$$(\partial B_{t+1}) \quad \mathbb{E}_t\left[(1-\Gamma(\bar{\omega}_{t+1},\sigma_t))\frac{R^k_{t+1}}{R_{t+1}}+\eta_{t+1}\left[\frac{R^k_{t+1}}{R_{t+1}}(\Gamma(\bar{\omega}_{t+1},\sigma_t))-\mu G(\bar{\omega}_{t+1},\sigma_t)-1\right]\right]=0 \tag{1}$$

$$(\partial\bar{\omega}_{t+1}) \quad \mathbb{E}_t\left[\eta_{t+1}-\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}\right]=0 \tag{2}$$

By combinig the two FOCs I get

$$\left(1-\Gamma(\bar{\omega}_{t+1},\sigma_t)+\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}\right)\frac{1+R^k_{t+1}}{1+R_{t+1}}-\frac{\Gamma'(\bar{\omega}_{t+1},\sigma_t)}{\Gamma'(\bar{\omega}_{t+1},\sigma_t)-\mu G'(\bar{\omega}_{t+1},\sigma_t)}=0 \tag{3}$$

To linearize GAMMA function

$$\Gamma_t=\bar{\omega}_{t+1}\left[1-F_t\right]+G_t \tag{4}$$

which results in the following log linear form

$$\Gamma(1+\hat{\Gamma}_t)=\bar{\omega}(1-F)\left[1+\hat{\omega}_{t+1}+\widehat{1-F_t}\right]+G(1+\hat{G}_t) \tag{5}$$

Then

$$\widehat{1-F_t}=\frac{F}{F-1}\hat{F}_t \tag{6}$$

and then I proceed by applying taylor expansion on F and G which are two-variables functions The log linear FOC reads

$$\mathbb{E}_t\left\{\frac{1+R^k}{1+R}(1-\Gamma)\left[1+\frac{R^k}{1+R^k}\hat{R}^k{}_{t+1}+\frac{\Gamma}{\Gamma-1}\hat{\Gamma}_t-\frac{R}{1+R}\hat{R}_{t+1}\right]+\right. \tag{7}$$

$$\frac{\Gamma'}{\Gamma'-\mu G'}\left[1+\hat{\Gamma}'_t-\frac{\Gamma'\hat{\Gamma}'_t-\mu G'\hat{G}'_t}{\Gamma'-\mu G'}\right]\times \tag{8}$$

$$\left.\left(\frac{1+R^k}{1+R}(\Gamma-\mu G)\left[1+\frac{R^k}{1+R^k}\hat{R}^k{}_{t+1}+\frac{\Gamma\hat{\Gamma}_t-\mu G\hat{G}_t}{\Gamma-\mu G}-\frac{R}{1+R}\hat{R}_{t+1}\right]-1\right)\right\} \tag{9}$$

To log linearize G′

$$\hat{G}'_t=\frac{G''_{\omega\omega}}{G'_\omega}\hat{\bar{\omega}}_{t+1}\bar{\omega}+\frac{G''_{\omega\sigma}}{G'_\omega}\hat{\sigma}_t\sigma \tag{10}$$

Log linear Γ is

$$\Gamma(1+\hat{\Gamma}_t)=\bar{\omega}(1-F)\left[1+\hat{\omega}_{t+1}+\widehat{1-F_t}\right]+G(1+\hat{G}_t) \tag{11}$$

with

$$\widehat{1-F_t}=\frac{F}{F-1}\hat{F}_t \tag{12}$$

How to express $\hat{F}_t$
For the CDFs of the log normal random variable $\bar{\omega}$ we have

$$G(\bar{\omega})=\Phi\left(\frac{ln(\bar{\omega})-\frac{1}{2}\sigma^2}{\sigma}\right) \tag{13}$$

and

$$\Gamma(\bar{\omega})=\bar{\omega}\left[1-\Phi\left(\frac{ln(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}\right)\right]+G(\bar{\omega}) \tag{14}$$

To compute derivatives of G we have

$$G'_{\bar{\omega}} = \frac{1}{\sigma}\phi\left(\frac{\ln(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right) \tag{15}$$

$$G''_{\bar{\omega}\bar{\omega}} = -\frac{\frac{\ln(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}}{\bar{\omega}\sigma^2}\phi\left(\frac{\ln(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right) \tag{16}$$

$$G'_{\sigma} = -\frac{\frac{\ln(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}}{\sigma}\phi\left(\frac{\ln(\bar{\omega}) - \frac{1}{2}\sigma^2}{\sigma}\right) \tag{17}$$

$$G''_{\bar{\omega}\sigma} = -\frac{\phi\left(\frac{\ln(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}\right)}{\sigma^2}\left[1 - \frac{(\ln(\bar{\omega}) + \frac{1}{2}\sigma^2)}{\sigma}\left(\frac{\ln(\bar{\omega}) - \frac{1}{2}\sigma^2}{\sigma}\right)\right] \tag{18}$$

To compute derivatives of Γ, we have

$$\Gamma'_{\bar{\omega}} = 1 - \Phi\left(\frac{\ln(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right) \tag{19}$$

$$\Gamma''_{\bar{\omega}\bar{\omega}} = -\frac{1}{\bar{\omega}\sigma}\phi\left(\frac{\ln(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right) \tag{20}$$

$$\Gamma'_{\sigma} = -\phi\left(\frac{\ln(\bar{\omega}) - \frac{1}{2}\sigma^2}{\sigma}\right) \tag{21}$$

$$\Gamma''_{\bar{\omega}\sigma} = \left(\frac{\left(\frac{\ln(\bar{\omega})+\frac{1}{2}\sigma^2}{\sigma}\right)}{\sigma} - 1\right)\phi\left(\frac{\ln(\bar{\omega}) + \frac{1}{2}\sigma^2}{\sigma}\right) \tag{22}$$