model does not restrict the production function to have a constant return to scale technology together with the fact that the estimation procedure can hit the upper bound of the prior distribution. So in this case, one might find results without economic sense. This observation gives rise to an augmented estimation procedure

Where $\varepsilon_{p,t}$ is a ARMA(1,1) process of the form $\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1}$, where $e_{p,t} \sim \mathcal{N}(0, \sigma_p^2)$

in order to avoid this situation, especially, one can propose a narrower restriction on the upper bound of prior distribution on output elasticities, define shortly. Second, the first estimate of oil's output elasticity when θ is calibrated, suggests an estimated $\hat{\alpha}_e$ similar to its first prior value. According to the first identification analysis around $\alpha_e = 0.015$, the identification strength of this parameter, advocates a weak identification. This intuition is confirmed using Figure 9, the prior and the posterior distribution match, therefore the identification issue previously raised is confirmed since this parameter is only explained by its prior distribution.

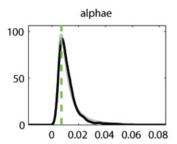


Figure 9: Posterior (solid black kine) and prior (solid grey line) distribution of α_e . The dashed green line stands for the posterior empirical mode

D.0.1 Restricted Estimation

Table 7 refers to the upper bound restriction limits for the first three estimations of Table 6 with respect to their own stars superscripts.

Table 7: Prior's upper bound restriction on output elasticities parameters

Elasticity	0.015	0.3	0.5
$lpha_e$	0.4	0.4	0.3
α_k	0.3	0.35	0.3
$lpha_\ell$	0.7	0.75	0.7

As shown in Table 8, once we restrict the model, the log-marginal density drops to a lower level. We can conclude that the model where θ is estimated and where the first prior parameter of α_e is equal to 0.6, corresponding to the forth column of the Table 6, outperforms these latest.

Table 8: Restricted estimation results

α_e prior value	Log marg. density	\hat{lpha}_e	Sum of α_i
0.015	-591.24	0.0798	1.0666
0.3	-594.37	0.0727	1.0681
0.5	-620.28	0.1458	1.1341

D.1 Results

As for the results obtained regarding the estimates of the stochastic processes resumed in Table 9, one can extract important observations. Concerning standard deviation estimates, most of the variance is driven by the demand shock (σ_g) and real price of oil (σ_{se}) in both estimates.³ The high standard deviation for the price of oil can be interpreted as being the resulting of a financial asset trade in a volatile stock market. For the case θ calibrated, we find a high persistency on AR(1) coefficients for government spending (0.93), price markup (0.96), technology (0.94) and the real price of oil (0.98), whereas for the other case, only the first two, together with the monetary policy (0.9308) have a high autoregressive parameter.

³Note that standard deviations describe in Table 9 are in percentage, meaning that if $\sigma = 1$, then 1 stands for 1%

Table 9: Prior and Posterior Distribution of Shock Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
θ estimated						
Autoregressive parameters						
Technology	$ ho_a$	Beta(0.5, 0.2)	0.8619	0.8481	0.7960	0.8999
Real oil price	$ ho_{se}$	$\mathrm{Beta}(0.5, 0.2)$	0.5761	0.5611	0.4629	0.6669
Real capital price	ρ_{sk}	Beta(0.5, 0.2)	0.7210	0.7080	0.6647	0.7524
Price markup1	$ ho_p$	$\mathrm{Beta}(0.5, 0.2)$	0.9418	0.9283	0.8955	0.9640
Price markup2	$ u_p$	Beta(0.5,0.2)	0.9796	0.9760	0.9610	0.9913
Government	$ ho_g$	Beta(0.5, 0.2)	0.9058	0.8995	0.8712	0.9258
Tech. in Gov.	$ ho_{ag}$	$\mathrm{Beta}(0.5, 0.2)$	0.6904	0.6127	0.3549	0.9472
Monetary	$ ho_i$	$\mathrm{Beta}(0.5, 0.2)$	0.9399	0.9308	0.9035	0.9581
Standard deviations						
Technology	σ_a	IGamma(1,2)	0.4361	0.4435	0.3901	0.4942
Real oil price	σ_{se}	IGamma(1,2)	2.0000	1.9373	1.8652	2.000
Real capital price	σ_{sk}	IGamma(1,2)	0.7740	0.7675	0.6379	0.8781
Price markup	σ_p	IGamma(1,2)	0.1814	0.1854	0.1615	0.2094
Government	σ_g	IGamma(1,2)	2.0000	1.7921	1.5508	1.9998
Monetary	σ_i	IGamma(1,2)	0.5410	0.4566	0.3859	0.5205
θ calibrated						
Autoregressive parameters						
Technology	$ ho_a$	Beta(0.5, 0.2)	0.9605	0.9401	0.9033	0.9774
Real oil price	$ ho_{se}$	Beta(0.5, 0.2)	0.9934	0.9872	0.9754	0.9977
Real capital price	ρ_{sk}	Beta(0.5, 0.2)	0.8940	0.8924	0.8483	0.9314
Price markup1	$ ho_p$	Beta(0.5,0.2)	0.9839	0.9621	0.9299	0.9971
Price markup2	$ u_p$	Beta(0.5, 0.2)	0.1652	0.1711	0.0593	0.2758
Government	$ ho_g$	Beta(0.5, 0.2)	0.9373	0.9312	0.9061	0.9560
Tech. in Gov.	$ ho_{ag}$	Beta(0.5, 0.2)	0.7129	0.6589	0.3808	0.9541
Monetary	$ ho_i$	Beta(0.5,0.2)	0.1914	0.2104	0.1249	0.2856
Standard deviations						
Technology	σ_a	IGamma(1,2)	0.4538	0.4542	0.3981	0.5078
Real oil price	σ_{se}	IGamma(1,2)	2.0000	1.9475	1.8842	2.000
Real capital price	σ_{sk}	IGamma(1,2)	0.5459	0.5750	0.4722	0.6714
Price markup	σ_p	IGamma(1,2)	0.4235	0.4645	0.2868	0.6602
Government	σ_g	IGamma(1,2)	2.0000	1.8359	1.6425	2.000
Monetary	σ_i	IGamma(1,2)	0.4778	0.4769	0.4062	0.54555

References

- [WB1, 2013] (2013). World bank report. Technical report, World Bank.
- [Andrle, 2010] Andrle, M. (2010). A note on identification patterns in DSGE models. Working Paper Series 1235, European Central Bank.
- [Ayres and Voudouris, 2014] Ayres, R. and Voudouris, V. (2014). The economic growth enigma: Capital, labour and useful energy? *Energy Policy*, 64:16–28.
- [Beato and Mas-Colell, 1985] Beato, P. and Mas-Colell, A. (1985). On marginal cost pricing with given tax-subsidy rules. *Journal of Economic Theory*, 37(2):356–365.
- [Blanchard and Galí, 2009] Blanchard, O. and Galí, J. (2009). "The Macroeconomic Effects of Oil Price Shocks: Why are the 2000s so Different from the 1970s?". In J. Galí and M. Gertler (eds.), International Dimensions of Monetary Policy. University of Chicago Press, pages 373–428.
- [Blanchard and Riggi, 2013] Blanchard, O. J. and Riggi, M. (2013). Why are the 2000s so different from the 1970s'? A structural interpretation of changes in the macroeconomic effects of oil prices. *Journal of the European Economic Association*, 11(5):1032–1052.
- [Blinder et al., 1998] Blinder, A. S., Canetti, E., and Lebow, D. (1998). Asking about Prices: A new Approach to understand price stickiness. Russel stage Foundation Publications.
- [Bonnet et al., 2014] Bonnet, O., Bono, P., Chapelle, C., and Wasmer, E. (2014). Does housing capital contribute to inequality? a comment on thomas piketty?s capital in the 21st century. *Sciences-Po Discussion paper 2014-07*.
- [Bonnisseau and Cornet, 1990] Bonnisseau, J.-M. and Cornet, B. (1990). Existence of marginal cost pricing equilibria in economies with several nonconvex firms. *Econometrica*, 58(3):661–82.
- [BP, 2013] BP (2013). Bp statistical review of world energy. Technical report, British Petroleum Company.
- [Brooks and Gelman, 1998] Brooks, S. P. and Gelman, A. (1998). General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics*, 7(4):434–455.
- [Browning et al., 1999] Browning, M., Hansen, L. P., and Heckman, J. J. (1999). Micro Data and General Equilibrium Models. Discussion Papers 99-10, University of Copenhagen. Department of Economics.
- [Calvo, 1983] Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. Journal of Monetary Economics, 12(3):383–398.
- [Canova and Sala, 2009] Canova, F. and Sala, L. (2009). Back to square one: Identification issues in dsge models. *Journal of Monetary Economics*, 56(4):431–449.
- [Cornet, 1988] Cornet, B. (1988). General equilibrium theory and increasing returns: Presentation. *Journal of Mathematical Economics*, 17(2-3):103–118.

- [Dehez and Dreze, 1988a] Dehez, P. and Dreze, J. (1988a). Competitive equilibria with quantity-taking producers and increasing returns to scale. *Journal of Mathematical Economics*, 17(2-3):209–230.
- [Dehez and Dreze, 1988b] Dehez, P. and Dreze, J. (1988b). Distributive production sets and equilibria with increasing returns. *Journal of Mathematical Economics*, 17(2-3):231–248.
- [Dynare, 2011] Dynare (2011). Dynare: Reference manual version 4. Dynare Working Papers 1, CEPREMAP.
- [EIA, 2012] EIA (2012). Annual energy outlook. Technical report, Energy Infrmation Administration.
- [Fouré et al., 2012] Fouré, J., Bénassy-Quéré, A., and Fontagné, L. (2012). The great shift: Macroeconomic projections for the world economy at the 2050 horizon. Working Papers 2012-03, CEPII research center.
- [Galbraith and Ferguson, 1999] Galbraith, J. K. and Ferguson, T. (1999). The american wage structure, 1920-1947. Research in Economic History, 19:205–257.
- [Galí, 2008] Galí, J. (2008). Introduction to monetary policy, inflation, and the business cycle: An introduction to the new keynesian framework. In *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Introductory Chapters. Princeton University Press.
- [Giraud, 2001] Giraud, G. (2001). An algebraic index theorem for non-smooth economies. Journal of Mathematical Economics, 36(4):215–283.
- [Hurwicz and Reiter, 1973] Hurwicz, L. and Reiter, S. (1973). On the boundedness of the feasible set without convexity assumptions. *International Economic Review*, 14 (3):580–86.
- [Iskrev, 2010] Iskrev, N. (2010). Local identification in dsge models. *Journal of Monetary Economics*, 57(2):189–202.
- [J. and Guthrie, 1952] J., E. W. and Guthrie, G. E. (1952). The shape of the average cost curve. *American Economic Review*, 42(5):832–8.
- [Kahraman and Giraud, 2014] Kahraman, Z. and Giraud, G. (2014). How dependent is growth from primary energy? the dependency ratio of energy in 33 countries (1970-2011). Working Paper Chair Energy and Prosperity.
- [Khramov, 2012] Khramov, V. (2012). Assessing DSGE models with capital accumulation and indeterminacy. IMF Working Papers 12/83, International Monetary Fund.
- [Kilian, 2008] Kilian, L. (2008). The economic effects of energy price shocks. *Journal of Economic Literature*, 46(4):871–909.
- [Kim and Sims, 2003] Kim, J., S. K. E. S. and Sims, C. (2003). "Calculating and using second order accurate solutions of discrete time dynamic equilibrium models". Finance and Economics Discussion Series 2003-61, Board of Governors of the Federal Reserve System (U.S.).

- [Kümmel et al., 2010] Kümmel, R., Ayres, R. U., and Lindenberger, D. (2010). Thermodynamic laws, economic methods and productive power of energy. *Journal of Non-Equilibrium Thermodynamics.*, 35:145–179.
- [Quinzii, 1992] Quinzii, M. (1992). "Increasing Returns and Efficiency". Oxford University Press.
- [Ratto, 2008] Ratto, M. (2008). "Analysing DSGE Models with Global Sensitivity Analysis". Computational Economics, 31(2):115–139.
- [Ratto and Pagano, 2010] Ratto, M. and Pagano, A. (2010). "Using recursive algorithms for the efficient identification of smoothing spline ANOVA models". *AStA Advances in Statistical Analysis*, 94(4):367–388.
- [Rudebusch, 2006] Rudebusch, G. (2006). "Monetary Policy Inertia: Fact or Fiction?". International Journal of Central Bankin, 2(4).
- [Samuelson, 1966] Samuelson, P. (1966). "A Summing-up". Quarterly Journal of Economics, 80(4):568–583.
- [Samuelson, 1979] Samuelson, P. (1979). "Paul Douglas's Measurement of Production Functions and Marginal Productivities". *Journal of Political Economy*, 87:923–939.
- [Sánchez, 2008] Sánchez, M. (2008). "Oil shocks and endogenous markups results from an estimated euro area DSGE model". Working Paper Series 860, European Central Bank.
- [Schilling et al., 1977] Schilling, W., Spirić, V., Rehn, L., and Robrock, K. (1977). "An elastic relaxation due to single self-interstitial atoms in electron-irradiated Al". *Phys. Rev. B*, 15:672–679.
- [Sims, 2011] Sims, E. (2011). "Permanent and Transitory Technology Shocks and the Behavior of Hours: A Challenge for DSGE Models". Seminar in series: Econ 310 Macroeconomics. Standford University.
- [Smets and Wouters, 2007] Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.
- [Taylor, 1993] Taylor, J. (1993). "Discretion versus policy rules in practice". Carnegie-Rochester Conference Series on Public Policy, 39(1):195–214.
- [Woodford, 2003] Woodford, M. (2003). "Interest and prices". Princeton Univ. Press, Princeton, NJ [u.a.].