Problem 1

$$\max \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{\xi L_t^{1+1/\eta}}{1+1/\eta}\right)\right]$$

subject to

$$\begin{aligned}
Y_t &= Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} \\
Y_t &= C_t + I_t + \frac{\phi}{2}\left(\frac{I_t}{K_{t-1}}-\delta\right)^2 K_{t-1} \\
K_t &= (1-\delta)K_{t-1} + I_t \\
Z_t &= e^{\varepsilon_t} Z_{t-1}^{\rho}
\end{aligned}$$

a)

The F.O.C.s are

$$\begin{aligned}
C_t^{-\sigma} &= \lambda_t \\
\frac{\xi L_t^{1/\eta}}{(1-\alpha)Z_t\left(\frac{K_{t-1}}{L_t}\right)^{\alpha}} &= \lambda_t \\
1+\phi\left(\frac{I_t}{K_{t-1}}-\delta\right) &= \frac{\mu_t}{\lambda_t} \\
\beta\mathbb{E}_t\left[\lambda_{t+1}\left(\alpha Z_{t+1}\left(\frac{K_t}{L_{t+1}}\right)^{\alpha-1}-\frac{\phi}{2}\left(\frac{I_{t+1}}{K_t}-\delta\right)^2+\phi\frac{I_{t+1}}{K_t}\left(\frac{I_{t+1}}{K_t}-\delta\right)\right)+(1-\delta)\mu_{t+1}\right] &= \mu_t
\end{aligned}$$

b)

Define

$$q_t \equiv \frac{\mu_t}{\lambda_t} = 1+\phi\left(\frac{I_t}{K_{t-1}}-\delta\right)$$

We have

$$C_t^{-\sigma}(1-\alpha)Z_t\left(\frac{K_{t-1}}{L_t}\right)^{\alpha} = \xi L_t^{1/\eta} \tag{1}$$

$$\beta\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left((1-\delta)q_{t+1}K_t+\alpha Z_{t+1}K_t^{\alpha}L_{t+1}^{1-\alpha}+\frac{\phi}{2}\left(\frac{I_{t+1}^2}{K_t}-\delta^2 K_t\right)\right)\right] = q_t K_t \tag{2}$$

$$(1-\delta)K_{t-1}+I_t = K_t \tag{3}$$

$$C_t + I_t + \frac{\phi}{2}\left(\frac{I_t}{K_{t-1}}-\delta\right)^2 K_{t-1} = Z_t K_{t-1}^{\alpha}L_t^{1-\alpha} \tag{4}$$

$$Z_t = e^{\varepsilon_t}Z_{t-1}^{\rho} \tag{5}$$