Detrended Equilibrium Conditions:

Propensity to consume of young, middle-aged workers and retirees:

$$\frac{1}{\chi_t^y} = 1 + \beta^\sigma (\Omega_{t+1}^{my} R_t)^{\sigma-1} \frac{1}{\chi_{t+1}^y} \quad (1)$$

$$\frac{1}{\chi_t^m} = 1 + \beta^\sigma (\Omega_{t+1}^{rm} R_t)^{\sigma-1} \frac{1}{\chi_{t+1}^m} \quad (2)$$

$$\frac{1}{\chi_t^r} = 1 + s_{t+1} \beta^\sigma R_t^{\sigma-1} \frac{1}{\chi_{t+1}^r} \quad (3)$$

where Ω_t^{rm} and Ω_t^{my} equal:

$$\Omega_t^{my} = p_t^y + (1 - p_t^y)(\frac{\chi_t^y}{\chi_t^m})^{\frac{1}{\sigma-1}} \quad (4)$$

$$\Omega_t^{rm} = p_t^m + (1 - p_t^m)(\frac{\chi_t^m}{\chi_t^r})^{\frac{1}{\sigma-1}} \quad (5)$$

Aggregate human capital:

$$\bar{H}_t^y = W_t^y + \frac{p_{t+1}^y}{\Omega_{t+1}^{my} R_t} \bar{H}_{t+1}^y + (1 - \frac{p_{t+1}^y}{\Omega_{t+1}^{my}}) \frac{\bar{H}_{t+1}^m}{R_t} \quad (6)$$

$$\bar{H}_t^m = W_t^m + \frac{p_{t+1}^m}{\Omega_{t+1}^{rm} R_t} \bar{H}_{t+1}^m \quad (7)$$

Aggregate consumption levels:

$$\bar{C}_t^y = \chi_t^y \left(\frac{R_{t-1} \bar{A}_{t-1}^y}{n_t} + \bar{H}_t^y \right) \quad (8)$$

$$\bar{C}_t^m = \chi_t^m \left(R_{t-1} \bar{A}_{t-1}^m \frac{N_{t-1}^m}{N_t^m} + \bar{H}_t^m \right) \quad (9)$$

$$\bar{C}_t^r = \chi_t^r \left(R_{t-1} \bar{A}_{t-1}^r \frac{N_{t-1}^r}{N_t^r} \right) \quad (10)$$

where $\frac{N_t^y}{N_{t-1}^y} = n_t = \bar{n}_t + p_t^y$.

Assets:

$$\bar{A}_t^y = p_{t+1}^y \left(\frac{R_{t-1}\bar{A}_{t-1}^y}{n_t} + W_t^y - \bar{C}_t^y \right) \tag{11}$$

$$\bar{A}_t^m = p_{t+1}^m \left(R_{t-1}\bar{A}_{t-1}^m \frac{N_{t-1}^m}{N_t^m} + W_t^m - \bar{C}_t^m \right) + (\frac{1-p_{t+1}^y}{p_{t+1}^y})\bar{A}_t^y MY_t^{-1} \tag{12}$$

$$\bar{A}_t^r = (1-\chi_t^r)R_{t-1}A_{t-1}^r \frac{N_{t-1}^r}{N_t^r} + \frac{(1-p_{t+1}^m)}{p_{t+1}^m} \left(\bar{A}_t^m - (\frac{1-p_{t+1}^y}{p_{t+1}^y})\bar{A}_t^y MY_t^{-1} \right) RM_t^{-1} \tag{13}$$

Production side:

$$W_t^y = (1-\alpha)\bar{Y}_t \gamma^{\frac{1}{\theta}} (\bar{L}_t)^{\frac{1-\theta}{\theta}} \tag{14}$$

$$W_t^m = (1-\alpha)\bar{Y}_t (1-\gamma)^{\frac{1}{\theta}} (\bar{L}_t)^{\frac{1-\theta}{\theta}} (MY_t)^{\frac{\theta-1}{\theta}} \tag{15}$$

$$R_t \bar{K}_{t-1} = \alpha n_t \bar{Y}_t \tag{16}$$

where:

$$\bar{L}_t = (\gamma^{\frac{1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} MY_t^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}} \tag{17}$$

$$\bar{Y}_t = \frac{Z_t}{n_t^\alpha} \bar{K}_{t-1}^\alpha \bar{L}_t^{1-\alpha} \tag{18}$$

Market clearing:

$$\bar{Y}_t = \bar{C}_t + \bar{K}_t - (1-\delta)\bar{K}_{t-1} \tag{19}$$

$$\bar{C}_t = \bar{C}_t^y + \bar{C}_t^m + \bar{C}_t^r \tag{20}$$

$$\bar{A}_t = \bar{A}_t^y + \bar{A}_t^m + \bar{A}_t^r \tag{21}$$

$$\bar{A}_t = \bar{K}_t \tag{22}$$

Population dynamics:

The laws of motion for MY_t and RM_t:

$$MY_t = \frac{(1-p_t^y)}{n_t} + \frac{p_t^m}{n_t} MY_{t-1} \tag{23}$$

(24)

$$RM_t = \frac{(1-p^m) + s_t RM_{t-1}}{((1-p_t^y)/MY_{t-1}) + p_t^m}$$

Growth rates:

$$\frac{N_t^y}{N_{t-1}^y} = n_t \tag{25}$$

(26)

$$\frac{N_t^m}{N_{t-1}^m} = \frac{1-p_t^y}{MY_{t-1}} + p_t^m$$

(27)

$$\frac{N_t^r}{N_{t-1}^r} = \frac{1-p_t^m}{RM_{t-1}} + s_t$$

Steady-state equations:

$$MY = \frac{1-p^y}{n-p^m} \tag{28}$$

$$RM = \frac{1-p^m}{n-s} \tag{29}$$

$$\chi^y = 1 - \beta^\sigma(\Omega^{my}R)^{\sigma-1} \tag{30}$$

$$\chi^m = 1 - \beta^\sigma(\Omega^{rm}R)^{\sigma-1} \tag{31}$$

$$\chi^r = 1 - s\beta^\sigma R^{\sigma-1} \tag{32}$$

$$\Omega^{my} = p^y + (1-p^y)(\frac{\chi^y}{\chi^m})^{\frac{1}{\sigma-1}} \tag{33}$$

$$\Omega^{rm} = p^m + (1-p^m)(\frac{\chi^m}{\chi^r})^{\frac{1}{\sigma-1}} \tag{34}$$

$$\bar{L} = (\gamma^{\frac{1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} MY^{\frac{\theta-1}{\theta}})^{\frac{\theta}{\theta-1}} \tag{35}$$

$$\bar{K} = (\frac{R}{\alpha})^{\frac{1}{\alpha-1}} n\bar{L} \tag{36}$$

$$\bar{Y} = n^{-\alpha}\bar{K}^\alpha \bar{L}^{1-\alpha} \tag{37}$$

$$W^y = (1-\alpha)\bar{Y}\gamma^{\frac{1}{\theta}}(\bar{L})^{\frac{1-\theta}{\theta}} \tag{38}$$

$$W^m = (1-\alpha)\bar{Y}(1-\gamma)^{\frac{1}{\theta}}(\bar{L})^{\frac{1-\theta}{\theta}}(MY)^{\frac{\theta-1}{\theta}} \tag{39}$$

$$\bar{H}^y = \frac{W^y}{1-\frac{p^y}{\Omega^{my}R}} + \frac{(\Omega^{my}-p^y)\Omega^{rm}RW^m}{(\Omega^{my}R-p^y)(\Omega^{rm}R-p^m)} \tag{40}$$

$$\bar{H}^m = \frac{W^m}{(1-\frac{p^m}{\Omega^{rm}R})} \tag{41}$$

(42)

$$\bar{A}^y = \frac{W^y - \chi^y \bar{H}^y}{1 - (1 - \chi^y)\frac{p^y R}{n}} \tag{43}$$

$$\bar{A}^m = \frac{p^m (W^m - \chi^m \bar{H}^m) + \frac{1-p^y}{p^y} \bar{A}^y M Y^{-1}}{1 - (1 - \chi^m)\frac{p^m R}{n}} \tag{44}$$

$$\bar{A}^r = \frac{\frac{1-p^m}{p^m} (\bar{A}^m - \frac{1-p^y}{p^y} \bar{A}^y M Y^{-1})}{1 - (1 - \chi^r)\frac{R}{n}} R M^{-1} \tag{45}$$

$$\bar{C}^y = \chi^y \left(\frac{R \bar{A}^y}{n} + \bar{H}^y \right) \tag{46}$$

$$\bar{C}^m = \chi^m \left(\frac{R \bar{A}^m}{n} + \bar{H}^m \right) \tag{47}$$

$$\bar{C}^r = \chi^r \left(\frac{R \bar{A}^r}{n} \right) \tag{48}$$

$$\bar{C} = \bar{Y} - \delta \bar{K} \tag{49}$$

$$\bar{C} = \bar{C}^y + \bar{C}^m + \bar{C}^r \tag{50}$$

$$\bar{A} = \bar{A}^y + \bar{A}^m + \bar{A}^r = \bar{K} \tag{51}$$