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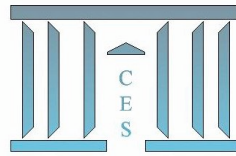
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# The Effects of Oil Price Shocks in a New-Keynesian Framework with Capital Accumulation

Verónica ACURIO VASCONEZ\*

Gaël GIRAUD†

Florent Mc ISAAC‡

Ngoc Sang PHAM§

Centre d'Economie de la Sorbonne  
Paris School of Economics

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## Abstract

The economic implications of oil price shocks have been extensively studied since the 1970s'. Despite this huge literature, no dynamic stochastic general equilibrium model was available that captures two well-known stylized facts: 1) the stagflationary impact of an oil price shock, together with 2) the influence of the energy productivity of capital on the depth and length of this impact. We build, estimate and simulate a New-Keynesian model with capital accumulation, which takes the case of an economy where oil is imported from abroad, and where these stylized facts can be accounted for. Moreover, the Bayesian estimation of the model on the US economy (1984-2007) suggests that the output elasticity of oil might have been above 10%, stressing the role of oil use in US growth at this time. Finally, our simulations confirm that an increase in energy efficiency significantly attenuates the effects of an oil shock — a possible explanation of why the third oil shock (1999-2008) did not have the same macro-economic impact as the first two ones. *JEL Codes : C68, E12, E23, Q43*

**Keywords:** New-Keynesian model, DSGE, oil, capital accumulation, stagflation, energy productivity.

## 1 Introduction

The two episodes of low growth, high unemployment, low real wages and high inflation that characterized most industrialized economies in the mid and late 1970s' are usually

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\*Email: [veronica.acurio-vasconez@malix.univ-paris1.fr](mailto:veronica.acurio-vasconez@malix.univ-paris1.fr), Université Paris-1, Panthéon-Sorbonne, Maison des Sciences Economiques, 106-112 Boulevard de l'Hôpital, 75013 Paris

†CNRS, Paris School of Economics, Email: [gael.giraud@parisschoolofeconomics.eu](mailto:gael.giraud@parisschoolofeconomics.eu)

‡Email: [florent.mc-isaac@univ-paris1.fr](mailto:florent.mc-isaac@univ-paris1.fr), Université Paris-1.

§Email: [pns.pham@gmail.com](mailto:pns.pham@gmail.com), Université Paris-1.

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viewed as the paradigmatic consequences of large price shocks that affect various countries simultaneously. Despite the huge literature devoted to the implications of oil prices, to the best of our knowledge, no dynamic general equilibrium model was available that captures the next two stylized facts : 1) the stagflationary impact of an oil price shock, together with 2) the various impacts of capital accumulation: in addition to the well-known hysteresis effect [Khr12], the potential role of capital as a new channel for monetary policy through the non-arbitrage relation involving the rental rate of capital and the Central Bank's interest rate and, above all, the role of capital energy efficiency in dampening the impact of an oil shock.

The present paper introduces energy into an otherwise standard New Keynesian model in the same way as [BG09] and [BR13], to which it adds capital accumulation. The latter addition is important not just because it adds realism to the modeling approach, but also because it improves the reliability of the model's empirical estimation. Energy is understood as being just oil, which is imported from abroad at an exogenous world price. Oil imports are paid for with exports of output, with trade being balanced at every date. Oil is consumed by households and used as an input in the production of intermediate goods. As a matter of fact, and this might be viewed as the main contribution of this paper, when estimated on the US (1984-2007), the output elasticity of oil use turns out to be significantly larger than what is currently assumed in the macro-economic literature. More specifically, we find an elasticity between 11% and 12%. In particular, this is much higher than the cost share of oil, which is usually less than 3%. Our finding confirms the standpoint that has been defended by several authors, including Ayres, Kümmel, Lindenberger and Voudouris (see [KL10], [K11] and [AV14]), according to whom the importance of energy in the fabric of economic growth is amply underestimated in the traditional Solovian approach.

As a result, our specification does react to an oil shock by a short-run decrease in real GDP and some inflation.<sup>1</sup> Next, the introduction of capital accumulation turns out not to impair the stylized facts just alluded to. Capital even amplifies the response of the economy to a shock. Our third, and most important, conclusion is that a reduction of output elasticity of energy suffices to imply a significant reduction of the effect of a shock on macroeconomic performances. This is the way the reduction of the sensitivity of industrialized countries to the oil shock in the 2000s' is accounted for in this paper.

When addressing these issues, we keep an eye on the events of the past decade that seem to call into question the relevance of oil price changes as a significant source of economic fluctuations. Since the late 1990s' indeed, the global economy has experienced an oil shock of sign and magnitude comparable to those of the 1970s' but, in contrast with the latter episodes, GDP growth and inflation have remained relatively stable in much of the industrialized world until the financial turbulences of 2007-2009 (cf. e.g., Sanches [S08], Blanchard and Galí, 2009 [BG09] ; Kilian, 2007 [Kil08]). In Blanchard and Galí [BG09], a structural VAR analysis suggests that the effects of oil price shocks have recently weakened because of the decrease in real wage rigidities, a smaller oil share in production and consumption, and improvements in the credibility of monetary policy. While these three properties did most probably play a role, this paper explores the explanatory power of yet another channel —namely the change in energy productivity in the industrial sector during the 80s', as a consequence of the first two oil shocks. At first glance, it seems that the impact of energy productivity is already taken into account through the decline of energy

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<sup>1</sup>For simplicity, we did not add a growth trend to our model. Were we to do so, our results would be a short-run decrease of the real GDP with respect to the long-run growth trend.

share in added value, analyzed in [BG09].<sup>2</sup> As we argue in the next section, however, these two parameters —cost share and energy productivity— should be viewed as decoupled variables, in general. Consequently, if the energy productivity of a country can no more be captured through its energy cost share, we need an explicit modeling of the efficiency of capital. This is yet another motivation for having added a capital accumulation dynamics to the standard DSGE model<sup>3</sup>. The decoupling of energy productivity from the energy share cost then opens the door for a reexamination of why the 2000s' have been so different from the 70s'. Our finding is that the improvement of energy productivity may well be a powerful explanatory factor for the muted impact of the oil shock experienced during the early 2000's.

Among the 2000s' oil shock literature, recently, in addition to [BG09], a last contribution is worth noticing, namely Blanchard and Raggi [BR13]. The latter performs an estimation of a Macroeconomic DSGE model, and confirms that a large decrease of real wage rigidities and an increase of the credibility of the monetary policy must have contributed to dampening the shock. Together with an estimation based on indirect inference, Blanchard and Raggi [BR13] *calibrate* the production function as being constant return to scale with an output elasticity of oil set equal to 0.015 for the period pre-1984 and 0.012 for the post-1984 period, the output elasticity of labor being therefore 0.985 and 0.988, respectively.

By contrast, in the present paper, using a Bayesian likelihood approach, most parameters are estimated, including oil's output elasticity. The latter turns out to lie between 0.11 and 0.12. That is, a 10% increase of oil consumption leads to a 1.1% or 1.2% increase of output —an elasticity 10 times larger than the one supposed by [BR13]. Our finding also contrasts with the literature where oil output elasticity is usually identified with the energy cost share, hence close to 0.03. Where does the gap between our output elasticity,  $\alpha_e$ , and the cost share come from? In the present DSGE model, it arises from the GDP definition and Calvo viscosity of prices (whose value, as usual, is calibrated around 0.65) which prevents prices from reflecting the standard first order conditions from which the cost share theorem is derived. At the stationary state, the Calvo friction vanishes, and one has:

$$\begin{aligned} \text{Oil's cost share} &:= \frac{P_e E}{P_y Y} \\ &= \frac{P_e E}{P_q Q - P_e E} \\ &= \frac{\alpha_e}{\mathcal{M}_p - \alpha_e} \end{aligned} \tag{1}$$

where  $Y$  stands for GDP,  $Q$  for domestic output,  $E$  for oil and  $\mathcal{M}_p$  for the price markup in the (imperfectly competitive) production sector. So that, even though they never coincide, the cost share and output elasticity remain somewhat close to each other. But along the transitional dynamics *towards* the steady state, the Calvo friction *does* enter in the scene. And this transitional dynamics is crucial for the Bayesian estimation of the output elasticity

<sup>2</sup>Profit-maximization and perfect competition in frictionless markets imply indeed the equality of energy output elasticity with the cost share of energy. Since the inverse of output elasticity may be taken as a proxy for energy productivity, it might seem that the improvement of energy productivity is reflected through the decline of energy share.

<sup>3</sup>See, e.g., [Khr12] and the references therein.

of energy. Together with the fact that, contrary to a large body of the literature, we do not restrict returns to scale to be *a priori* constant, this explains why our Bayesian estimation does not lead to an elasticity close to the empirically observed cost share. Conversely, (1) implies that, along the steady state, absent any price friction, the cost share should be close to 10%, which is obviously at odds with historical data. This simply confirms that the US economy evolved rather far from its steady state during the period under scrutiny. The elasticity parameter,  $\alpha_e$ , being constant, its value does not depend upon whether the economy remained in the vicinity of its steady-state path, or not.

The paper is organized as follows. The next section presents the conceptual framework, in particular the decoupling issue just alluded to. Section 3 describes the model. Section 4 provides the estimation procedure. Section 5 gives our main findings by analyzing at length how our model reacts to a real oil price shock. We leave the complete methodological details and numerical simulations to an extensive on-line Appendix.

## 2 Conceptual framework

Apart from the introduction of energy as an input in the aggregate production function, our New-Keynesian framework is rather standard. Three conceptual issues are worth being addressed: the possible decoupling between the cost share and output elasticity of energy (subsection 2.1.), the introduction of increasing returns to scale (section 2.2.) and the very definition of a global “price level” (subsection 2.3.)

### 2.1 Decoupling the cost share from output elasticity

Direct estimates of the returns to physical capital suggest an output elasticity with respect to physical capital around 0.7. It seems to contradict the marginal productivity theory of distribution which asserts that, in competitive markets and under constant returns to scale, the profit share should be equal to the elasticity of output: the profit share in the developed countries is indeed well-known to stay around 0.4 since several decades (and tends rather to decline in the last years). To quote just the seminal papers, Romer (1987) suggested that the difference is explained by externality in physical capital. Barro and Sala-i-Martin (1992) and Mankiw, Romer and Weil (1992, p. 432) argued that this decoupling could be explained by the omission of variables: investment in education as part of the accumulable capital in the first paper, human capital in the second. Of course, dropping the assumption of perfect competition and/or constant returns to scale also helps understand the possible decoupling.

Following, e.g., [KL10], let us mention yet another reason why output elasticity might not always equal the profit share, even in competitive markets, under constant returns to scale and absent any externality of omitted variables. Denoting  $x = (x_i)_i$  the input vector,  $Y(\cdot)$  the production function, and  $p = (p_i)_i$  the real price of inputs, the profit maximization program of the producer,<sup>4</sup>

$$\max_x Y(x) - p \cdot x \tag{2}$$

leads to:

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<sup>4</sup>In the following argument, the output is taken as numéraire.

$$\varepsilon_i := \frac{x_i}{Y(x)} \times \frac{\partial Y}{\partial x_i}(x) = \frac{p_i x_i}{p \cdot x} \quad (3)$$

where  $\varepsilon_i$  is the output elasticity of the production factor,  $x_i$ . This textbook argument rests on the assumption that the producer's maximization program (2) faces *no* constraint apart from the very definition of  $Y(\cdot)$ . Suppose, on the contrary, that (2) must be written, somewhat more realistically:

$$\max_x Y(x) - p \cdot x \quad \text{s.t. } f(x) = 0 \quad (4)$$

where  $f(\cdot)$  is some smooth function. Whenever the input,  $x_i$ , is interpreted as energy, we can think of  $f(\cdot)$  as capturing geological resource restrictions on fossil energies, geopolitical or climatic constraints, the bargaining power of labor forces, institutional rigidities of the labor market, etc. The cost-share identity (3) now involves a shadow price given by the (normalized) Lagrange multiplier,  $\lambda$ , of the additional constraint,  $f(x) = 0$ :

$$\varepsilon_i = \frac{x_i(p_i - \lambda \frac{\partial f(x)}{\partial x_i})}{p \cdot x - \lambda x_i \frac{\partial f(x)}{\partial x_i}}. \quad (5)$$

It follows that shadow prices may be responsible for the decoupling between the energy share,  $p_i x_i / p \cdot x$ , and its output elasticity,  $\varepsilon$ . Suppose, for instance, that the cost share remains small, while  $\lambda \rightarrow +\infty$ . Then,  $\varepsilon_i \rightarrow 1$ .<sup>5</sup> Similarly,  $\varepsilon$  may take any real value between  $x_i p_i / x \cdot p$  and  $-\infty$  whenever  $0 < \lambda < (p \cdot x) \frac{\partial x_i}{\partial f(x)}$ . So that a large share  $x_i p_i / x \cdot p$  is compatible with a small  $\varepsilon$ . The strength of this latter argument for decoupling is that it prevents us from concluding that one factor's return is underpaid (when the profit share is below its output elasticity) or overpaid (in the opposite situation): both might well exhibit a "fair return" once all the constraints in the production sector have been taken into account.<sup>6</sup>

In a companion paper, [KG14], the elasticity of primary energy use is estimated through an error correction model for 33 countries, along time series from 1970 to 2011. Estimated elasticities are robustly located between 0.4 (France) and 0.7 (U.S.), with an average around 0.6. This empirical finding is confirmed by the Bayesian estimation of the present model, performed in the same paper —where, again, the output elasticity of energy turns out to be close to 0.6. According to the previous argument, this seems to suggest that economic actors face binding constraints regarding the use of primary energy. Similarly, in [KG14], it is suggested that the output elasticity of capital could be much lower than is suggested by the capital share. As already noticed, this is perfectly compatible with (5).

Put otherwise, one purpose of this paper is to confirm the relevance of the reduction of the dependence of the industrialized economies (especially the U.S. economy) to oil in the 2000s' as compared to the 1970s' as an explanatory factor for why the last decade was so different from the 70s', and to show that this can be illustrated and modeled without relying on any *a priori* coupling between the output elasticity and cost share.<sup>7</sup>

<sup>5</sup>A similar observation is made in [KL08] and [Kĭ1].

<sup>6</sup>One may be tempted to replicate that, at least in the perfectly flexible case, prices should already reflect the constraint,  $f(x) = 0$ , so that there would be no need to make it explicitly in 4. However, for prices to convey publicly this information, some individual producers must hold it privately, that is, they must have taken it into account in their individual profit-maximization programme. Consequently, it must show up as well at the aggregate level.

<sup>7</sup>Thus, this work complements [BG09] whose analysis is based on this coupling.

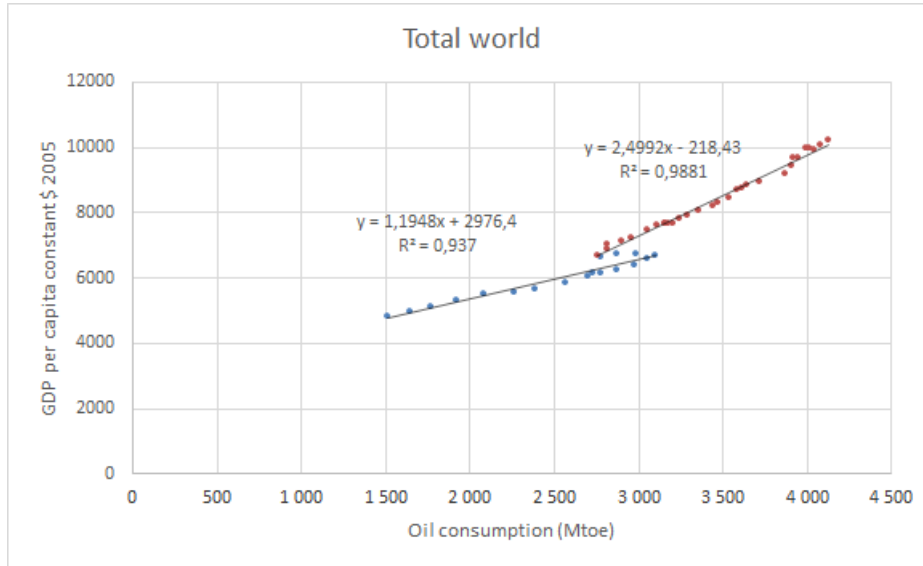


Figure 1: **Productivity of oil, world, 1960-2011**

Sources: BP statistical review, 2013 [BP13] for oil series and World Bank for World GDP, 2012[Ban12]

As shown by Figure 1, this dependency significantly decreased during the 80s', most probably as a consequence of the adaptation of the industrial sector to the shocks received in the previous decade. The x-axis is the world consumption of oil in millions of tones of oil-equivalent (toe), the y-axis is the world GDP per capita in 2011 constant US \$. The first point on the south-west stands for 1960, the last one (in the north-east), for 2011. The four points in the turning phase between the first segment and the second represent the years 1973 to 1979. The closeness of the red points, when compared to the distance separating the green points illustrates the slow down of growth and oil consumption per capita after the second oil shock. A complete independence between the world GDP and oil consumption at the world level would imply that the resulting segments be vertical. Clearly, there was an improvement in the energy productivity from the 80s' on as compared to the 60's and early 70s': the slope (2.49) of the affine segment posterior to the second oil shock twice as large as that of the green segment prior to the first oil shock (1.19).<sup>8</sup> In this paper, we capture such a shift by decreasing the output elasticity of oil.<sup>9</sup> The main question addressed in this paper is whether such a change in the structure of the production sector can be responsible for the muted impact of the third oil shock on Western economies.

## 2.2 Increasing returns ?

On the analytical side, the main consequence of allowing for such non-conventional elasticities is to force us to relax the textbook constant returns assumption which underlies

<sup>8</sup>Of course, the large  $R^2$  (between 0.93 and 0.99) obtained in the two OLS performed before 1973 and after 1979 suggest a strong endogeneity between GDP growth and oil consumption. This issue is addressed in [KG14].

<sup>9</sup>Indeed, whenever  $\partial Y(x)/x$  is a decreasing function of  $x$  (as in our model), an increase of the productivity,  $Q/x$ , of some input  $x$  together with an increase of the demand for  $x$ , must translate into a decrease of  $\varepsilon$  according to (5).



(3). There are various motivations for not imposing *a priori* the constant returns to scale restriction. Firstly, as is well known, the fact that empirical investigations often conclude that, at the aggregate level, returns to scale seem constant is but an econometric artifact.<sup>10</sup> Second, it is equally well-known that a production sector with strictly decreasing returns to scale cannot exhibit indefinite growth.<sup>11</sup> Hence, endogenous growth must rely, at some stage or another, on some non-convexity in the production sector. Third, empirical inquiries unambiguously conclude that, at the micro level, most industrial sectors exhibit increasing returns to scale.<sup>12</sup>

Our purpose is not to revisit here the pros and cons of exogenous versus endogenous growth theory. More simply, we aim at letting the data speak by estimating a New Keynesian model that is compatible with every kinds of returns to scale. Of course, when dealing with a non-concave production function, the main challenge is to define the producer's behavior, as the mere profit maximization program (2) or (4) may no more have any solution (or may admit several solutions). We adopt the most commonly used behavioral framework found in the literature devoted to increasing returns, namely marginal cost pricing:<sup>13</sup> At equilibrium, the representative producer chooses a production plan,  $x$ , such that the price,  $p$ , of inputs, equals their marginal cost,  $\partial Y(x)/\partial x_i$ . This seems to be the less onerous way of dealing with the lack of decreasing returns. Indeed, marginal cost pricing readily leads to the familiar first-order conditions that are otherwise instrumental for the numerical simulations of the response to exogenous shocks in the DSGE tradition. In the context of the DSGE literature, our departure with the standard practice is therefore that first-order conditions are necessary but need no more be sufficient for profit-maximization.

As a matter of fact, our estimation concludes unambiguously that returns to scale are strictly increasing. This should not come as a surprise since we did not introduce any *ad hoc* technological progress that would fuel exogenous growth.

### 2.3 GDP deflator and CPI

With no capital accumulation and zero public expenditures, our model reduces to the one first introduced by [BG09] with two changes, in the monetary policy and in the definition of the GDP deflator. In Blanchard and Galí [BG09], indeed, the CPI is defined as  $P_{c,t}$ , the core CPI, as  $P_{q,t}$  and the GDP deflator, as  $P_{y,t}$ . The three indices are related by the following equations:

$$P_{q,t} := P_{y,t}^{1-\alpha_e} P_{e,t}^{\alpha_e}; \quad (6)$$

$$P_{c,t} := P_{q,t}^{1-x} P_{e,t}^x; \quad (7)$$

and

$$P_{y,t} := P_{q,t}^\beta P_{e,t}^{1-\beta} \quad (8)$$

where  $P_{e,t}/P_{q,t}$  is the (exogenous) real price of energy at time  $t$ ,  $\alpha_e, x \in (0, 1)$ , but — and this turns out to be crucial —,  $\beta > 1$ .

<sup>10</sup>Cf. [Sam79].

<sup>11</sup>Cf. [HR73].

<sup>12</sup>[BL98] and [EG52].

<sup>13</sup>Cf. [Cor88], [Qui92], [BMC85], [BC90], [DD88a] or [Gir01].

These conventions have the paradoxical consequence that, when the energy price experiences an upward shock, the GDP deflator decreases (everything else being kept fixed) as can be seen from (8). We fix this problem by imposing  $P_{c,t} \equiv P_{y,t}$  while keeping (6), (7), and the following budget identity, which defines GDP, in the left-hand side, as the aggregation of domestic product minus energy import:

$$P_{y,t}Y_t = P_{q,t}Q_t - P_{e,t}E_t.$$

### 3 A New-Keynesian economy with imported energy

The real prices of oil and capital relative to the price of final good are

$$S_{e,t} := \frac{P_{e,t}}{P_{q,t}} \quad (9)$$

$$S_{k,t} := \frac{P_{k,t}}{P_{q,t}}. \quad (10)$$

They both are assumed to follow AR(1) processes:

$$\ln(S_{e,t}) = (1 - \rho_{se})\ln(\bar{S}_e) + \rho_{se} \ln(S_{e,t-1}) + e_{se,t} \quad (11)$$

$$\ln(S_{k,t}) = (1 - \rho_{sk})\ln(\bar{S}_k) + \rho_{sk} \ln(S_{k,t-1}) + e_{sk,t}. \quad (12)$$

where  $e_{se,t} \sim \mathcal{N}(0, \sigma_{se}^2)$  and  $e_{sk,t} \sim \mathcal{N}(0, \sigma_{sk}^2)$ ,  $\bar{S}_e$  and  $\bar{S}_k$  respectively stand for the steady states of real price of oil and capital.

Let us now briefly describe how capital accumulation and imported oil enter into the model.

#### 3.1 Household

The representative infinitely-lived household works, invests in government bonds and capital, pays taxes and consumes both oil and the final good. Its instantaneous utility function is

$$u(C_t, L_t) = \ln(C_t) - \frac{L_t^{1+\phi}}{1+\phi},$$

where  $C_t$  is the consumption at time  $t$ ,  $L_t$  is labor and  $\phi$  is the inverse of the Frisch elasticity. Let  $W_t$  denote the nominal wage,  $P_{k,t}$ , the nominal price of capital, and  $r_{t+1}^k$ , the real rental rate of capital. The dynamics of capital accumulation follows, as usually,

$$I_t := K_{t+1} - (1 - \delta)K_t,$$

where  $\delta \in (0, 1)$  is the depreciation rate. At variance with several DSGE models, the capital price is not identified with the consumption price but is rather viewed as exogenous. Indeed, the custom to identify both consumption and capital prices arises, as is well-known from the Cambridge controversy, from the lack of an equilibrium condition that would permit pinning down the market value of capital.<sup>14</sup> But this mere identification prevents from

<sup>14</sup>Cf. [Sam66].

capturing decoupled bubble phenomena, such as the housing bubble that affects most Western countries since the middle of the 90s'.<sup>15</sup>

The nominal short-run interest rate,  $i_t$ , is set by the Central Bank. At time  $t$ ,  $T_t$  denotes the tax paid by the household. Being the shareholder of the firms, the household receives the global dividend  $D_t := \int_0^1 D_t(j) dj$ , i.e., the sum of dividends of all intermediate good firms.

The problem of the household is

$$\begin{aligned} \max \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u(C_t, L_t)], \quad 0 < \beta < 1, \\ \text{subject to :} \quad & P_{e,t} C_{e,t} + P_{q,t} C_{q,t} + P_{k,t} (K_{t+1} - (1 - \delta) K_t) + B_t \\ & \leq (1 + i_{t-1}) B_{t-1} + W_t L_t + D_t + r_t^k P_{k,t} K_t + T_t, \end{aligned} \quad (13)$$

where the consumption flow is defined as:

$$C_t := \Theta_x C_{e,t}^x C_{q,t}^{1-x}, \quad (14)$$

with  $x \in (0, 1)$  being the share of oil in consumption,  $\Theta_x := x^{-x} (1-x)^{-(1-x)}$  and  $C_{q,t} := \left( \int_{[0,1]} C_{q,t} \frac{\epsilon-1}{\epsilon} (i) di \right)^{\frac{\epsilon}{\epsilon-1}}$  a CES index of domestic goods. The transversality condition prevents Ponzi schemes, hence guarantees existence of solutions to the household's program:

$$\lim_{k \rightarrow \infty} \mathbb{E}_t \left( \frac{B_{t+k}}{t+k-1 \prod_{s=0}^{k-1} (1+i_{s-1})} \right) \geq 0, \quad \forall t. \quad (15)$$

The optimal allocation of expenditures among different domestic goods yields:

$$P_{q,t} C_{q,t} = (1-x) P_{c,t} C_t \quad (16)$$

$$P_{e,t} C_{e,t} = x P_{c,t} C_t \quad (17)$$

$$\text{where } P_{c,t} = P_{e,t}^x P_{q,t}^{1-x} \text{ is the CPI index.} \quad (18)$$

### 3.2 Final Good Firm

There is a continuum,  $[0, 1]$ , of intermediate goods that serve in producing the consumption commodity. A representative final good producing firm maximizes its profit with no market power. Its CES production function is given by<sup>16</sup>

$$Q_t = \left( \int_{[0,1]} Q_t(i) \frac{\epsilon-1}{\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (19)$$

where  $\epsilon > 0$  is the elasticity of substitution among intermediate goods.

<sup>15</sup>See, e.g., [BW14].

<sup>16</sup>For simplicity, no oil is needed to produce the final commodity out of the intermediate goods.

The final good firm chooses quantities of intermediate goods  $(Q_t(i))_{i \in [0,1]}$  in order to maximize its profit:

$$\max_{Q_t(\cdot)} P_{q,t} Q_t - \int_{[0,1]} P_{q,t}(i) Q_t(i) di \quad (20)$$

$$\text{subject to : } Q_t = \left( \int_{[0,1]} Q_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (21)$$

Therefore, the demand of good  $i$  is given by  $Q_t(i) = \left( \frac{P_{q,t}(i)}{P_{q,t}} \right)^{-\epsilon} Q_t$ . The production function of the final good firm exhibits constant return to scale, so that, at equilibrium, it makes zero profit. The price of the final good will therefore be:

$$P_{q,t} = \left( \int_{[0,1]} P_{q,t}(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \quad (22)$$

### 3.3 Intermediate Goods Firms

Each intermediate commodity is produced through a Cobb-Douglas technology involving oil:

$$Q_t(i) = A_t E_t(i)^{\alpha_e} L_t(i)^{\alpha_\ell} K_t(i)^{\alpha_k} \quad (23)$$

$$\alpha_e, \alpha_\ell, \alpha_k \geq 0,$$

where  $A_t$  is a total productivity factor (TFP) so that its logarithm follows an  $AR(1)$  process,  $\ln(A_t) = \rho_a \ln(A_{t-1}) + e_{a,t}$ , where  $e_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$ .

The strategy of firm  $i$  can be decomposed in two steps: First, taking prices  $P_{e,t}$ ,  $P_{k,t}$ ,  $r_t^k$ ,  $W_t$ , and demand  $Q_t(i)$  as given, firm  $i$  chooses quantities of oil  $E_t(i)$ , labor  $L_t(i)$ , and capital  $K_t(i)$  so as to minimize its cost. Since returns to scale need not be decreasing,  $\alpha_e + \alpha_\ell + \alpha_k$  will possibly be larger than 1. As a consequence, in this paper, the producer is assumed to follow the marginal cost pricing behavior, which is characterized by the (standard) first-order conditions:

$$\text{marginal cost} = mc_t(i) := \frac{W_t}{\alpha_\ell \frac{Q_t(i)}{L_t(i)}} = \frac{r_t^k P_{k,t}}{\alpha_k \frac{Q_t(i)}{K_t(i)}} = \frac{P_{e,t}}{\alpha_e \frac{Q_t(i)}{E_t(i)}}. \quad (24)$$

In order to keep compact notations, we denote  $F_t := \left( \frac{A_t \alpha_e^{\alpha_e} \alpha_\ell^{\alpha_\ell} \alpha_k^{\alpha_k}}{P_{e,t}^{\alpha_e} W_t^{\alpha_\ell} (r_t^k P_{k,t})^{\alpha_k}} \right)^{\frac{-1}{\alpha_e + \alpha_\ell + \alpha_k}}$ , so that

$$\text{cost function: } cost(Q_t(i)) = (\alpha_e + \alpha_\ell + \alpha_k) F_t Q_t(i)^{\frac{1}{\alpha_e + \alpha_\ell + \alpha_k}}, \quad (25)$$

$$\text{marginal cost: } mc_t(i) = F_t Q_t(i)^{\frac{1}{\alpha_e + \alpha_\ell + \alpha_k} - 1}. \quad (26)$$

In the second step, each firm sets the price,  $P_{q,t}(i)$ , so as to maximize its net profit. We assume that prices are set à la Calvo. A fraction,  $\theta$ , of intermediate good firms cannot reset their prices at time  $t$ :

$$P_{q,t}(i) = P_{q,t-1}(i).$$

and a fraction,  $1 - \theta$ , sets its prices optimally:

$$P_{q,t}(i) = P_{q,t}^o(i).$$

Clearly,  $P_{q,t}^o(i)$  does not depend upon  $i$ , and we can write  $P_{q,t}^o(i) = P_{q,t}^o$ , for every  $i$ . Therefore we have the following ‘‘Aggregate Price Relationship’’

$$P_{q,t} = \left( \theta P_{q,t-1}^{1-\epsilon} + (1-\theta)(P_{q,t}^o)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \quad (27)$$

At date  $t$ , denote  $Q_{t,t+k}(i)$  the output at date  $t+k$  for firm  $i$  that last resets its price in period  $t$ . Firm  $i$ 's problem is

$$\begin{aligned} \max_{P_{q,t}(i)} \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \theta^k d_{t,t+k} [P_{q,t}(i)Q_{t,t+k}(i) - \text{cost}(Q_{t,t+k}(i))] \right] \\ \text{subject to } Q_{t,t+k}(i) = \left( \frac{P_{q,t}(i)}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}, \quad \forall k \geq 0. \end{aligned} \quad (28)$$

Again, this problem does not depend on  $i$ , hence  $P_{q,t}(i) = P_{q,t}^o$ . From the first order condition for  $P_{q,t}^o$  we have

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k d_{t,t+k} Q_{t,t+k}^o [P_{q,t}^o - \mu^p m c_{t,t+k}^o] = 0, \quad (29)$$

where  $m c_{t,t+k}^o := F_{t+k}(Q_{t,t+k}^o)^{\frac{1}{\alpha_e + \alpha_\ell + \alpha_k} - 1}$ ,  $Q_{t,t+k}^o = \left( \frac{P_{q,t}^o}{P_{q,t+k}} \right)^{-\epsilon} Q_{t+k}$  for every  $k \geq 0$  and  $\mathcal{M}^p := \frac{\epsilon}{\epsilon-1}$  is the price markup.

### 3.4 Monetary Policy

Let  $\Pi_{q,t} := \frac{P_{q,t}}{P_{q,t-1}}$  be the core inflation. As is usual, the Central Bank sets the nominal short-term interest rate according to the following monetary policy

$$1 + i_t = \frac{1}{\beta} (\Pi_{q,t})^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \varepsilon_{i,t}, \quad (30)$$

where  $\bar{Y}$  represents the steady state of  $Y_t$  and  $\ln(\varepsilon_{i,t}) = \rho_i \ln(\varepsilon_{i,t-1}) + e_{i,t}$ , where  $e_{i,t} \sim \mathcal{N}(0, \sigma_i^2)$ .

### 3.5 Government

The Government budget constraint is:

$$(1 + i_{t-1})B_{t-1} + G_t = B_t + T_t, \quad (31)$$

where  $G_t$  is the nominal government spending. We assume that the real government spending  $G_{r,t} = \frac{G_t}{P_{q,t}}$  is an exogenous process given by

$$\ln(G_{r,t}) = (1 - \rho_g) \ln(\omega \bar{Q}) + \rho_g \ln(G_{r,t-1}) + \rho_{ag} e_{a,t} + e_{g,t}$$

with  $\omega$ , the share of output that the government takes for its own spending,  $\bar{Q}$  represents the steady state of the domestic output and  $e_{g,t} \sim \mathcal{N}(0, \sigma_g^2)$ .

At equilibrium, each economic agent solves its maximization problem, all markets clear, and the government budget constraint is fulfilled.

### 3.6 GDP and GDP Deflator

We define real GDP ( $Y_t$ ) as follows:

$$P_{y,t}Y_t := P_{q,t}Q_t - P_{e,t}E_t$$

where  $P_{y,t}$  is the GDP deflator, that we assume to be equal to the CPI,  $P_{c,t}$ <sup>17</sup>.

## 4 Model Estimation

### 4.1 Setting

A log-linear version of the model around its steady state is presented in the online Appendix. Note that for estimation purposes we add an ad-hoc shock for the New-Keynesian Phillips Curve  $\varepsilon_{p,t}$ <sup>18</sup>, that can be interpreted as being a markup shock. Using Bayesian techniques, we estimate the model<sup>19</sup>. We choose this estimation technique, because as pointed out by the literature, the Bayesian approach takes advantage of the general equilibrium approach and outperforms GMM and ML in small samples.

For the estimation, the data set is made of six macroeconomic quarterly US variables: real GDP in chained dollars, real private fixed investment, hours worked, inflation, oil use in production and the Federal Funds rate<sup>20</sup>. The sample goes from 1984:Q1 to 2007:Q1. The sample time range is motivated by the well-known structural change in 1984 and the 2007-2008 crisis. Due to the stationary specification of the model, we detrend the first two series, which are not original stationary, using linear detrending<sup>21</sup>. The remaining series are stationary, so we do not detrend them, but we take out their respective mean in the estimation period.

There are 26 parameters, including parameters that characterize the exogenous shocks. Of the 26 parameters, we fix 5 according to the literature. The discount factor  $\beta$  is calibrated at 0.99. The depreciation rate  $\delta$  is calibrated at 0.025. We set the government spending output share  $\omega$  to 0.18 and we calibrate  $\epsilon$  to 8, that generate a steady state markup approximately equal to 1.14. These values are in line with empirical results. Finally, following Blanchard and Galí [BG09], we calibrate the share of oil consumption for the households  $x$ , at 0.023. The calibration of these parameters are resumed in Table 1. Those parameters are calibrated due to their well-known lack of identification in macro-data,

### 4.2 Identification Analysis

Before estimating, and due to the fact that there is no consensus about the value of the oil output elasticity  $\alpha_e$ , we perform an identification study of the model defined SUPRA, recently developed by Ratto [Rat08], Ratto and Pagano [RP10], Andrieu (2010) [And10],

<sup>17</sup>See subsection 2.2. for a discussion of this convention.

<sup>18</sup>Where  $\varepsilon_{p,t}$  is a  $ARMA(1,1)$  process of the form  $\varepsilon_{p,t} = \rho_p \varepsilon_{p,t-1} + e_{p,t} - \nu_p e_{p,t-1}$ , where  $e_{p,t} \sim \mathcal{N}(0, \sigma_p^2)$

<sup>19</sup>All estimations are done with Dynare version 4.4.1 [Dyn11]. Two tests are available to check the stability of the sample generating using MCMC algorithm, both implemented in Dynare. The MCMC Diagnostics: Univariate convergence diagnostic, Brooks and Gelman [BG98] and a comparison between and within moments of multiple chains.

<sup>20</sup>For a further explanation about the series sources and transformation, please refer to the online Appendix.

<sup>21</sup>We do not use HP-filter techniques because linear detrending implies more persistent deviation from trend than one-sided HP-filtered data.

Table 1: Calibrated Parameters

$\beta$	$\delta$	$\omega$	$x$	$\epsilon$
0.99	0.025	0.18	0.023	8

Canova and Sala [CS09] and Iskrev [Isk10] among others, and implemented in the Dynare's identification toolbox. This methodology is based on sensitivity analysis of the two first moments implied by the model together with the data. One can analyze the perturbation generated by a small change of one or few parameters with respect to the moment.

In order to run an identification analysis, we need to specify starting values for all parameters to identify. We first initialize our parameters as in Table 2,

Table 2: Starting values for the first identification

$\alpha_e$	$\alpha_\ell$	$\alpha_k$	$\phi$	$\phi_\pi$	$\phi_y$	$\theta$	$\rho_j$	$\sigma_j$
0.015	0.7	0.3	1.17	1.2	0.5	0.65	0.5	1

where  $j \in \{a, se, sk, g, p, i\}$ , so that  $\rho_j$  denotes all the autoregressive parameters in the model and  $\sigma_j$ , all the standard deviations.

The identification results point out that there exist a lack of identifiability strength for the output elasticity of oil,  $\alpha_e$ , when its starting value is around 0.015<sup>22</sup>. This last observation gives rise to test whether if this identification strength issue could be fixed using different initial values for the elasticities. One might propose a set of elasticities parameters values in order to check local identifiability strength, resume on Table 3.

Table 3: Set of starting values

<i>Elasticity</i>	(1)	(2)	(3)	(4)	(5)	(6)
$\alpha_e$	0.1	0.2	0.3	0.4	0.5	0.6
$\alpha_k$	0.3	0.3	0.2	0.2	0.2	0.1
$\alpha_\ell$	0.7	0.6	0.6	0.5	0.4	0.4

These starting values are built so that  $\alpha_e + \alpha_\ell + \alpha_k = 1.1$ , meaning that we allow for increasing return to scale technology.

The identification analysis, available on the Appendix, gives us the following results. Firstly, the highest  $\alpha_e$  is, the highest identification strength it has. Secondly, one can note that in this experimentation parameter  $\theta$ , loses nearly all its identification strength, with respect to the other variables. This last comment gives rise to estimate and compare the model with and without estimating  $\theta$ .

<sup>22</sup>For a more extensive explanation of this phenomena, please refer you to the appendix.

### 4.3 Estimation Results

Most priors are borrowed from Smets and Wouters [SW07]. However, we use an inverse-gamma distribution for all the elasticity parameters. We use this prior for three reasons: first to rely on positive values, second to concentrate the probability mass around the first parameter value and third to allow an asymmetry in the estimation.

Here we present a resume table for all estimations and the results obtained with the conditions that give us the best log-marginal density in both cases: (a)  $\theta$  estimated and (b)  $\theta$  calibrated. However, the complete results for all the different 14 estimations are available on the Appendix. The estimation of both cases is motivated by three facts. First, as explain in the last section, the parameter  $\theta$  loses identification strength as soon as we change starting values for elasticity parameters. Second, the New-Keynesian Philips Curve equation, mixes parameters  $\theta$ ,  $\alpha_e$ ,  $\alpha_l$  and  $\alpha_k$ , therefore in the estimation process, the inference of the Calvo parameter can interfere the inference on the elasticities parameters. Third, when estimating  $\theta$  the posterior mean obtained suggests that  $\hat{\theta} \in [0.9320; 0.9751]$  (except for one inference, which gives us  $\hat{\theta} = 0.5250$ ). Regarding to the literature on price stickiness, this interval indicates a much more higher degree of stickiness that the one usually found in the literature. For the case where  $\theta$  is calibrated, one can set this value at 0.65, consistent with previous findings in the literature.

Table 4 reports the prior and the posterior distributions for each parameter along with the mode, the mean and the 10 and 90 percentiles of the posterior distribution for both cases. In the same way, Table 5 refers to the estimates of the prior and posterior distributions of the shock processes. Finally, Table 6 presents a summarize of all estimations. It is structured as follows. The first column presents the oil's output elasticity first parameter of the inverse-gamma distribution used as a prior. The second column displays the log-marginal density of each estimation. The third column shows the posterior of the oil's output elasticity. Finally, the fourth column summarizes the sum of output elasticities. The priors distribution and priors' parameters of other variables remain unchanged except for the prior's parameters of  $\alpha_k$  and  $\alpha_l$  that change along with  $\alpha_e$ , as shown in Table 3.

Table 6 is ranking (ascending) with respect to the log-marginal density values. Several observations can emerge from this table. First, the first (best) three estimations when  $\theta$  is estimated give us the sum of elasticities greater than the steady state markup ( $\varepsilon/(\varepsilon - 1) \approx 1.14$ ). This gives rise to problematic economic interpretation due to the fact that one can show that if  $\sum_{i \in \{e, l, k\}} \alpha_i > \varepsilon/(\varepsilon - 1)$ , the steady state value of firm's profit is negative. This is not surprising, since the model does not restrict the production function to have a constant return to scale technology together with the fact that the estimation procedure can hit the upper bound of the prior distribution. So in this case, one might find results without economic sense. This observation gives rise to an augmented estimation procedure in order to avoid this situation, especially, one can propose a narrower restriction on the upper bound of prior distribution on output elasticities, define shortly. Second, the first estimate of oil's output elasticity when  $\theta$  is calibrated, suggests an estimated  $\hat{\alpha}_e$  similar to its first prior value. According to the first identification analysis around  $\alpha_e = 0.015$ , the identification strength of this parameter, advocates a weak identification. This intuition is confirmed using Figure 2, the prior and the posterior distribution match, therefore the identification issue previously raised is confirmed since this parameter is only explained by its prior distribution.



Table 4: Prior and Posterior Distribution of Structural Parameters

Parameter	Prior distribution	Posterior distribution				
		Mode	Mean	10%	90%	
<i>θ</i> estimated						
Capital elasticity	$\alpha_k$ IGamma(0.1,2)	0.3728	0.3599	0.3380	0.3822	
Labor elasticity	$\alpha_\ell$ IGamma(0.4,2)	0.6424	0.6411	0.6111	0.6745	
Oil elasticity	$\alpha_e$ IGamma(0.6,2)	0.1234	0.1254	0.1051	0.1460	
Inverse Frisch elasticity	$\phi$ IGamma(1.17,0.5)	0.6209	0.6308	0.4736	0.8019	
Taylor rule response to inflation	$\phi_\pi$ Normal(1.2,0.1)	1.2235	1.2253	1.0686	1.3558	
Taylor rule response to output	$\phi_y$ Normal(0.5,0.1)	0.8020	0.7882	0.6884	0.8876	
Calvo price parameter	$\theta$ Beta(0.5,0.1)	0.9812	0.9812	0.9380	0.9883	
<i>θ</i> calibrated						
Capital elasticity	$\alpha_k$ IGamma(0.2,2)	0.3918	0.3809	0.3624	0.3989	
Labor elasticity	$\alpha_\ell$ IGamma(0.4,2)	0.5947	0.5966	0.5622	0.6305	
Oil elasticity	$\alpha_e$ IGamma(0.5,2)	0.1132	0.1177	0.0915	0.1434	
Inverse Frisch elasticity	$\phi$ IGamma(1.17,0.5)	1.2562	1.2625	0.9073	1.6069	
Taylor rule response to inflation	$\phi_\pi$ Normal(1.2,0.1)	1.5236	1.5307	1.3883	1.6722	
Taylor rule response to output	$\phi_y$ Normal(0.5,0.1)	0.0265	0.0214	0.0001	0.0402	

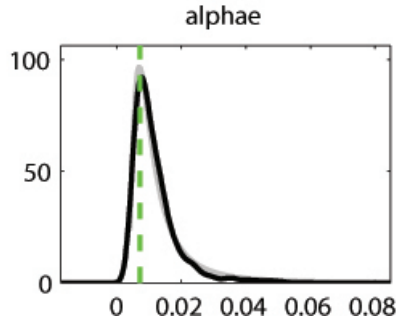


Figure 2: Posterior (solid black line) and prior (solid grey line) distribution of  $\alpha_e$ . The dashed green line stands for the posterior empirical mode

#### 4.3.1 Restricted Estimation

Table 7 refers to the upper bound restriction limits for the first three estimations of Table 6 with respect to their own stars superscripts.

As shown in Table 8, once we restrict the model, the log-marginal density drops to a lower level. We can conclude that the model where  $\theta$  is estimated and where the first prior parameter of  $\alpha_e$  is equal to 0.6, corresponding to the fourth column of the Table 6, outperforms these latest.

Table 5: Prior and Posterior Distribution of Shock Parameters

Parameter		Prior distribution	Posterior distribution			
			Mode	Mean	10%	90%
$\theta$ estimated						
<i>Autoregressive parameters</i>						
Technology	$\rho_a$	Beta(0.5,0.2)	0.8619	0.8481	0.7960	0.8999
Real oil price	$\rho_{se}$	Beta(0.5,0.2)	0.5761	0.5611	0.4629	0.6669
Real capital price	$\rho_{sk}$	Beta(0.5,0.2)	0.7210	0.7080	0.6647	0.7524
Price markup1	$\rho_p$	Beta(0.5,0.2)	0.9418	0.9283	0.8955	0.9640
Price markup2	$\nu_p$	Beta(0.5,0.2)	0.9796	0.9760	0.9610	0.9913
Government	$\rho_g$	Beta(0.5,0.2)	0.9058	0.8995	0.8712	0.9258
Tech. in Gov.	$\rho_{ag}$	Beta(0.5,0.2)	0.6904	0.6127	0.3549	0.9472
Monetary	$\rho_i$	Beta(0.5,0.2)	0.9399	0.9308	0.9035	0.9581
<i>Standard deviations</i>						
Technology	$\sigma_a$	IGamma(1,2)	0.4361	0.4435	0.3901	0.4942
Real oil price	$\sigma_{se}$	IGamma(1,2)	2.0000	1.9373	1.8652	2.000
Real capital price	$\sigma_{sk}$	IGamma(1,2)	0.7740	0.7675	0.6379	0.8781
Price markup	$\sigma_p$	IGamma(1,2)	0.1814	0.1854	0.1615	0.2094
Government	$\sigma_g$	IGamma(1,2)	2.0000	1.7921	1.5508	1.9998
Monetary	$\sigma_i$	IGamma(1,2)	0.5410	0.4566	0.3859	0.5205
$\theta$ calibrated						
<i>Autoregressive parameters</i>						
Technology	$\rho_a$	Beta(0.5,0.2)	0.9605	0.9401	0.9033	0.9774
Real oil price	$\rho_{se}$	Beta(0.5,0.2)	0.9934	0.9872	0.9754	0.9977
Real capital price	$\rho_{sk}$	Beta(0.5,0.2)	0.8940	0.8924	0.8483	0.9314
Price markup1	$\rho_p$	Beta(0.5,0.2)	0.9839	0.9621	0.9299	0.9971
Price markup2	$\nu_p$	Beta(0.5,0.2)	0.1652	0.1711	0.0593	0.2758
Government	$\rho_g$	Beta(0.5,0.2)	0.9373	0.9312	0.9061	0.9560
Tech. in Gov.	$\rho_{ag}$	Beta(0.5,0.2)	0.7129	0.6589	0.3808	0.9541
Monetary	$\rho_i$	Beta(0.5,0.2)	0.1914	0.2104	0.1249	0.2856
<i>Standard deviations</i>						
Technology	$\sigma_a$	IGamma(1,2)	0.4538	0.4542	0.3981	0.5078
Real oil price	$\sigma_{se}$	IGamma(1,2)	2.0000	1.9475	1.8842	2.000
Real capital price	$\sigma_{sk}$	IGamma(1,2)	0.5459	0.5750	0.4722	0.6714
Price markup	$\sigma_p$	IGamma(1,2)	0.4235	0.4645	0.2868	0.6602
Government	$\sigma_g$	IGamma(1,2)	2.0000	1.8359	1.6425	2.000
Monetary	$\sigma_i$	IGamma(1,2)	0.4778	0.4769	0.4062	0.54555

Table 6: Summary of estimation results

$\alpha_e$ prior value		Log marg. density		$\hat{\alpha}_e$		Sum of $\alpha_i$	
$\theta$ estim.	$\theta$ calib.	$\theta$ estim.	$\theta$ calib.	$\theta$ estim.	$\theta$ calib.	$\theta$ estim.	$\theta$ calib.
0.015*	0.015	-567.16	-570.93	0.1178	0.0117	1.3648	1.1077
0.3**	0.5	-567.65	-589.99	0.085	0.1177	1.3622	1.0952
0.5***	0.2	-579.18	-591.80	0.1138	0.0533	1.2002	1.0913
0.6	0.1	-586.98	-592.99	0.1254	0.0356	1.1264	1.1188
0.1	0.6	-592.84	-593.28	0.082	0.1304	1.1168	1.0966
0.4	0.3	-596.08	-596.51	0.1090	0.0625	1.0226	1.1023
0.2	0.4	-596.92	-600.66	0.0839	0.1055	1.1322	1.0915

Table 7: Prior's upper bound restriction on output elasticities parameters

<i>Elasticity</i>	0.015*	0.3**	0.5***
$\alpha_e$	0.4	0.4	0.3
$\alpha_k$	0.3	0.35	0.3
$\alpha_\ell$	0.7	0.75	0.7

Table 8: Restricted estimation results

$\alpha_e$ prior value	Log marg. density	$\hat{\alpha}_e$	Sum of $\alpha_i$
0.015*	-591.24	0.0798	1.0666
0.3**	-594.37	0.0727	1.0681
0.5***	-620.28	0.1458	1.1341

#### 4.4 Estimation Analysis

Concerning behavioral parameters, summarized in Table 4, several key informations are worth making. Firstly, we find evidence for increasing return to scale technology i.e. capital's output elasticity is on average 0.37, labor's output elasticity is 0.62 and oil's output elasticity is 0.12. Leading to an average sum of 1.1. Likewise, these findings break the oil's output elasticity of 0.015 adopted by Blanchard and Galí [BG09]. Furthermore, the key feature of our findings is that oil's output elasticity at 0.12 is robust regarding both estimates. Secondly, both monetary policy response coefficients ( $\phi_\pi$  and  $\phi_y$ ) are significantly different. On the one hand, when  $\theta$  is estimated, we find a lower core inflation coefficient ( $\phi_\pi$ ) than Taylor [Tay93] originally stated, whereas the response to output gap ( $\phi_y$ ) is higher. These results are in line with findings in Rudebusch [Rud06]. On the other hand, for the case where  $\theta$  is calibrated, we find a response to core inflation close to Taylor [Tay93] estimation, nevertheless the response coefficient to output gap is nearly non-significant. One can analyze this difference due to the inference of  $\theta \approx 0.96$  in the

first case, meaning a high rigidity on core CPI price re-setting. Hence core inflation have low probability to reach a high level compares to the calibrated case ( $\theta = 0.65$ ). Since inflation is highly controlled by the Calvo parameter in the first case, the Central Bank has no reason to overreact to inflation, in contrast to its counterpart. Thirdly, findings on the inverse of Frisch elasticity are truly different. This is not surprising, since there is no consensus in the literature, see Browning, Hansen and Heckman [BH99] among others. The first estimate leads to a Frisch elasticity equal to 1.585 ( $\approx 1/0.6308$ ) and the second estimate is 0.79 ( $\approx 1/1.2625$ ). It is worth emphasizing that Smets and Wouters [SW07] finding of this parameter is 0.52 ( $\approx 1/1.92$ ), close to what we find for the case where  $\theta$  is calibrated to the Smets and Wouters [SW07] Calvo parameter posterior mean, i.e. 0.65.

Turning to the estimates of the stochastic processes, Table 5, one can extract important observations. Concerning standard deviation estimates, most of the variance is driven by the demand shock ( $\sigma_g$ ) and real price of oil ( $\sigma_{se}$ ) in both estimates<sup>23</sup>. The high standard deviation for the price of oil can be interpreted as being the resulting of a financial asset trade in a volatile stock market. For the case  $\theta$  calibrated, we find a high persistency on  $AR(1)$  coefficients for government spending (0.93), price markup (0.96), technology (0.94) and the real price of oil (0.98), whereas for the other case, only the first two, together with the monetary policy (0.9308) have a high autoregressive parameter. The stylized fact from the previous observation is the strong difference between monetary policy autoregressive parameters over our estimates, indeed, on the one hand we have a high persistency whereas on the other hand we have a value close to the one find by Smets and Wouters [SW07]. This can be explain by the fact that when  $\theta$  has a high level, hence the core CPI is low, then monetary authorities will set its interest rate by a persistent impulse, i.e. a shock with a high memory.

## 5 Simulations and Results

### 5.1 The effects of an oil shock

There are six sources of potential exogenous shocks in our economy: the real price of oil, the real price of capital, the government expenditure, the monetary policy, the price markup and the technology. Having estimated the model, we study the impulse response functions (thereafter IRFs), using the mean of the posterior estimation. We will concentrate on the real price of oil shock.

We make the analysis for both estimation protocols, namely the situation where  $\theta$  is estimated along with the other parameters, and its counterpart where  $\theta$  is calibrated.

Let us begin with the case where  $\theta$  is estimated. The estimated value of  $\theta$  being 0.96, this implies a high stickiness level in prices. The corresponding IRFs are represented in Figure 3. As expected, an increase of the price of oil generates a immediate decrease of oil consumption but a limited reaction on domestic prices (which are too viscous to react instantaneously). Consequently, intermediate firms do not reduce their production, but prefer substitute capital and labor to oil. Real wages, therefore, increase as well as the rental rate of capital. Because domestic consumption is in any case affected and the rental rate of capital is high, the representative household prefers to invest more than to consume. However, despite the increase on domestic production, GDP is affected negatively because

<sup>23</sup>Note that standard deviations describe in Table 5 are in percentage, meaning that if  $\sigma = 1$ , then 1 stands for 1%

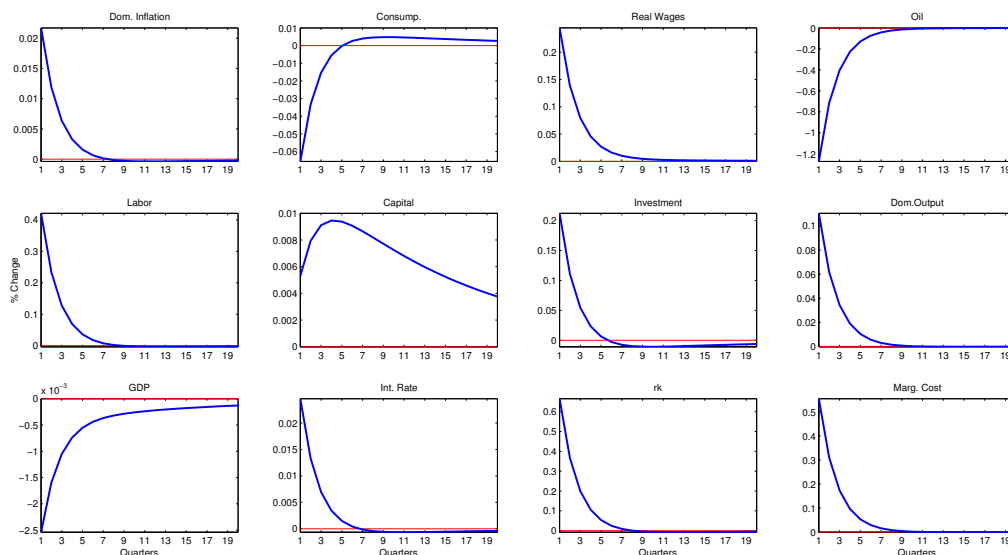


Figure 3: Response to one standard deviation shock on real price of oil. Case:  $\theta$  estimated

of the growing cost of importing oil. Finally, the small inflationary pressure induces a weak monetary reaction of the Central Bank.

Let us now study the case where  $\theta$  is calibrated at 0.65. Figure 4 presents the corresponding IRFs. Now, a larger fraction of firms can reset optimally their prices instantaneously, so that the inflationary effect of an oil shock is more pronounced. Therefore, the shock provokes a large decrease in consumption. The latter hits the domestic producers, who therefore reduce their production. Due to the lower demand, no substitution effect takes place, so that firms decrease their demand for capital, labor and oil. This reduced inputs' demand depressed both real wages and the rental rate of capital. Consequently investment decreases. The reduction of production lowers also the marginal cost of inputs, which provokes a deflation. In an attempt to re-launch the economy, the Central Bank then decreases its interest rate. GDP is also more negatively affected in this case than in the previous simulations, due to the reduction of domestic output. Moreover, the negative impact of the oil shock is much more persistent in the calibrated case. This comes from the fact that the posterior value of the autoregressive shock drops from 0.9872 to 0.56. Note also that when  $\theta$  is calibrated, the posterior value of the Taylor rule response to output parameter,  $\phi_y$ , drops from 0.78 to 0.02, meaning that the Central Bank practically does not react to the GDP reduction.

The overall conclusion is that, contrary to what intuition would perhaps suggest, a higher price flexibility does not imply that the economy is more immune to an oil shock.

## 5.2 Reducing the oil dependency ?

Let us now analyse the reaction to an oil shock of an economy that has decreased its dependence with respect to oil. To capture this feature, we reduce the oil's output elasticity from 0.11 (or 0.12) to 0.05. Figure 5 and Figure 6 present the IRFs for a 1% increase in the real price of oil in the  $\theta$ -calibrated case and the  $\theta$ -estimated situation. In both cases, the impact of an oil shock is significantly reduced by the smaller dependency of the economy

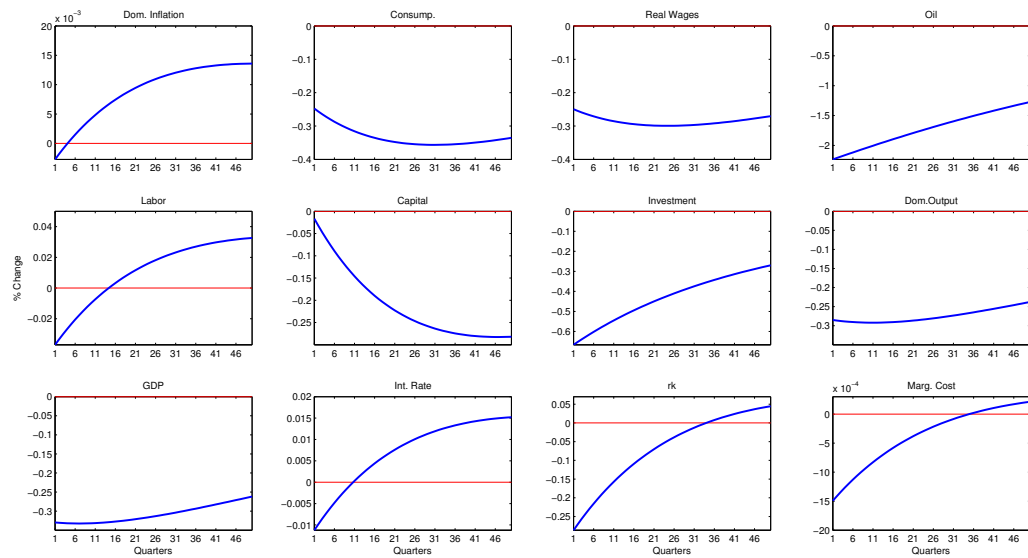


Figure 4: Response to one standard deviation shock on real price of oil. Case:  $\theta$  calibrated

with respect to oil: domestic inflation, reduction of real wages and of GDP are attenuated by the reduction of  $\alpha_e$ . This is quite logical and should not come as a surprise. How intuitive this finding might be, it sheds some important light on two issues: Firstly, this provides a good explanatory candidate for the muted impact of the third oil shock which lasted from 2000 to 2007. Our empirical estimation of  $\alpha_e$  is based on the whole period 1984-2007, and arises therefore as a time average. This does not preclude the true elasticity of oil from having decreased across time during this very period, as it is convincingly suggested by Figure 3 above. Therefore, one reason why we did not observe the stagflationary effect we could have expected for during the early 2000s' may have been the successful reduction of the US economy's dependency towards oil consumption. The second insight is forward-looking. As we have seen, more flexibility does not mean a better immunization against an oil shock, at least if flexibility refers to domestic price flexibility. Reducing the output elasticity of oil, by means of increasing the oil efficiency of production, turns out to be a much more promising policy recommendation.

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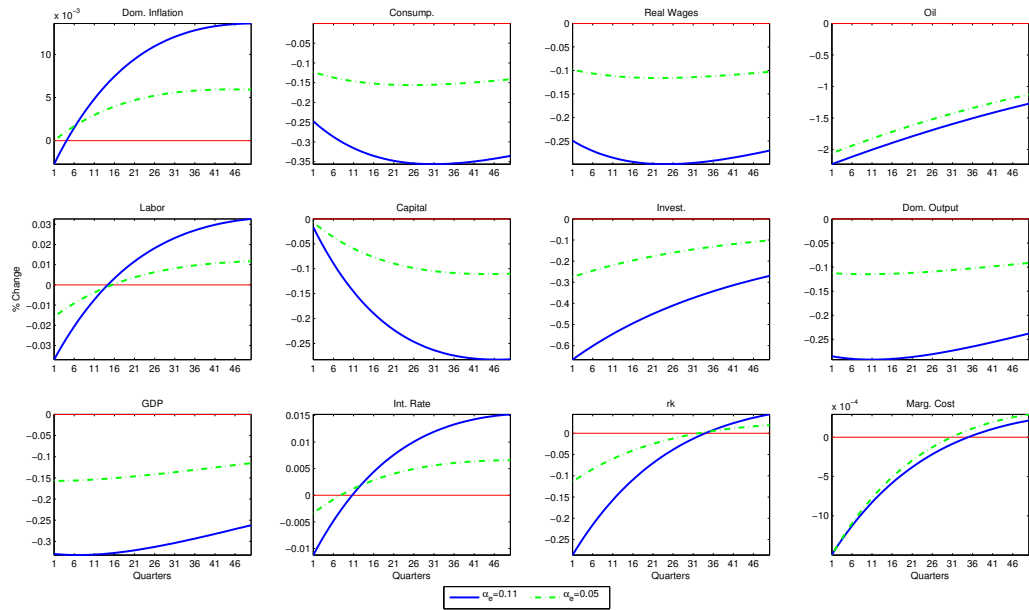


Figure 5: The ecological transition effect. Case:  $\theta$  calibrated

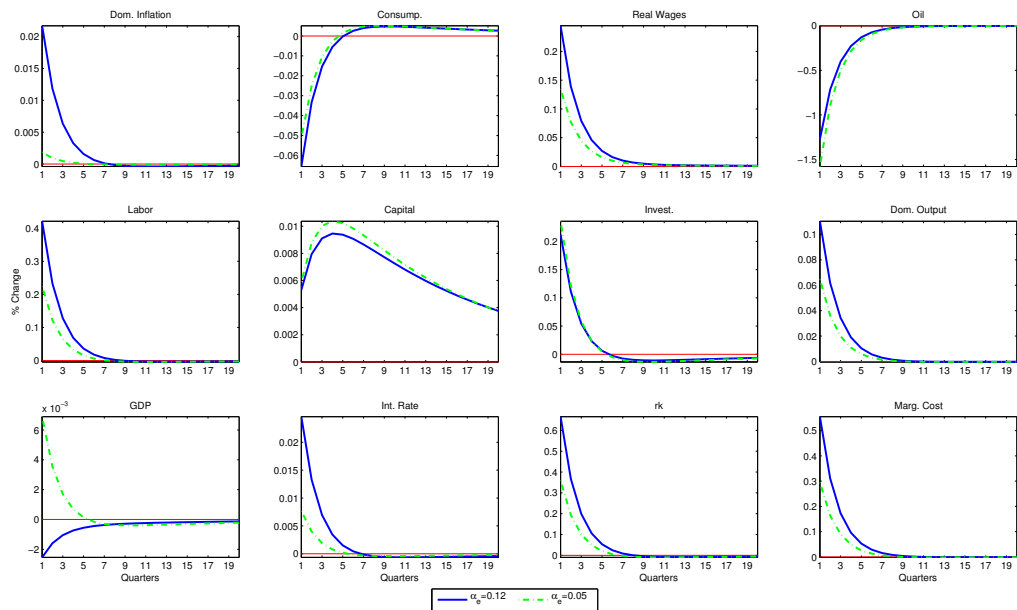


Figure 6: The ecological transition effect. Case:  $\theta$  estimated

## References

- [And10] M. Andrieu, “A note on identification patterns in DSGE models”, Working Paper Series 1235, European Central Bank, August 2010.
- [AV14] R. Ayres and V. Voudouris, “The economic growth enigma: Capital, labour and useful energy?”, *Energy Policy* **64** (2014), 16–28.
- [Ban12] World Bank, *Annual report*, Tech. report, World Bank, 2012.
- [BC90] J.M. Bonnisseau and B. Cornet, “Existence of Marginal Cost Pricing Equilibria in Economies with Several Nonconvex Firms”, *Econometrica* **58** (1990), no. 3, 661–82.
- [BG98] S. Brooks and A. Gelman, “General Methods for Monitoring Convergence of Iterative Simulations”, *Journal of Computational and Graphical Statistics* **7** (1998), no. 4, 434–455.
- [BG09] O. Blanchard and J. Galí, “the macroeconomic effects of oil price shocks: Why are the 2000s so different from the 1970s?”, In J. Galí and M. Gertler (eds.), *International Dimensions of Monetary Policy*. University of Chicago Press (2009).
- [BH99] L.P. Hansen, M. Browning, and J. Heckman, “Micro Data and General Equilibrium Models”, Discussion Papers 99-10, University of Copenhagen. Department of Economics, June 1999.
- [BL98] E. Canetti, A. S. Blinder, and D. Lebow, “Asking about Prices: A new Approach to understand price stickiness”, Russel stage Foundation Publications, 1998.
- [BMC85] P. Beato and A. Mas-Colell, “On marginal cost pricing with given tax-subsidy rules”, *Journal of Economic Theory* **37** (1985), no. 2, 356–365.
- [BP13] BP, *Bp statistical review of world energy*, Tech. report, British Petroleum Company, 2013.
- [BR13] O. Blanchard and M. Riggi, “Why are the 2000s so different from the 1970s? A structural interpretation of changes in the macroeconomic effects of oil prices”, *Journal of the European Economic Association* **11** (2013), no. 5, 1032–1052.
- [BW14] P.H. Bono, Ch. Chapelle Bonnet, O. and E. Wasmer, “Does housing capital contribute to inequality? A comment on Thomas Piketty’s *Capital in the 21st Century*”, Sciences-Po Discussion paper 2014-07 (2014).
- [Cal83] G. Calvo, “Staggered prices in a utility-maximizing framework”, *Journal of Monetary Economics* **12** (1983), no. 3, 383–398.
- [Cor88] B. Cornet, “General equilibrium theory and increasing returns : Presentation”, *Journal of Mathematical Economics* **17** (1988), no. 2-3, 103–118.
- [CS09] F. Canova and L. Sala, *Back to square one: Identification issues in DSGE models*, *Journal of Monetary Economics* **56** (2009), no. 4, 431–449.



- [DD88a] P. Dehez and J. Dreze, “*Competitive equilibria with quantity-taking producers and increasing returns to scale*”, *Journal of Mathematical Economics* **17** (1988), no. 2-3, 209–230.
- [DD88b] ———, “*Distributive production sets and equilibria with increasing returns*”, *Journal of Mathematical Economics* **17** (1988), no. 2-3, 231–248.
- [Dyn11] Dynare, “*Dynare: Reference Manual Version 4*”, Dynare Working Papers 1, CEPREMAP, April 2011.
- [EG52] W. J. Eiteman and G. E. Guthrie, “*The Shape of the average cost curve*”, *American Economic Review* **42(5)** (1952), 832–8.
- [EIA12] EIA, *Annual energy outlook*, Tech. report, Energy Information Administration, 2012.
- [FF12] A. Bénassy-Quéré Fouré, J. and L. Fontagné, “*The Great Shift: Macroeconomic projections for the world economy at the 2050 horizon*”, Working Papers 2012-03, CEPII research center, Feb 2012.
- [Gal08] J. Galí, “*Introduction to Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*”, “Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework”, Introductory Chapters, Princeton University Press, December 2008.
- [Gir01] G. Giraud, “*An Algebraic Index Theorem for Non-smooth Economies*”, *Journal of Mathematical Economics* **36** (2001), no. 4, 215–283.
- [HR73] L. Hurwicz and S. Reiter, “*On the Boundedness of the Feasible Set Without Convexity Assumptions*”, *International Economic Review* **14 (3)** (1973), 580–86.
- [Isk10] N. Iskrev, “*Local identification in DSGE models*”, *Journal of Monetary Economics* **57** (2010), no. 2, 189–202.
- [K11] R. Kümmel, “*The Second Law of Economics - Energy, Entropy, and the Origins of Wealth*”, Springer Verlag, 2011.
- [KG14] Z. Kahraman and G. Giraud, “*How Dependent is Growth from Primary Energy ? The Dependency ratio of Energy in 33 Countries (1970-2011)*”, Working Paper Chair Energy and Prosperity (2014).
- [Khr12] V. Khramov, “*Assessing DSGE Models with Capital Accumulation and Indeterminacy*”, IMF Working Papers 12/83, International Monetary Fund, Mar 2012.
- [Kil08] L. Kilian, “*The Economic Effects of Energy Price Shocks*”, *Journal of Economic Literature* **46** (2008), no. 4, 871–909.
- [KL08] J. Schmid-R. Ayres Kümmel, R. and D. Lindenberger, “*Cost Shares, Output Elasticities, and Substitutability Constraints*”, EWI Working Paper, No 08.02 (2008).

- [KL10] R. Ayres Kümmel, R. and D. Lindenberger, “*Thermodynamic laws, economic methods and the productive power of energy*”, *Journal of Non-equilibrium Thermodynamics* **35** (2010), 145–179.
- [KS03] S. Kim-E. Schaumburg Kim, J. and C. Sims, “*Calculating and using second order accurate solutions of discrete time dynamic equilibrium models*”, Finance and Economics Discussion Series 2003-61, Board of Governors of the Federal Reserve System (U.S.), 2003.
- [LK11] D. Lindenberger and R. Kümmel, “*Energy and the state of nations*”, *Energy* (2011), 6010 – 6018.
- [OEC13] OECD, *Oecd data lab*, Tech. report, OECD, 2013.
- [Qui92] M. Quinzii, “*Increasing Returns and Efficiency*”, Oxford University Press, 1992.
- [Rat08] M. Ratto, “*Analysing DSGE Models with Global Sensitivity Analysis*”, *Computational Economics* **31** (2008), no. 2, 115–139 (English).
- [RP10] M. Ratto and A. Pagano, “*Using recursive algorithms for the efficient identification of smoothing spline ANOVA models*”, *AStA Advances in Statistical Analysis* **94** (2010), no. 4, 367–388.
- [Rud06] G. Rudebusch, “*Monetary Policy Inertia: Fact or Fiction?*”, *International Journal of Central Bankin* **2** (2006), no. 4.
- [S08] M. Sánchez, “*Oil shocks and endogenous markups - results from an estimated euro area DSGE model*”, Working Paper Series 860, European Central Bank, Jan 2008.
- [Sam66] P.A. Samuelson, “*A Summing-up*”, *Quarterly Journal of Economics* **80(4)** (1966), 568–583.
- [Sam79] P.A. Samuelson, “*Paul Douglas’s Measurement of Production Functions and Marginal Productivities*”, *Journal of Political Economy* **87** (1979), 923–939.
- [Sim11] E. Sims, “*Permanent and Transitory Technology Shocks and the Behavior of Hours: A Challenge for DSGE Models*”, Seminar in series: Econ 310 Macroeconomics. Stanford University (2011).
- [SR77] V. Spiric L. E Rehn Schilling, W. and K.H Robrock, “*An elastic relaxation due to single self-interstitial atoms in electron-irradiated Al*”, *Phys. Rev. B* **15** (1977), 672–679.
- [SW07] F. Smets and R. Wouters, “*Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach*”, *American Economic Review* **97** (2007), no. 3, 586–606.
- [Tay93] J. Taylor, “*Discretion versus policy rules in practice*”, *Carnegie-Rochester Conference Series on Public Policy* **39** (1993), no. 1, 195–214.
- [Woo03] M. Woodford, “*Interest and prices*”, Princeton Univ. Press, Princeton, NJ [u.a.], 2003.