Equilibrium System - Simpler version

Households:

Euler equation:

$$u'(c_t^e) = \beta \mathbf{E}_t \left(\frac{1+i_{t+1}}{\Pi_{t+1}} \left[e_{t+1} u'(c_{t+1}^e) + u_{t+1} u'(c_{t+1}^u) + (1-l_{t+1}) u'(c_{t+1}^n) \right] \right) \tag{1}$$

Constraints:

$$x_t = \frac{b_{t+1}}{P_t} + (1+i_t)\frac{a_t}{P_t} - (1+i_t)\frac{b_t}{P_t} \tag{2}$$

$$c_t^u = x_t + \frac{\tau_t^u}{P_t} \tag{3}$$

$$c_t^n = x_t \tag{4}$$

$$\frac{a_{t+1}}{P_t} = x_t + (1-\tau_t)[e_t \frac{w_t}{P_t} + d_t e_t] + u_t \frac{\tau_t^u}{P_t} - [e_t c_t^e + u_t c_t^u + (1-l_t) c_t^n]$$

where $d_t = \frac{D_t}{e_t}$.

Employment accumulation:

$$e_t = \rho_t e_{t-1} + f_t^s s_t \tag{5}$$

with searchers s_t given by:

$$s_t = l_t - \rho_t e_{t-1} \tag{6}$$

Participation condition:

$$MRS_{N,C,t} - \Omega_t = f_t^s \left[(1-\tau_t)\frac{w_t}{P_t} - e_t \frac{\bar{\chi}(e_t)^\iota}{u'(c_t^e)} \right] - f_t^s \Upsilon_t + \tag{7}$$

$$+ f_t^s \beta \mathbf{E}_t \frac{u'(c_{t+1}^e)}{u'(c_t^e)} \rho_{t+1} \frac{(1-f_{t+1}^s)}{f_{t+1}^s} (MRS_{N,C,t+1} - \Omega_{t+1}) \tag{8}$$

$$\tag{9}$$

where:

$$MRS_{N,C,t} = \frac{(1-l_t)[\bar{\xi} * (1-l_t)^{-\phi}]}{u'(c_t^e)} \tag{10}$$

and:

$$\Omega_t = \frac{\tau_t^u}{P_t} + (c_t^n - c_t^u) + \frac{u(c_t^u) - (u(c_t^n) + \xi_t)}{u'(c_t^e)} \tag{11}$$

$$\Upsilon_t = \frac{\tau_t^u}{P_t} + (c_t^e - c_t^u) + \frac{u(c_t^u) - (u(c_t^e) - \chi_t)}{u'(c_t^e)} \tag{12}$$

$$\chi_t = \bar{\chi}\frac{e_t^{1+\iota}}{1+\iota} \tag{13}$$

$$\xi_t = \bar{\xi}\frac{(1-l_t)^{1-\phi}}{1-\phi} \tag{14}$$

Asset market equilibrium:

$$\frac{b_{t+1}}{P_t} = \frac{a_{t+1}}{P_t} = \bar{b}_t \tag{15}$$

Firms:

Optimal hiring:

$$\frac{k_t}{f_t^v} = \frac{q_t}{P_t}z_t - \frac{w_t}{P_t} + \mathbf{E}_t\left\{\Lambda_{t,t+1}\rho_{t+1}\frac{k_{t+1}}{f_{t+1}^v}\right\} \tag{16}$$

Dividends definition:

$$d_t^w = \frac{q_t}{P_t}z_t e_t - \frac{w_t}{P_t}e_t - k_t v_t \tag{17}$$

Desired price:

$$p_{x,t}^* = \frac{p_t^A}{p_t^B} \tag{18}$$

with:

$$p_t^A = \frac{\varepsilon_p - 1}{\varepsilon_p} q_t Y_t + \mathbf{E}_t \Lambda_{t,t+1} (1-\theta) \Pi_{t+1,t}^{\epsilon_p - 1} p_{t+1}^A \tag{19}$$

and:

$$p_t^B = Y_t + \mathbf{E}_t \Lambda_{t,t+1} (1-\theta) \Pi_{t+1,t}^{\epsilon_p - 1} p_{t+1}^B \tag{20}$$

Inflation:

$$\pi_t = \left(\frac{1-\theta}{1-\theta(\frac{p_t^*}{P_t})^{1-\epsilon_p}} \right)^{\frac{1}{1-\epsilon_p}} \tag{21}$$

Output:

$$\varsigma_t Y_t = z_t e_t \tag{22}$$

Price dispersion:

$$\varsigma_t = \theta(\frac{p_t^*}{P_t})^{-\epsilon_p} + (1-\theta)\pi_t^{\epsilon_p} \varsigma_{t-1} \tag{23}$$

Total dividends:

$$D_t = Y_t - \frac{q_t}{P_t} z_t e_t + d_t^w \tag{24}$$

Government

Government budget constraint:

$$\tau_t(e_t(\frac{w_t}{P_t}) + D_t) = u_t \frac{\tau_t^u}{P_t} \tag{25}$$

Taylor rule:

$$1 + i_{t+1} = (1+\bar{i})(\frac{P_t}{P_{t-1}})^{\psi} e^{\epsilon_{it}}$$

Labor Market:

Job finding rate:

$$f^s_{x,t} = \alpha_m(\frac{v_t}{s_t})^{1-\alpha} \tag{26}$$

Job filling rate:

$$f^v_t = \alpha_m(\frac{v_t}{s_t})^{-\alpha} \tag{27}$$

Wages:

Bargained wage:

$$\frac{w^*_t}{P_t} = \vartheta\left(\frac{q_t}{P_t}z_t + \mathbf{E}_t\Lambda_{t,t+1}\rho_{t+1}f^s_{t+1}\frac{k_{t+1}}{f^v_{t+1}}\right) + (1-\vartheta)\frac{1}{1-\tau_t}(e_t\frac{\chi(e_t)^\iota}{u'(c^e_t)} + \Upsilon_t) \tag{28}$$

Shocks:

Productivity:

$$log(z_t) = (1-\rho_z)log(\bar{z}) + \rho_z log(z_{t-1}) + \sigma_z\varepsilon_{z,t} \tag{29}$$

Separation:

$$log(\rho_t) = (1-\rho_\rho)log(\bar{\rho}) + \rho_\rho log(\rho_{t-1}) + \sigma_\rho\varepsilon_{\rho,t} \tag{30}$$

Borrowing:

$$\bar{b}_t = (1-\rho_b)(\bar{b}) + \rho_b\bar{b}_{t-1} + \sigma_b\varepsilon_{b,t} \tag{31}$$

Monetary policy:

$$\epsilon_{it} = \rho_i\epsilon_{i,t-1} + \sigma_i\varepsilon_{it} \tag{32}$$