

Consider a standard neoclassical model of investment

$$\begin{aligned} V_t &= \max_{I_t, K_{t+1}} D_t + \mathbb{E}_t[e^{m_{t+1}} V_{t+1}] \\ D_t &= Y_t - I_t - \frac{\theta}{2} \left( \frac{I_t}{K_t} \right)^2 K_t \\ Y_t &= e^{x_t + z_t} K_t^\alpha \\ K_{t+1} &= (1 - \delta) K_t + I_t \end{aligned}$$

Exogenous variables are specified as follows

$$\begin{aligned} x_{t+1} &= (1 - \rho_x) \bar{x} + \rho_x x_t + \sigma_x \varepsilon_{x,t+1} \\ z_{t+1} &= \rho_z z_t + \sigma_z \varepsilon_{z,t+1} \\ m_{t+1} &= -r_f - \frac{1}{2} (\lambda_x^m \sigma_x)^2 - \lambda_x^m \sigma_x \varepsilon_{x,t+1} \end{aligned}$$

The first-order condition, combined with the envelop theorem, is given by

$$1 + \theta \frac{I_t}{K_t} = \mathbb{E}_t \left[ e^{m_{t+1}} \left[ \alpha \frac{Y_t}{K_t} + \frac{\theta}{2} \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 + (1 - \delta) \left( 1 + \theta \frac{I_{t+1}}{K_{t+1}} \right) \right] \right]$$

Marginal Q equals

$$MQ_t = 1 + \theta \frac{I_t}{K_t}$$

Stock returns are given by

$$R_{t+1} = \frac{V_{t+1}}{V_t - D_t}$$

The standard timing convention is that, upon observing the shocks at the beginning of period  $t$ , firms make optimal investment decisions. See, for example, Zhang (2005 JF)

How to deal with the timing of variables in Dynare?

**Steady-state:** In the steady state, all variables are constant. That is,  $K^* = (1 - \delta)K^* + I^*$ . Therefore,  $I_t = \delta K_t$  or  $I_t/K_t = \delta$ . In the steady state, the stochastic discount factor is just the risk-free rate,  $e^{m_{t+1}} = e^{-r_f} = 1/R_f$ . The steady-state value of capital stock is given by

$$\begin{aligned} 1 + \theta\delta &= \frac{1}{R_f} \left[ \alpha K^{\alpha-1} e^{\bar{x} + \bar{z}} + \frac{\theta}{2} \delta^2 + (1 - \delta)(1 + \theta\delta) \right] \\ \alpha K^{\alpha-1} e^{\bar{x} + \bar{z}} &= (1 + \theta\delta)(R_f - 1 + \delta) - \frac{\theta}{2} \delta^2 \end{aligned}$$

$$\log \alpha + (\alpha - 1)k + \bar{x} + \bar{z} = \log \left[ (1 + \theta\delta)(R_f - 1 + \delta) - \frac{\theta}{2} \delta^2 \right]$$

$$k^* = \left[ \log \left[ (1 + \theta\delta)(R_f - 1 + \delta) - \frac{\theta}{2} \delta^2 \right] - \log \alpha - (\bar{x} + \bar{z}) \right] / (\alpha - 1)$$