model version 1

Consider a ramsey problem subject to the following constraints

$$\begin{aligned} f^1(X_{t+1}, Y_t) &= 0 \\ f^2(X_t, Y_t) &= 0 \\ &\vdots \end{aligned}$$

The planner's FOCs are

$$\frac{\partial L}{\partial X_t} : 0 = \lambda^1_{t-1} f^1_X(X_t, Y_{t-1}) + \lambda^2_t f^2_X(X_t, Y_t) + ... \tag{1}$$

$$\frac{\partial L}{\partial Y_t} : 0 = \lambda^1_t f^1_Y(X_{t+1}, Y_t) + \lambda^2_t f^2_Y(X_t, Y_t) + ... \tag{2}$$

Dynare evaluates them as follows in the first period $t = 1$

$$\frac{\partial L}{\partial X_t} : 0 = \lambda^1_{SS} f^1_X(X_1, Y_{SS}) + \lambda^2_1 f^2_X(X_1, Y_1) + ... \tag{3}$$

$$\frac{\partial L}{\partial Y_t} : 0 = \lambda^1_1 f^1_Y(X_2, Y_1) + \lambda^2_1 f^2_Y(X_1, Y_1) + ... \tag{4}$$

model version 2

Consider an equivalent ramsey problem subject to these constraints:

$$
\begin{aligned}
f^1(\tilde{X}_t, Y_t) &= 0 \qquad (5)\\
f^2(X_t, Y_t) &= 0 \qquad (6)\\
\tilde{X}_t &= X_{t+1} \qquad (7)\\
&\vdots
\end{aligned}
$$

Here we have defined $\tilde{X}$ to be the expectation of next period X and used this definition in the first constraint. With exogenous policy these 2 constraints define identical equilibria.

The planner's FOCs are

$$
\begin{aligned}
\frac{\partial L}{\partial X_t} : 0 &= \lambda_t^2 f_X^2(X_t, Y_t) - \lambda_{t-1}^3 \qquad (8)\\
\frac{\partial L}{\partial Y_t} : 0 &= \lambda_t^1 f_Y^1(\tilde{X}_t, Y_t) + \lambda_t^2 f_Y^2(X_t, Y_t) + ... \qquad (9)\\
\frac{\partial L}{\partial \tilde{X}_t} : 0 &= \lambda_t^1 f_{\tilde{X}}(\tilde{X}_t, Y_t) + \lambda_t^3 ... \qquad (10)
\end{aligned}
$$

Dynare evaluates them as follows in the first period $t = 1$

$$
\begin{aligned}
\frac{\partial L}{\partial X_t} : 0 &= \lambda_1^2 f_X^2(X_1, Y_1) - \lambda_{SS}^3 \qquad (11)\\
\frac{\partial L}{\partial Y_t} : 0 &= \lambda_1^1 f_Y^1(\tilde{X}_1, Y_1) + \lambda_1^2 f_Y^2(X_1, Y_1) + ... \qquad (12)\\
\frac{\partial L}{\partial \tilde{X}_t} : 0 &= \lambda_1^1 f_{\tilde{X}}^1(\tilde{X}_1, Y_1) + \lambda_1^3 ... \qquad (13)
\end{aligned}
$$

Using the steady state version of (7) and (10) we can write

$$
-\lambda_{SS}^3 = \lambda_{SS}^1 f_{\tilde{X}}(X_{SS}, Y_{SS}) + ... \qquad (14)
$$

and use this to eliminate λ^3 and $\tilde{X}_1$ from (11)-(13):

$$
\begin{aligned}
\frac{\partial L}{\partial X_t} : 0 &= \lambda_{SS}^1 f_{\tilde{X}}(X_{SS}, Y_{SS}) + \lambda_1^2 f_X^2(X_1, Y_1) + ... \qquad (15)\\
\frac{\partial L}{\partial Y_t} : 0 &= \lambda_1^1 f_Y^1(\tilde{X}_1, Y_1) + \lambda_1^2 f_Y^2(X_1, Y_1) + ... \qquad (16)
\end{aligned}
$$

Comparison

First compare the two systems (1)-(2) and (8)-(10). Its easy to show that they are identical for t>1.

Second, compare the two systems (3)-(4) and (15)-(16). (4) and (16) are identical. But (3) and (15) are not identical. One features X_1 the other X_{SS}.

Hence the "timeless" optimal policy will be different across the two model versions even though they behave identically for exogenous policy. time0 optimal policy will however be identical, since thenλ^1_{SS} is replaced by 0.