Households maximise utility from consumption and money

$$\max E_0 \sum_{t=0}^{\infty} \beta_t^H [lnC_t^H + \ln(M_t)] \quad (1)$$

$$C_t^H + D_t + SB_t + M_t = \frac{R_{t-1}^D}{\pi_t} D_{t-1} + [1 - F(\overline{w}_t)] \frac{R_{t-1}^{SB}}{\pi_t} SB_{t-1} + Div_t + M_{t-1} \quad (2)$$

Budget constraint: LHS, consumption, deposit, shadow banking assets, real money
RHS: interest earnings on deposit, interest earnings on shadow banking assets ($F(\overline{w}_t)$:default probability), lump-sum transfer from retailer, real money.

The F. O. Cs are,

$$\partial C_t^H: \qquad \frac{1}{C_t^H} = \lambda_t^H \quad (3)$$

$$\partial M_t: \qquad \frac{1}{M_t} + \beta^H \lambda_{t+1}^H = \lambda_t^H \quad (4)$$

$$\partial D_t: \qquad \beta^H \frac{R_t^D}{\pi_{t+1}} \lambda_{t+1}^H = \lambda_t^H \quad (5)$$

$$\partial SB_t: \quad \beta^H [1 - F(\overline{w}_t)] \frac{R_t^{SB}}{\pi_{t+1}} \lambda_{t+1}^H = \lambda_t^H \quad (6)$$

$$\partial \lambda_t^H: \frac{R_{t-1}^D}{\pi_t} D_{t-1} + [1 - F(\overline{w}_t)] \frac{R_{t-1}^{SB}}{\pi_t} SB_{t-1} + Div_t + M_{t-1} - M_t - C_t^H - D_t - SB_t = 0 \quad (7)$$

Combining (3)&(4), (3)&(5), (3)&(6)

$$\frac{1}{M_t} + \beta^H \frac{1}{C_{t+1}^H} = \frac{1}{C_t^H} \quad (8)$$

$$\frac{C_{t+1}^H}{C_t^H} = \beta^H \frac{R_t^D}{\pi_{t+1}} \quad (9)$$

$$\frac{C_{t+1}^H}{C_t^H} = \beta^H [1 - F(\overline{w}_t)] \frac{R_t^{SB}}{\pi_{t+1}} \quad (10)$$

Combining (8) and (9),

$$\frac{C_t^H}{M_t} = \frac{R_t^D - \pi_{t+1}}{R_t^D} \quad (11)$$

Combining (8) and (10),

$$\frac{C_t^H}{M_t} = \frac{[1 - F(\overline{w}_t)]R_t^{SB} - \pi_{t+1}}{[1 - F(\overline{w}_t)]R_t^{SB}} \tag{12}$$

Bankers choose consumption to maximise utility.

$$\max E_0 \sum_{t=0}^{\infty} \beta_t^B lnC_t^B \tag{13}$$

$$\begin{aligned} C_t^B + R_{t-1}^D D_{t-1} + [1 - F(\overline{w}_t)]R_{t-1}^{SB} SB_{t-1} + L_t^{SOE} + L_t^{POE} \\ = D_t + SB_t + R_{t-1}^L L_{t-1}^{SOE} + [1 - F(\overline{w}_{t-1})]R_{t-1}^{SB} L_{t-1}^{POE} \\ + (1-\mu)\int_0^{\overline{w}} wdF(w)\,(R_t^K Q_{t-1}^{POE} K_t^{POE} + R_t^H Q_{t-1}^H H_t) \end{aligned} \tag{14}$$

$$L_t^{SOE} \leq (1 - ve^{\varepsilon_t^v})hD_t \tag{15}$$

$$L_t^{POE} \leq SB_t \tag{16}$$

$$\begin{aligned} [1 - F(\overline{w})]R_{t-1}^{SB} L_{t-1}^{POE} + (1-\mu)\int_0^{\overline{w}} wdF(w)\,(R_t^K Q_{t-1}^{POE} K_t^{POE} + R_t^H Q_{t-1}^H H_t) \\ = R_{t-1}^L L_{t-1}^{POE} \end{aligned} \tag{17}$$

Equation (14), LHS: consumption (C_t^B), interest payment on deposit ($R_{t-1}^D D_{t-1}$), interest payment on shadow banking products ($[1 - F(\overline{w}_t)]R_{t-1}^{SB} SB_{t-1}$), loans to SOEs ($L_t^{SOE}$), loans to POE ($L_t^{POE}$)
RHS: deposit, shadow banking assets, interest earning from loans to SOEs, interest earnings from loans to POEs if there is no default ($[1 - F(\overline{w}_{t-1})]R_{t-1}^{SB} L_{t-1}^{POE}$), asset left from the bankrupt POEs ($(1-\mu)\int_0^{\overline{w}} wdF(w)\,(R_t^K Q_{t-1}^{POE} K_t^{POE} + R_t^H Q_{t-1}^H H_t)$).

Equation (15), loans to SOEs subject to bank regulation, reseave requirement (v), loan-to-deposit ratio (h).

Equation (16), there is no restriction on shadow banking loans.
Equation (17) expected return from loans to POEs equals the opportunity cost.

The lagrangian is,

$$\begin{aligned} E_0 \sum_{t=0}^{\infty} \beta_t^B \Bigg\{ lnC_t^B + \lambda_t^B \Bigg[D_t + R_{t-1}^L (1 - ve^{\varepsilon_{t-1}^v})hD_{t-1} + (1 \\ - \mu)\int_0^{\overline{w}_t} w_t dF(w)\,(R_t^K Q_{t-1}^{POE} K_t^{POE} + R_t^H Q_{t-1}^H H_t) - C_t^B - R_{t-1}^D D_{t-1} - (1 \\ - ve^{\varepsilon_t^v})hD_t \Bigg] \Bigg\} \end{aligned}$$

The F. O. Cs are

$$\partial C_t^B:\ \frac{1}{C_t^B} = \lambda_t^B \tag{18}$$

$$\partial D_t:\ \left[R_t^D - R_t^L(1 - ve^{\varepsilon_t^v})h\right] = \frac{\lambda_t^B}{\beta^B \lambda_{t+1}^B}\left[1 - (1 - ve^{\varepsilon_t^v})h\right] \tag{19}$$

Combining (18) and (19), yields,

$$R_t^D - R_t^L\left(1 - ve^{\varepsilon_t^v}\right)h = \frac{C_{t+1}^B}{\beta^B C_t^B}\left[1 - (1 - ve^{\varepsilon_t^v})h\right] \tag{20}$$

$$R_t^L = \frac{R_t^D}{\left(1 - ve^{\varepsilon_t^v}\right)h} - \frac{C_{t+1}^B}{\beta^B C_t^B}\frac{\left[1 - (1 - ve^{\varepsilon_t^v})h\right]}{\left(1 - ve^{\varepsilon_t^v}\right)h} \tag{21}$$