with the firm-level inflation $\pi_{f,t} = \frac{P_{f,t}}{P_{f,t-1}}$ and $\pi_t = \frac{P_t}{P_{t-1}}$ the aggregate product inflation.

1.3 Wages determination

Nominal wages are determined through a Nash bargaining scheme between workers and employers who maximise the joint surplus of employment by choosing the nominal wages. Formally, they solve:

$$\arg\max_{W_{f,t}} \left(\mu^W_{f,t}\right)^{\varepsilon^L_t \eta} \left(\mu^L_{f,t}\right)^{1-\eta\varepsilon^L_t},$$

with $\eta \in [0,1]$ the negociation power according to workers associated with an exogenous shock ε^L_t. $\mu^W_{f,t}$ is the employment surplus from the household perspective defined in Eq.B.12. Note that we used the suberscript f for $\mu_{f,t}$. Due to constant returns, all workers are the same at the margin and the wage negociation is between the firm and the marginal worker. Finally, $\mu^L_{f,t}$ is the employment surplus from the producer perspective defined in Eq.B.14.

The FOC with respect to nominal wage $W_{f,t}$ implies the following sharing rule,

$$\left(1-\omega_t\right)\mu^W_{f,t} = \omega_t \mu^L_{f,t}, \tag{B.22}$$

where ω_t is the time-varying negotiation power defined as:

$$\omega_t = \varepsilon^N_t \eta \left(\varepsilon^N_t \eta - \left(1 - \varepsilon^N_t \eta\right) \frac{\partial \mu^L_{f,t}}{\partial W_{f,t}} \frac{1}{\frac{\partial \mu^W_{f,t}}{\partial W_{f,t}}} \right)^{-1}.$$

Since the firms is subject to adjustment cost *à la* Rotemberg (1982) for adjusting the nominal wage $W_{f,t}$. Formally, they have to pay:

$$AC^W_{f,t} = \frac{\kappa^W}{2} \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W \left(\pi_{t-1} - 1\right) \right)^2,$$

where $\kappa^W \geq 0$ is the degree of rigidity and $\lambda^W \in [0,1]$ is the indexation on past inflation π_{t-1}. Derive the employment surplus for the marginal worker with respect to $W_{f,t}$ gives:

$$\frac{\partial \mu^W_{f,t}}{\partial W_{f,t}} = \frac{1}{P^C_t},$$

and for the employment surplus for the producers,

$$\frac{\partial \mu^W_{f,t}}{\partial W_{f,t}} = -\partial \frac{(1+AC^W_{f,t})W_{f,t}}{\partial W_{f,t}} \frac{1}{P^C_t} + \mathbb{E}_t \left\{ \beta_{t,t+1} \left(1-\delta^L\right)\left(1-\delta^N\right) \frac{\partial \mu^L_{f,t+1}}{\partial W_{f,t}} \right\}$$

$$\Leftrightarrow = -(1+AC^W_{f,t} + W_{f,t}\partial \frac{AC^W_{f,t}}{\partial W_{f,t}}) \frac{1}{P^C_t} - \mathbb{E}_t \left\{ \beta_{t,t+1} \left(1-\delta^L\right)\left(1-\delta^N\right) \partial \frac{AC^W_{f,t+1} W_{f,t+1}}{\partial W_{f,t}} \frac{1}{P^C_{t+1}} \right\},$$

where $\partial \frac{AC^W_{f,t}}{\partial W_{f,t}} = \frac{1}{W_{f,t-1}} \kappa^W \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right)$ and
$\partial \frac{AC^W_{f,t+1}}{\partial W_{f,t}} = -\frac{W_{f,t+1}}{(W_{f,t})^2} \kappa^W \left(\frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W (\pi_t - 1) \right)$. Then we can rewrite the FOC of the employment surplus as:

$$\frac{\partial \mu^W_{f,t}}{\partial W_{f,t}} = \begin{array}{c} -\left(1 + AC^W_{f,t} + \frac{W_{f,t}}{W_{f,t-1}} \kappa^W \left(\frac{W_{f,t}}{W_{f,t-1}} - 1 - \lambda^W (\pi_{t-1} - 1) \right)\right) \\ +\mathbb{E}_t \left\{ \beta_{t,t+1} \left(1-\delta^L\right)\left(1-\delta^N\right) \left(\frac{W_{f,t+1}}{W_{f,t}} \right)^2 \kappa^W \left(\frac{W_{f,t+1}}{W_{f,t}} - 1 - \lambda^W (\pi_t - 1) \right) \frac{P^C_t}{P^C_{t+1}} \right\} \end{array}.$$

Then the time-varying negociation power reads:

$$\omega_t = \frac{\varepsilon^N_t \eta}{\left(\varepsilon^N_t \eta + (1-\varepsilon^N_t \eta)\right)\left(1 + AC^W_{f,t} + \kappa^W \psi^W_{f,t}\right)},$$

where $\psi^W_{f,t}$ is the auxiliary variable that depends on the Rotemberg adjustment cost used:

$$\psi^W_{f,t} = \begin{array}{c} \pi^W_{f,t}\left(\pi^W_{f,t} - 1 - \lambda^W (\pi_{t-1} - 1)\right) \\ -\mathbb{E}_t \left\{ \beta_{t,t+1} \frac{(1-\delta^L)(1-\delta^N)}{\pi^C_{t+1}} \left(\pi^W_{f,t+1}\right)^2 \left(\pi^W_{f,t+1} - 1 - \lambda^W (\pi_t - 1)\right) \right\} \end{array},$$

where $\pi^W_{f,t} = \frac{w_t}{w_{t-1}} \pi_t$ is the nominal wage inflation rate at firm f.

After presenting the time-varying negociation power, we turn to the wage setting using the sharing rule (Eq.B.22) and replace with the expression of surplus from both perspectives (firms in Eq.B.14 and the marginal worker in Eq.B.12) we have:

$$\frac{W_t}{P_t^C}\left(1+\omega_t AC_{f,t}^W\right) = \begin{array}{c} \omega_t\left(\alpha\frac{mc_{f,t}y_{f,t}}{l_{f,t}} + \mathbb{E}_t\left\{\beta_{t,t+1}\left(1-\delta^L\right)\left(1-\delta^N\right)\mu_{f,t+1}^L\right\}\right) \\ +\left(1-\omega_t\right)b-\left(1-\omega_t\right)\mathbb{E}_t\left\{\beta_{t,t+1}\mu_{f,t+1}^W\left(1-\delta^N\right)\left(1-\delta^L\right)\left(1-f_{t+1}\right)\right\} \end{array}.$$

Since the sharing rule holds in $t+1$ ($\mathbb{E}_t\mu_{f,t+1}^W = \mathbb{E}_t\frac{\omega_{t+1}}{(1-\omega_{t+1})}\mu_{f,t+1}^L$), we can rewrite the previous equation as:

$$\frac{W_t}{P_t^C}\left(1+\omega_t AC_{f,t}^W\right) = \begin{array}{c} \omega_t\left(\alpha\frac{mc_{f,t}y_{f,t}}{l_{f,t}}\right)+\left(1-\omega_t\right)b \\ +\mathbb{E}_t\left\{\beta_{t,t+1}\left(1-\delta^L\right)\left(1-\delta^N\right)\mu_{f,t+1}^L\left(\omega_t-\frac{(1-\omega_t)\omega_{t+1}}{(1-\omega_{t+1})}\left(1-f_{t+1}\right)\right)\right\} \end{array},$$

and using the the free-entry condition for vacancies (Eq. B.15) to replace $\mu_{f,t+1}^L$:

$$\frac{W_t}{P_t^C}\left(1+\omega_t AC_{f,t}^W\right) = \begin{array}{c} \omega_t\left(\alpha\frac{mc_{f,t}y_{f,t}}{l_{f,t}}\right)+\left(1-\omega_t\right)b \\ +\mathbb{E}_t\left\{\beta_{t,t+1}\left(1-\delta^L\right)\left(1-\delta^N\right)\frac{f^V}{q_{t+1}}\left(\omega_t-\frac{(1-\omega_t)\omega_{t+1}}{(1-\omega_{t+1})}\left(1-f_{t+1}\right)\right)\right\} \end{array}.$$

Then in the case of a flexible wage (absence of exogenous shock i.e. ε_t^L and rigidity i.e. $\kappa^W = \lambda^W = 0$), we have:

$$\frac{W_t}{P_t^C} = \eta\left(\alpha\frac{mc_{f,t}y_{f,t}}{l_{f,t}}\right)+\left(1-\eta\right)b+\eta\mathbb{E}_t\left\{\beta_{t,t+1}\frac{f^V f_{t+1}}{q_{t+1}}\left(1-\delta^L\right)\left(1-\delta^N\right)\right\}.$$

1.4 Symetric equilibrium, new entrants and aggregation

1.4.1 Symetric equilibrium

Given Cobb-Douglas technology and perfect capital mobility, all producers choose the same capital/output ratio and, in turn, the same capital/labor and labor/output ratios. As a consequence, the marginal cost is symmetric across firms ($mc_{f,t} = mc_t$). Thus, equilibrium prices and quantities are identical across producers f.