

## 1 The model (equations and variables)

### 1.1 The model in brief

The model you will analyze is a simplified version of the New-Keynesian small open-economy (SOE) model in Justiniano and Preston (2010), which in turn is based on the model in Monacelli (2005) and Gali and Monacelli (2005). Compared to Justiniano and Preston's model, our simplified model assumes that the law of one price (LoP) holds for all imported retail goods and there is no price indexation for these imported goods. The model is also extended to include labor-supply shocks, which could be used as a proxy for the supply-side disruption of the COVID-19 pandemic. Aggregate fluctuations in our model are driven by 7 exogenous shocks: risk premium, monetary policy (money supply), preference (consumer spending), labor supply, foreign inflation, foreign output, and foreign interest rate.

The model can be derived from the ground up with micro-foundations, based on optimizing households, domestic firms and importers, etc., resulting in a set of *non-linear* equations. We will instead work directly with the **log-linearized equilibrium equations**, listed below.

### 1.2 The log-linearized equations

Consumption Euler-equation (the IS equation):

$$\hat{c}_t = \left(\frac{h}{1+h}\right)\hat{c}_{t-1} + \left(\frac{1}{1+h}\right)E_t\hat{c}_{t+1} - \frac{1}{\sigma}\left(\frac{1-h}{1+h}\right)\left[\hat{i}_t - E_t\hat{\pi}_{t+1}\right] + \gamma\hat{g}_t \quad (1)$$

Goods-market clearing condition:

$$\hat{y}_t = (1-\alpha)\hat{c}_t + \alpha\hat{y}_t^* + \alpha\eta(2-\alpha)\hat{S}_t \quad (2)$$

The link between terms of trade and real exchange rate:

$$\hat{q}_t = (1-\alpha)\hat{S}_t \quad (3)$$

Changes (growth rate) of the terms of trade:

$$\hat{S}_t - \hat{S}_{t-1} = \hat{\pi}_{F,t} - \hat{\pi}_{H,t} \quad (4)$$

Domestic-price inflation (the "Phillips curve"):

$$(\widehat{\pi}_{H,t} - \delta_H \widehat{\pi}_{H,t-1}) = \beta E_t (\widehat{\pi}_{H,t+1} - \delta_H \widehat{\pi}_{H,t}) + \frac{(1 - \theta_H)(1 - \theta_H \beta)}{\theta_H} \widehat{m}c_t \quad (5)$$

The real marginal cost:

$$\widehat{m}c_t = \varphi \widehat{y}_t + \alpha \widehat{S}_t + \sigma \widehat{c}_t + \widehat{\varepsilon}_{s,t} \quad (6)$$

The wedge between CPI- and PPI-inflation:

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + \alpha (\widehat{S}_t - \widehat{S}_{t-1}) \quad (7)$$

The uncovered interest-parity (UIP) condition:

$$\widehat{i}_t - \widehat{i}_t^* = E_t \widehat{c}_{t+1}^c - \chi (\widehat{a}_t + E_t \widehat{\phi}_{t+1}) \quad (8)$$

The net-foreign-assets position (the current account):

$$\widehat{y}_t - \widehat{c}_t = \widehat{a}_t - \beta^{-1} \widehat{a}_{t-1} + \frac{\alpha}{(1 - \alpha)} \widehat{q}_t \quad (9)$$

Imported-good inflation (based on the law of one price):

$$\widehat{\pi}_{F,t} = \widehat{c}_t^c + \widehat{\pi}_t^* \quad (10)$$

Monetary-policy (Taylor) rule:

$$\widehat{i}_t = \rho_i \widehat{i}_{t-1} + \psi_\pi \widehat{\pi}_t + \psi_y \widehat{y}_t + \psi_{\Delta y} \Delta \widehat{y}_t + \psi_e \widehat{c}_t^c - \varepsilon_{m,t} \quad (11)$$

Evolution of risk premium:

$$\widehat{\phi}_t = \rho_\phi \widehat{\phi}_{t-1} + \varepsilon_{\phi,t} \quad (12)$$

Evolution of foreign output:

$$\widehat{y}_t^* = \rho_{y^*} \widehat{y}_{t-1}^* + \varepsilon_{y^*,t} \quad (13)$$

Evolution of foreign inflation:

$$\widehat{\pi}_t^* = \rho_{\pi^*} \widehat{\pi}_{t-1}^* + \varepsilon_{\pi^*,t} \quad (14)$$

Evolution of foreign interest rate:

$$\widehat{i}_t^* = \rho_{i^*} \widehat{i}_{t-1}^* + \varepsilon_{i^*,t} \quad (15)$$

Evolution of preference shock:

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_{g,t} \quad (16)$$

Evolution of labor-supply shock:

$$\widehat{\varepsilon}_{s,t} = \rho_s \widehat{\varepsilon}_{s,t-1} + \eta_{s,t} \quad (17)$$

**Definition of variables and shocks** NOTE: all hatted variables are in terms of *log or percentage deviation* from the steady-state value, except for  $\widehat{i}_t$ ,  $\widehat{\pi}_t$ ,  $\widehat{\pi}_{H,t}$ ,  $\widehat{\pi}_{F,t}$ ,  $\widehat{\pi}_t^*$ , and  $\widehat{i}_t^*$ , which are in terms of *level deviation* from the steady state (e.g.  $\widehat{i}_t \equiv i_t - \bar{i}$ ).

$\widehat{c}_t$	consumption (per capita)
$\widehat{i}_t$	nominal interest rate
$\widehat{\pi}_t$	CPI inflation
$\widehat{y}_t$	output
$\widehat{S}_t$	terms of trade (price of exports in terms of imports)
$\widehat{q}_t$	real exchange rate
$\widehat{\pi}_{H,t}$	domestic-goods (PPI) inflation
$\widehat{\pi}_{F,t}$	imported-goods inflation
$\widehat{mc}_t$	real marginal cost
$\widehat{a}_t$	domestic-households' holding of foreign assets
$\widehat{c}_t^c$	domestic-currency depreciation rate (% change in the exchange rate)
$\widehat{y}_t^*$	foreign output
$\widehat{\pi}_t^*$	foreign inflation
$\widehat{i}_t^*$	foreign interest rate
$\widehat{\phi}_t$	relative risk premium
$\widehat{g}_t$	consumer preference
$\widehat{\varepsilon}_{s,t}$	exogenous labor-supply disruption
$\varepsilon_{m,t}$	monetary-policy shock (i.i.d.)
$\varepsilon_{\phi,t}$	risk-premium shock (i.i.d.)
$\varepsilon_{y^*,t}$	foreign-output shock (i.i.d.)
$\varepsilon_{\pi^*,t}$	foreign-inflation shock (i.i.d.)
$\varepsilon_{i^*,t}$	foreign interest-rate shock (i.i.d.)
$\varepsilon_{g,t}$	preference shock (i.i.d.)
$\eta_{s,t}$	labor-supply shock (i.i.d.)

### Definition of parameters and their calibration

Parameters	Definition	Value
$\sigma$	inverse intertemporal elasticity of substitution	1
$\alpha$	openness parameter	0.25
$\eta$	elasticity of substitution between domestic and imported goods	0.80
$\beta$	subjective discount factor	0.99
$\theta_H$	probability of price fixity for domestic goods	0.70
$\chi$	risk-premium parameter	0.01
$\varphi$	inverse Frisch labor-supply elasticity	1.26
$h$	degree of habit formation	0.25
$\gamma$	consumption elasticity of preference shock	0.12
$\delta_H$	degree of price indexation	0.20
$\rho_i$	Taylor-rule interest smoothness	0.75
$\psi_\pi$	Taylor-rule response to inflation	1.90
$\psi_y$	Taylor-rule response to output	0.08
$\psi_{\Delta y}$	Taylor-rule response to output growth	0.67
$\psi_e$	Taylor-rule response to exchange-rate fluctuations	0
$\rho_\phi$	AR(1) coefficient of risk-premium shock	0.95
$\rho_{y^*}$	AR(1) coefficient of foreign-output shock	0.55
$\rho_{\pi^*}$	AR(1) coefficient of foreign-inflation shock	0.50
$\rho_{i^*}$	AR(1) coefficient of foreign interest-rate shock	0.50
$\rho_g$	AR(1) coefficient of preference shock	0.65
$\rho_s$	AR(1) coefficient of labor-supply shock	0.70
$\sigma_m, \sigma_\phi, \sigma_{y^*}, \sigma_{\pi^*}, \sigma_{i^*}, \sigma_g, \sigma_s$	standard deviations of shocks	1

## The Questions

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1. Solve the model described above using Dynare. Obtain the impulse response for *10 periods* to a one-time 1% shock to

- (a) money supply or the domestic interest-rate shock ( $\varepsilon_{m,t}$ );
- (b) preference ( $\varepsilon_{g,t}$ );
- (c) labor supply ( $\eta_{s,t}$ );
- (d) foreign output ( $\varepsilon_{y^*,t}$ ).

Analyze (i.e. explain the dynamics) and plot the effect of each of these shocks to domestic output ( $\hat{y}_t$ ), consumption ( $\hat{c}_t$ ), interest rate ( $\hat{i}_t$ ), inflation ( $\hat{\pi}_t$ ), domestic-currency *nominal depreciation* ( $\hat{\varepsilon}_t^c$ ), and the "shocked" variable (e.g. if it's a foreign output shock, plot  $\hat{y}_t^*$ ). **Plot these six variables in one 3x2 figure or plot (with 3 rows and 2 columns)**. Relate your analysis to what you have learned in the first half of the course (the *qualitative* AA-DD model). For the money supply or the domestic interest-rate shock, do you observe an overshooting of the nominal exchange rate?

[**Extra points:** plot the evolution of the *level* of nominal exchange rate and the current account in a separate figure and explain the dynamics.]

### 2. COVID-19 pandemic and monetary and exchange-rate policies.

Let's analyze the economic impact of the COVID-19 pandemic using this model, with several different assumptions on the central bank's monetary and exchange-rate policies. Here, the COVID-19 pandemic "shocks" are proxied by a combination of negative labor supply and negative preference shocks. The preference (or consumer-spending) shock in the model is a type of demand shock that influences household intertemporal consumption-saving decisions. A negative preference shock thus serves as a proxy for a reduction in aggregate demand during the pandemic, e.g. due to lost labor income or an increase in household income uncertainty which leads to a precautionary saving behaviour. A negative labor supply shock captures an aggregate supply reduction due to supply-chain disruptions and large-scale social and economic restrictions (lockdowns).

In particular, assume that the economy starts at period 0 (2019.Q4) at the steady state. Assume that the pandemic "shocks" occur for 4 periods or quarters, from periods 1-4 (2020.Q1-2020.Q4) with the following magnitudes:

Period (Quarter)	1 (2020.Q1)	2 (2020.Q2)	3 (2020.Q3)	4 (2020.Q4)
Labor supply ( $\eta_{s,t}$ )	-5%	-7%	-3%	2%
Preference ( $\varepsilon_{g,t}$ )	-3%	-3%	-2%	3%

There are positive labor supply and preference shocks in period 4, perhaps in response to the expectations that an effective COVID-19 vaccine is imminent and about to be approved and rolled out to the public.

- (a) Analyze the effect of these pandemic shocks under the current policy rule with  $\rho_i = 0.75$ ,  $\psi_\pi = 1.9$ ,  $\psi_y = 0.08$ ,  $\psi_{\Delta y} = 0.67$ , and  $\psi_e = 0$ . Plot the responses of  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $\hat{i}_t$ ,  $\hat{\pi}_t$ ,  $\hat{e}_t^c$  in one (3x2) figure. Explain their dynamics. In another (2x1) figure, plot the evolution of exogenous labor supply variable  $\hat{e}_{s,t}$  and consumer preference variable  $\hat{g}_t$ . Is this combination of shocks a realistic representation of the COVID-19 pandemic shocks? Why?
- (b) In the model above, we assume a fully-flexible (floating) exchange rate regime. Suppose that the central bank also directly intervenes in the foreign exchange market, i.e. it's operating under a *managed floating* exchange rate. This policy can be analyzed within our model by assuming that

$$\psi_e = 0.65 > 0$$

The rest of policy rule coefficients are unchanged. Analyze the effect of the pandemic shocks under this new assumption, **in comparison to the effect in part (a)**. Plot the responses of  $\hat{y}_t$ ,  $\hat{c}_t$ ,  $\hat{i}_t$ ,  $\hat{\pi}_t$ ,  $\hat{e}_t^c$  in one (3x2) figure. Is this policy more effective in terms of mitigating the effect of the pandemic shocks on  $\hat{y}_t$ ,  $\hat{\pi}_t$ , and  $\hat{e}_t^c$ ? Explain.

- (c) Now assume that the central bank is operating under a *fixed exchange-rate* regime. Specifically, the monetary policy rule in equation (11) is **replaced** with the following policy rule:

$$\hat{e}_t^c = 0$$

This policy rule effectively (and credibly) fixes the nominal exchange rate at a specified level. **Redo questions 2(a)**. Your answer and analysis should be in comparison to the floating exchange-rate regime (both when  $\psi_e = 0$  and  $\psi_e = 0.65$ ).

[Notes: (i) *Dynare does not plot the impulse response of a variable if that variable is always constant (zero deviation from the steady state)*, (ii) *since the foreign-debt holding,  $\hat{a}_t$ , enters the UIP condition in equation (8), you will generally not find  $\hat{i}_t = \hat{i}_t^*$  under the fixed-exchange rate regime, unless  $\chi = 0$ ).*]

[**Extra points:** plot the variables under the three policies in one figure, e.g. the plot for  $\hat{y}_t$  should include three different impulse responses.]

## References

- [1] Gali, J. and T. Monacelli. 2005. "Monetary policy and exchange rate volatility in a small open economy". *Review of Economic Studies* 72: 707-734.
- [2] Justiniano, A. and B. Preston. 2010. "Monetary policy and uncertainty in an empirical small open-economy model". *Journal of Applied Econometrics* 25: 93-128.
- [3] Monacelli, T. 2005. "Monetary policy in a low pass-through environment". *Journal of Money, Credit, and Banking* 37(6): 1019-1045.