1 Non-Linear Model: First Order Conditions and Steady State

After solving the log-linearized version, we implemented the non-linear version of the model of Christensen and Dib (2008) in Dynare. The model equations are[1]:

$$log\left(A_t\right) = \left(1-\rho_A\right)\, log\left(A_{ss}\right) + \rho_A\, log\left(A_{t-1}\right) + \epsilon_{At} \tag{1}$$

$$log\left(b_t\right) = \left(1-\rho_b\right)\, log\left(b_{ss}\right) + \rho_b\, log\left(b_{t-1}\right) + \epsilon_{bt} \tag{2}$$

$$log\left(e_t\right) = \rho_e\, log\left(e_{t-1}\right) + \epsilon_{et} \tag{3}$$

$$log\left(x_t\right) = \rho_x\, log\left(x_{t-1}\right) + \epsilon_{xt} \tag{4}$$

$$\frac{e_t\, {c_t}^{\frac{(-1)}{\gamma}}}{{c_t}^{\frac{\gamma-1}{\gamma}} + {b_t}^{\frac{1}{\gamma}}\, {m_t}^{\frac{\gamma-1}{\gamma}}} = \lambda_t \tag{5}$$

$$\left(\frac{b_t\, c_t}{m_t}\right)^{\frac{1}{\gamma}} = \frac{R_t - 1}{R_t} \tag{6}$$

$$\frac{\eta}{1-h_t} = \lambda_t\, w_t \tag{7}$$

$$\frac{\lambda_t}{R_t} = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{\pi_{t+1}} \tag{8}$$

$$y_t = {k_t}^{\alpha}\, \left(A_t\, h_t\right)^{1-\alpha} \tag{9}$$

$$z_t = \frac{y_t\, \alpha\, \xi_t}{k_t} \tag{10}$$

$$w_t = \frac{y_t\, \left(1-\alpha\right)\, \xi_t}{h_t} \tag{11}$$

$$c_t + i_t = y_t \tag{12}$$

$$1 = \phi\, \left(\frac{\pi_{ss}}{\pi_t}\right)^{1-\theta} + \left(1-\phi\right)\, {p_t^{rat}}^{1-\theta} \tag{13}$$

$$g_{1t} = \lambda_t\, y_t + \beta \mathbb{E}_t \frac{\pi_{ss}\, \phi}{\pi_{t+1}}\, g_{1t+1} \tag{14}$$

$$g_{2t} = \xi_t\, \lambda_t\, y_t + \beta \mathbb{E}_t\, \phi\, g_{2t+1} \tag{15}$$

$$g_{1t}\, {p^{rat}}_t\, \left(\theta-1\right) = \theta\, g_{2t} \tag{16}$$

$$\mathbb{E}_t f_{t+1} = \mathbb{E}_t \frac{R_t\, S_t}{\pi_{t+1}} \tag{17}$$

$$f_t = \frac{z_t + \left(1-\delta\right)\, q_t}{q_{t-1}} \tag{18}$$

[1]Note that k, n and S are pre-determined.

$$n_{t+1} = \nu\ (k_t\, f_t\, q_{t-1} - \mathbb{E}_{t-1} f_t\ (k_t\, q_{t-1} - n_t)) \tag{19}$$

$$k_{t+1} = x_t\, i_t + k_t\ (1 - \delta) \tag{20}$$

$$x_t\, q_t - 1 - \chi \left(\frac{i_t}{k_t} - \delta \right) = 0 \tag{21}$$

$$\frac{R_t}{(\bar{R})} = \left(\frac{\pi_t}{\pi_{ss}} \right)^{\rho_\pi} \left(\frac{y_t}{(\bar{y})} \right)^{\rho_y} \left(\frac{\mu_t}{(\bar{\mu})} \right)^{\rho_\mu} exp\,(\epsilon_{Rt}) \tag{22}$$

$$\mu_t = \frac{m_t\, \pi_t}{m_{t-1}} \tag{23}$$

$$S_t = 4\ (1 + S_{ss}\ (\psi - 1)) \left(\frac{n_{t+1}}{q_t\, k_{t+1}} \right)^2 - 2\ (3S_{ss}\psi + 2 - 2\, S_{ss})\, \frac{n_{t+1}}{q_t\, k_{t+1}} + 1 + 2S_{ss}\psi \tag{24}$$

Note that we re-wrote the infinite sum in equation (28) of Christensen and Dib (2008) recursively (see equations 14 to 16), as illustrated in Fernández-Villaverde and Rubio-Ramírez (2006) (see appendix C for derivation). Moreover, we had to specify a functional form for the external finance premium. We have the following information on the premium $S(\omega_t)$, with $\omega_t = \frac{n_{t+1}}{q_t k_{t+1}}$:

1. $S(1) = 1$
2. $S(\omega_{ss}) = S(\frac{1}{2}) = S_{ss} = 1.0075$
3. $-\frac{\partial S(\omega)}{\partial \omega}\Big|_{\omega_{ss}} \frac{\omega_{ss}}{S_{ss}} = -\frac{\partial S}{\partial \omega}\Big|_{\omega_{ss}} \frac{1}{2 \cdot 1.0075} = \psi, \quad \frac{\partial S(\omega)}{\partial \omega} < 0$

Consider a quadratic function $S(\omega) = c_1\omega^2 + c_2\omega + c_3$. The above conditions yield three equations in four unknowns, such that ψ remains to be determined:

$$\begin{aligned}
&1 = c_1 + c_2 + c_3 \ \Leftrightarrow\ c_3 = 1 - c_1 - c_2 \ \text{ (from point 1)} \\
&S_{ss} = \frac{1}{4}c_1 + \frac{1}{2}c_2 + c_3 = \frac{1}{4}c_1 + \frac{1}{2}c_2 + 1 - c_1 - c_2 = 1 - \frac{3}{4}c_1 - \frac{1}{2}c_2 \ \text{ (from point 2)} \\
&\psi = -\frac{c_1 + c_2}{2S_{ss}} \ \Leftrightarrow\ -2S_{ss}\psi - c_1 = c_2 \ \text{ (from point 3)} \ \Rightarrow \\
&S_{ss} = 1 - \frac{3}{4}c_1 + S_{ss}\psi + \frac{1}{2}c_1 \ \Leftrightarrow\ c_1 = 4 + 4S_{ss}(\psi - 1) \ \Rightarrow \\
&c_2 = -6S_{ss}\psi - 4 + 4S_{ss},\ c_3 = 1 + 2S_{ss}\psi \ \Rightarrow \\
&S(\omega) = 4[1 + S_{ss}(\psi - 1)]\omega^2 - 2[3S_{ss}\psi + 2 - 2S_{ss}]\omega + 1 + 2S_{ss}\psi
\end{aligned}$$

This yields equation (24). Note that Christensen and Dib (2008) apparently used $S_{ss} = 1$ in their code. However, meeting the calibration target of a risk spread of around 300 basis points requires to use a value of 1.0075, which the authors officially claim to use. Hence, we used this value in our computations.

As in Bernanke et al. (1999, p. 1347), we assumed that when entrepreneurs die, they "simply consume their accumulated resources and depart from the scene". This departs from the assumption of Christensen and Dib (2008) that entrepreneurs leave transfer or "seed" money when dying, which is then distributed among the new-born entrepreneurs. The reason is that the variable g is not pinned down in the model as specified by the authors.We do not know which share of the deceasing households' wealth is transferred. If deceasing households bequeathed all their net worth to the next generation, there would be no point in modeling death at all, since the total net worth of the continuum of households would be the same each period. However, if they only bequeath a certain fraction, we have no information how big this would be. Moreover, we don't know if the heirs have to pay back the debt of their ancestors, or if they get the return on their ancestors' investment. As there is nothing in the model that would pin down g, we decided to follow Bernanke et al. (1999). Since equation (19) then reads $1 = \nu f_{ss}$ in steady state, this requires to set $\nu = \frac{1}{f_{ss}} = \frac{S_{ss}R_{ss}}{\pi_{ss}} = \frac{S_{ss}}{\beta}$. This ν would imply a lifespan of entrepreneurs of around 17 years, as opposed to the 37 quarters implied by the value chosen by Bernanke et al. (1999) and Christensen and Dib (2008). Also note that Christensen and Dib (2008) forgot the expectation dated $t-1$ in equation (19) and committed an error when deriving the New Keynesian Phillips curve. A derivation of all log-linearized equations is given in appendix B and C. Moreover, we chose to use the contemporaneous version of equation (A.12) in Christensen and Dib (2008) and define this as the ex post return on capital held in t (see equation (18), in accordance with equation (C.10) in Christensen and Dib (2008). The equations for which there are differences from the equations used by Christensen and Dib (2008) are marked in red.

2 Impulse Response Functions

In a first step, we used the estimated and calibrated parameters of Christensen and Dib (2008) reported in tables 1 and 2 of their paper. Note that we divided χ by δ when using the authors' estimates, since the authors missed a δ in equation (C.11) of their paper (see Assignment 1 and appendix B). The corresponding IRFs to 1 % shocks are plotted in figures 1 - 5. They are identical to the IRFs obtained with the "manually" log-linearized model, as they should be[2]. However, they differ from those in the paper due to the mentioned forgotten expectations.

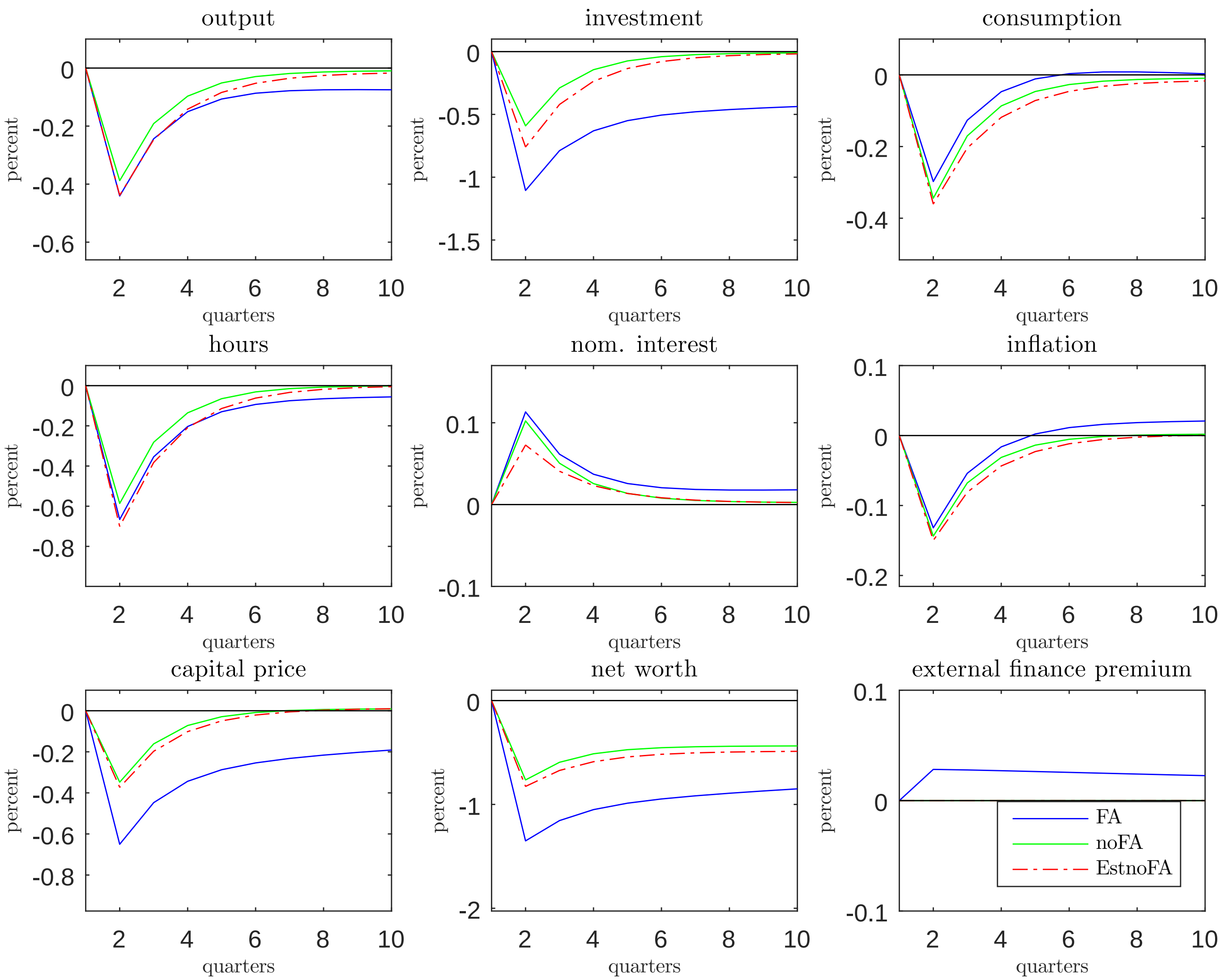

Figure 1: IRFs to Monetary Policy Shock

[2]We did all computations with the linear as well as the non-linear model. If not otherwise reported, the results were identical

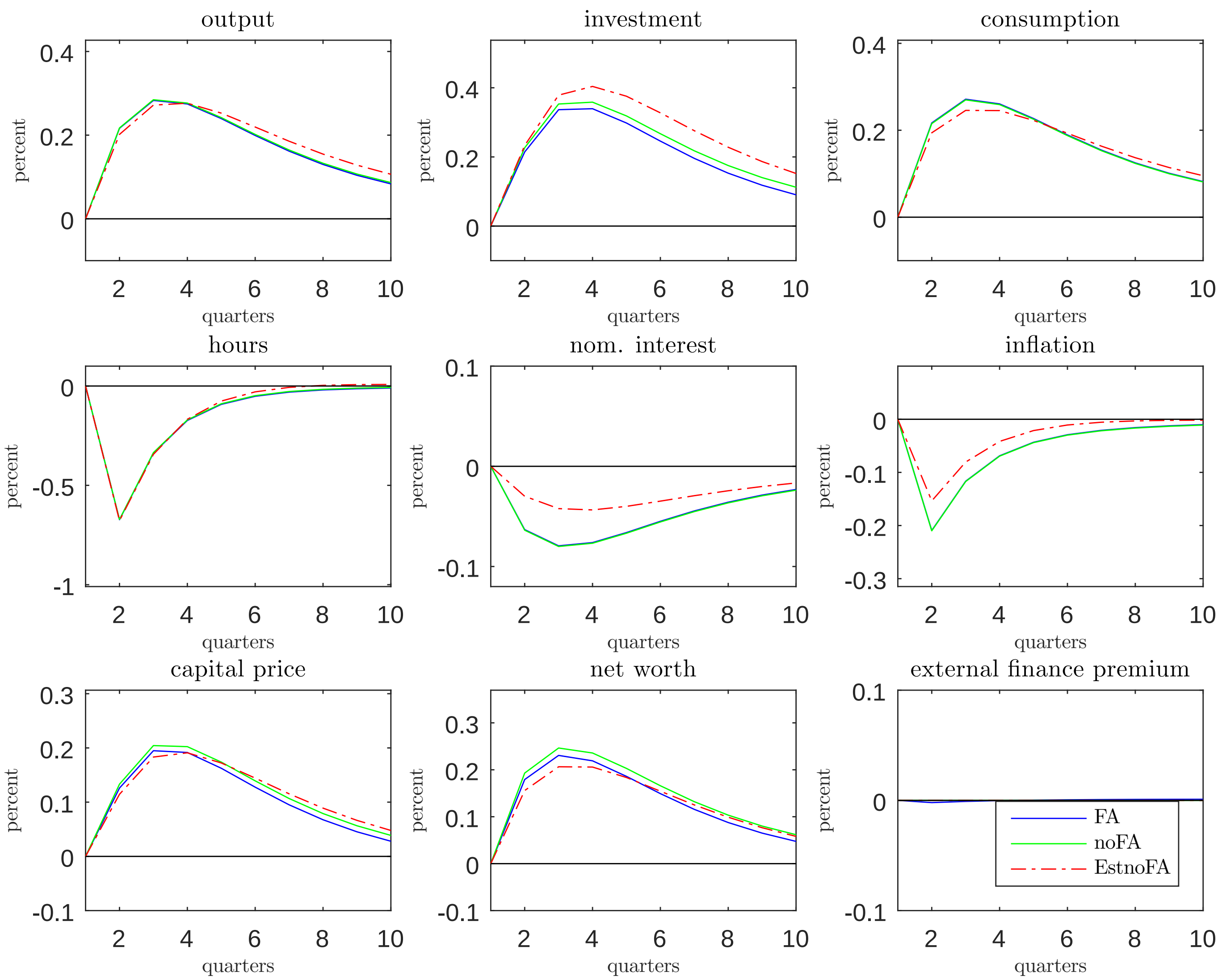

Figure 2: IRFs to Technology Shock

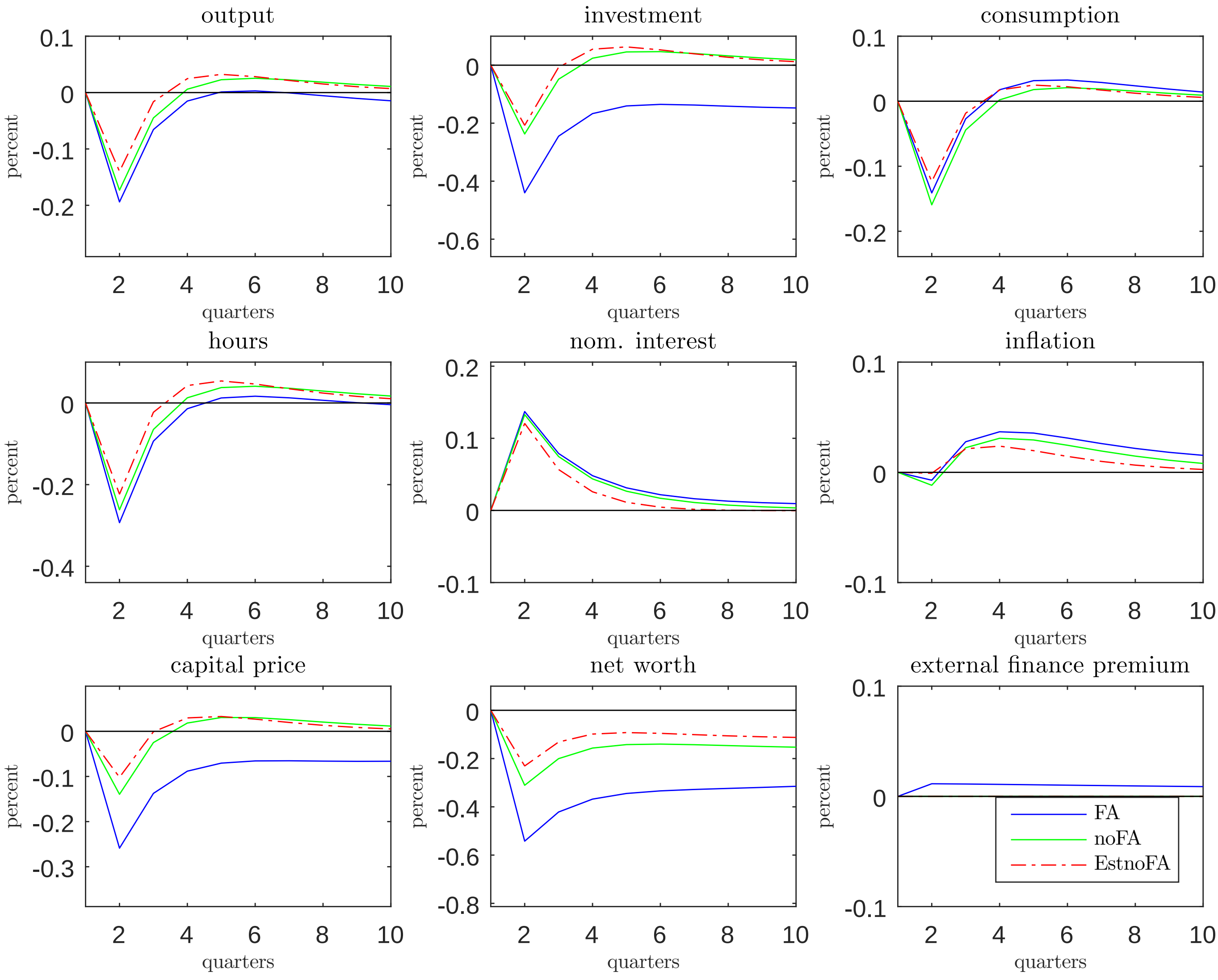

Figure 3: IRFs to Money Demand Shock

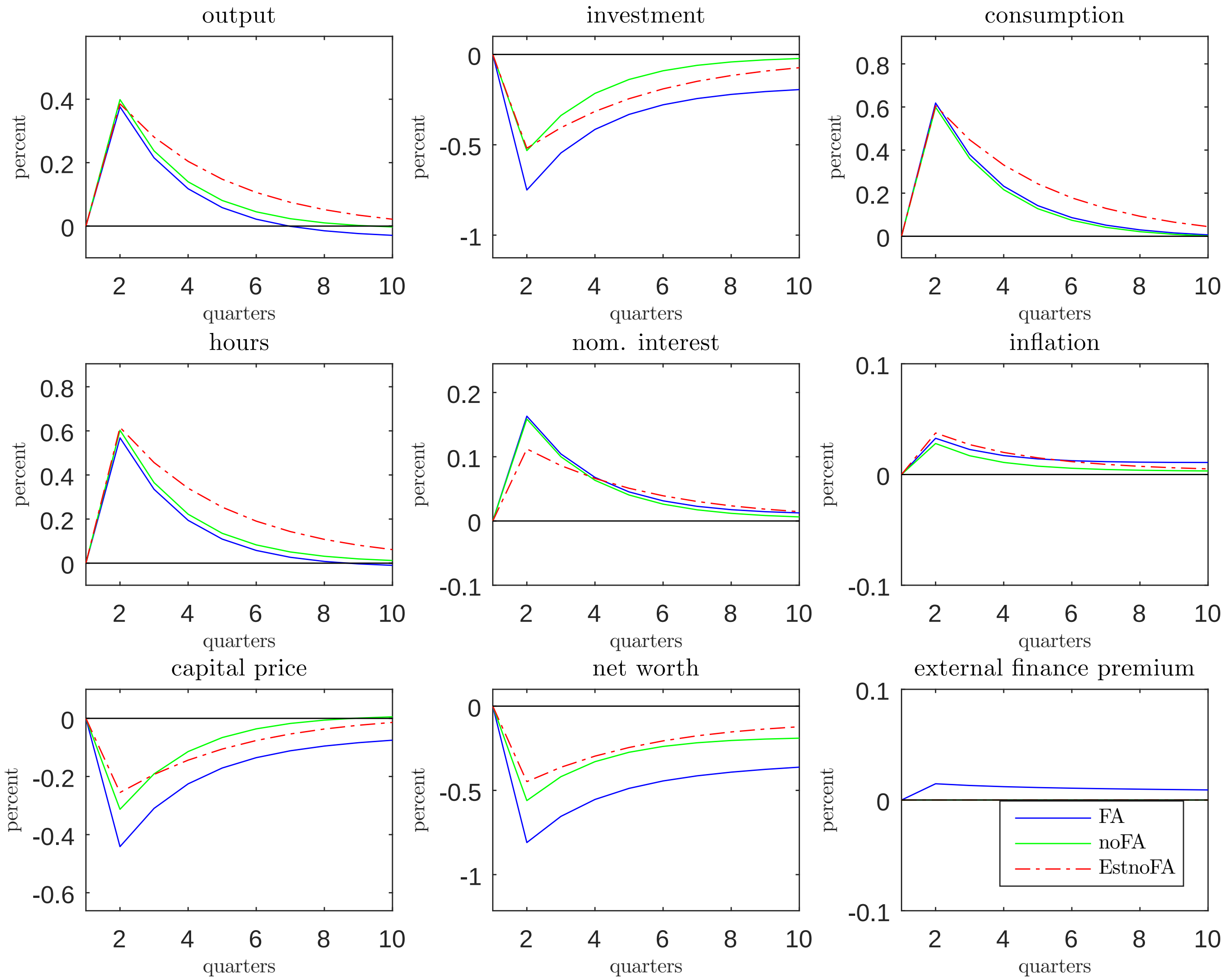

Figure 4: IRFs to Preference Shock

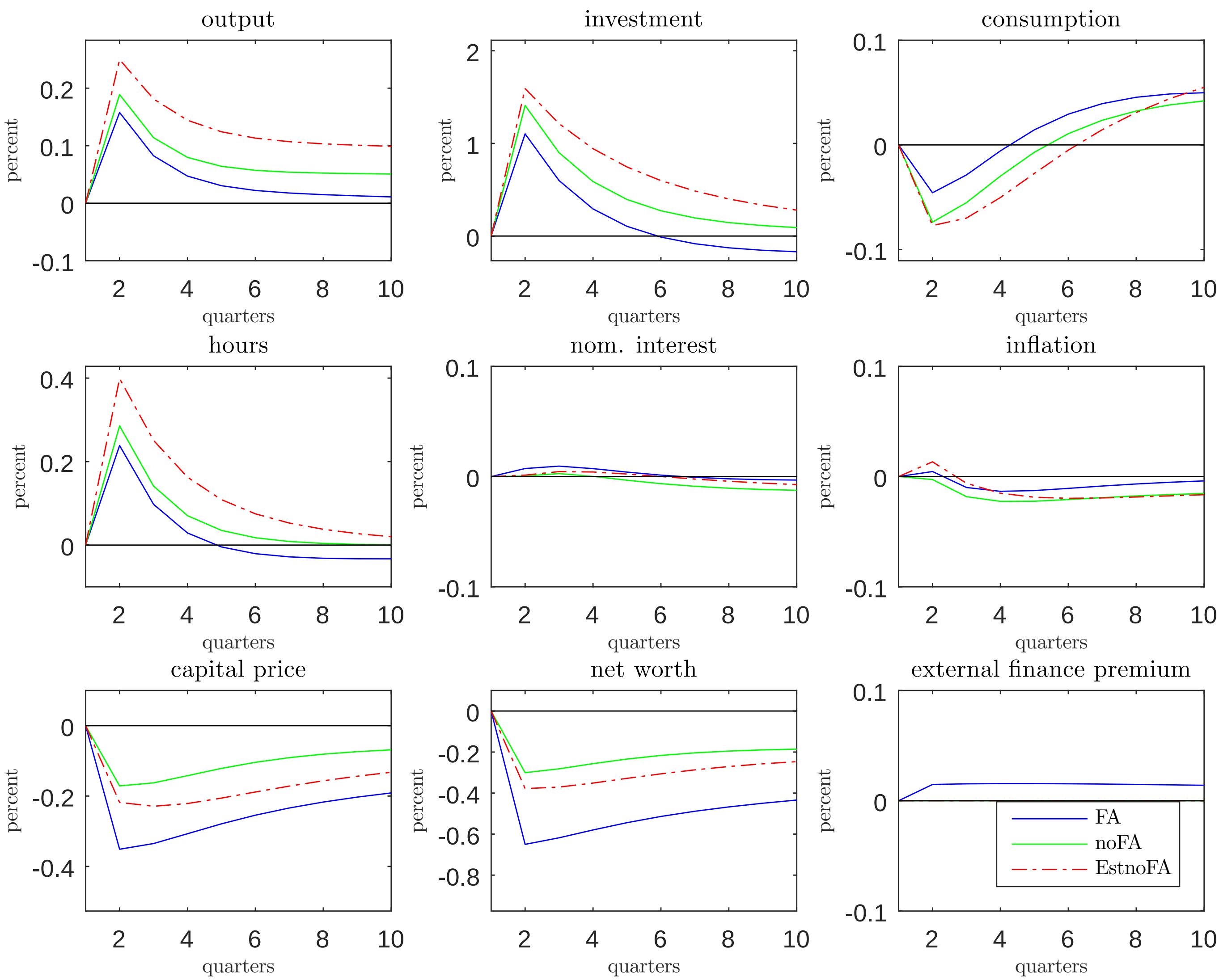

Figure 5: IRFs to Investment-Efficiency Shock

Comparing the IRFs in figures 1 to 5 to those in the original paper, we observe that the responses to the monetary policy shock are almost unchanged, though the responses of the capital price q and net worth n are a bit more pronounced. For the technology shock, there is almost no effect on the external finance premium. The IRF of net worth changes sign, and the three different specifications yield very similar responses of the capital price. Looking at the money demand and the preference shock, the IRFs are again very similar to those of Christensen and Dib (2008), with the IRFs of q and n a bit more pronounced. Last but not least, the IRFs to the investment efficiency shock are also qualitatively very similar, with a slight difference in scaling especially for output, consumption, hours and inflation. We checked whether the calibration targets discussed in problem set 1 are still satisfied. It turned out that this is the case. In problem set 1, we found a discrepancy for the ratio of real balances to consumption. However, we noticed that we get a value of 0.0833 when using the FRED series PCEC as an alternative measure for consumption instead of PCND. The model-implied value is 0.0797. Accordingly, we chose to stick to the calibrated parameters of Christensen and Dib (2008).