

The model used in the Sims (2002) paper is written as

$$\begin{aligned} w(t) &= \frac{1}{3}E_t [W(t) + W(t+1) + W(t+2)] - \alpha(u(t) - u_n) + \nu(t) \\ W(t) &= \frac{1}{3}(w(t) + w(t-1) + w(t-2)) \\ u(t) &= \theta u(t-1) + \gamma W(t) + \mu + \varepsilon(t) \end{aligned}$$

where $\nu(t), \varepsilon(t)$ are exogenous innovations. Now we have to rewrite this system of equations such that it fits into the canonical form

$$\Gamma_0 y(t) = \Gamma_1 y(t-1) + C + \Psi z(t) + \Pi \eta(t).$$

Sims (2002) defines the state vector

$$y(t) = \begin{pmatrix} w(t) \\ w(t-1) \\ W(t) \\ u(t) \\ E_t W(t+1) \end{pmatrix}.$$

Shifting the first equation one period backwards and rearranging yields

$$\frac{1}{3}E_{t-1}W(t) + \frac{1}{3}E_{t-1}W(t+1) = w(t-1) - \frac{1}{3}W(t-1) + \alpha u(t-1) - \alpha u_n - v(t-1).$$

If we define $\eta_1(t) = W(t) - E_{t-1}W(t)$ and $\eta_2(t) = E_t W(t+1) - E_{t-1}W(t+1)$ then follows

$$\frac{1}{3}[W(t) - \eta_1(t)] + \frac{1}{3}[E_t W(t+1) - \eta_2(t)] = w(t-1) - \frac{1}{3}W(t-1) + \alpha u(t-1) - \alpha u_n - v(t-1) \quad (1)$$

$$\Leftrightarrow \underline{\underline{\frac{1}{3}W(t) + \frac{1}{3}E_t W(t+1) = w(t-1) - \frac{1}{3}W(t-1) + \alpha u(t-1) - \alpha u_n - v(t-1) + \frac{1}{3}\eta_1(t) + \frac{1}{3}\eta_2(t)}}$$

From the second and third equation of the model we obtain

$$-\frac{1}{3}w(t) - \frac{1}{3}w(t-1) + W(t) = \frac{1}{3}w(t-2) \quad (2)$$

$$u(t) - \gamma W(t) = \theta u(t-1) + \mu + \varepsilon(t). \quad (3)$$

Finally, we have to add to the system the definitions for the auxiliary variables

$$w(t-1) = w(t-1) \quad (4)$$

$$W(t) = E_{t-1}W(t) + \eta_1(t) \quad (5)$$

Now we can cast eq. (1)-(5) into the canonical form

$$\Gamma_0 y(t) = \Gamma_1 y(t-1) + C + \Psi z(t) + \Pi \eta(t)$$

which gives

$$\begin{aligned} & \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & 1 & 0 & 0 \\ 0 & 0 & -\gamma & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} w(t) \\ w(t-1) \\ W(t) \\ u(t) \\ E_t W(t+1) \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & -\frac{1}{3} & \alpha & 0 \\ 0 & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} w(t-1) \\ w(t-2) \\ W(t-1) \\ u(t-1) \\ E_{t-1}W(t) \end{pmatrix} + \begin{pmatrix} -\alpha \cdot u_n \\ 0 \\ \mu \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} v(t-1) \\ \varepsilon(t) \end{pmatrix} + \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \eta_1(t) \\ \eta_2(t) \end{pmatrix} \end{aligned}$$