Version 1

The lagrangian of the problem, omitting all equations but the definition of the **expected** return on capital and the Phillips curve, is:

$$
\begin{aligned}
L &= \sum_t \beta^t ... & (1)\\
&+\sum_t \beta^t \lambda_{1,t}\left\{r_t - \frac{R_{t+1}-\delta q_{t+1}+\frac{q_{t+1}-q_t}{\Delta}}{q_t}\right\} \\
&+\sum_t \beta^t \lambda_{2,t}\left\{\left(r_t - \frac{Y_{t+1}-Y_t}{\Delta Y_t}\right)\pi_{t+1} - \frac{\varepsilon}{\theta}\left(m_t - m^*\right) - \frac{\pi_{t+1}-\pi_t}{\Delta}\right\}
\end{aligned}
$$

The FOCs are

$$
\begin{aligned}
\frac{\partial L}{\partial r_1}:0 &= \lambda_{1,t}+\lambda_{2,t}\pi_{t+1}+... & (2)\\
\frac{\partial L}{\partial R_1}:0 &= -\frac{\beta^{-1}\lambda_{1,t-1}}{q_{t-1}}+... & (3)
\end{aligned}
$$

The FOCs at t=1, taking into account that t=0 is the SS:

$$
\begin{aligned}
\frac{\partial L}{\partial r_1}:0 &= \lambda_{1,1}+\lambda_{2,1}\pi_2+... & (4)\\
\frac{\partial L}{\partial R_1}:0 &= -\frac{\beta^{-1}\lambda_{1,SS}}{q_{SS}}+... & (5)
\end{aligned}
$$

Eliminate $\lambda_{1,SS}$ using the SS version of (2)

$$
0 = \frac{\beta^{-1}\lambda_{2,SS}\pi_{2,SS}}{q_{SS}}+... \qquad (6)
$$

Version 2

The lagrangian of the problem, omitting all equations but the definition of the **current** return on capital and the Phillips curve, is:

$$
\begin{aligned}
L &= \sum_t ... & (7)\\
&+\sum_t \beta^t \lambda_{1,t}\left\{r_t - \frac{R_t-\delta q_t+\frac{q_t-q_{t-1}}{\Delta}}{q_{t-1}}\right\} \\
&+\sum_t \beta^t \lambda_{2,t}\left\{\left(r_{t+1} - \frac{Y_{t+1}-Y_t}{\Delta Y_t}\right)\pi_{t+1} - \frac{\varepsilon}{\theta}\left(m_t - m^*\right) - \frac{\pi_{t+1}-\pi_t}{\Delta}\right\}
\end{aligned}
$$

The FOCs are

$$
\begin{aligned}
\frac{\partial L}{\partial r_t}:0 &= \lambda_{1,t}+\beta^{-1}\lambda_{2,t-1}\pi_t+... & (8)\\
\frac{\partial L}{\partial R_t}:0 &= -\frac{\lambda_{1,t}}{q_{t-1}}+... & (9)
\end{aligned}
$$

The FOCs at t=1, taking into account that t=0 is the SS:

$$\frac{\partial L}{\partial r_1} : 0 = \lambda_{1,1} + \beta^{-1} \lambda_{2,SS} \pi_1 + ... \tag{10}$$

$$\frac{\partial L}{\partial R_1} : 0 = -\frac{\lambda_{1,1}}{q_{SS}} + ... \tag{11}$$

Eliminate $\lambda_{1,1}$

$$0 = \frac{\beta^{-1} \lambda_{2,SS} \pi_1}{q_{SS}} + ... \tag{12}$$

Comparison

Comparing (6) and (12) we see that in version 1 the optimality condition (6) includes π_{SS} but in version 2 the condition (12) inlcudes π_1. As (7) and (8) show this π_1 is the result of equating $\mathbb{E}_0 [\pi_1]$ to π_1.