Note that

$$G_t(\bar{\omega}_{t+1}, \sigma_t) = \int_0^{\bar{\omega}_{t+1}} \omega \, dF_t(\omega) = \Phi\left(\frac{\ln(\omega_{t+1}) + 0.5 \times \sigma_t^2}{\sigma_t} - \sigma_t\right) \tag{1}$$

where Φ denotes cumulative standard density function,

using Taylor linearization

$$G_t(\bar{\omega}_{t+1}, \sigma_t) = G(\bar{\omega}, \sigma) + G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma) \tag{2}$$

reordering

$$G_t(\bar{\omega}_{t+1}, \sigma_t) - G(\bar{\omega}, \sigma) = G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma) \tag{3}$$

where $G_{\bar{\omega}}$ y G_{σ} involve partial derivatives of Φ, so probability densities will appear

dividing by $G(\bar{\omega}, \sigma)$

$$\frac{G_t(\bar{\omega}_{t+1}, \sigma_t) - G(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} = \frac{G_{\bar{\omega}}(\bar{\omega}, \sigma) \times (\bar{\omega}_{t+1} - \bar{\omega}) + G_{\sigma}(\bar{\omega}, \sigma) \times (\sigma_t - \sigma)}{G(\bar{\omega}, \sigma)} \tag{4}$$

then

$$\hat{G}_t = \bar{\omega} \times \frac{G_{\bar{\omega}}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \hat{\bar{\omega}}_{t+1} + \sigma \times \frac{G_{\sigma}(\bar{\omega}, \sigma)}{G(\bar{\omega}, \sigma)} \times \hat{\sigma}_t \tag{5}$$

where $\hat{x}_t = (x_t - x)/x$.

with respect to $\Gamma_t(\bar{\omega}_{t+1}, \sigma_t)$ we have to

$$\Gamma_t(\bar{\omega}_{t+1}, \sigma_t) = \bar{\omega}_{t+1}[1 - F_t(\bar{\omega}_{t+1})] + G_t(\bar{\omega}_{t+1}, \sigma_t) \tag{6}$$

where

$$F_t(\bar{\omega}_{t+1}, \sigma_t) = \Phi\left(\frac{\ln(\omega_{t+1}) + 0.5 \times \sigma_t^2}{\sigma_t}\right) \tag{7}$$

and the procedure is similar