A summary of detrended equilibrium conditions is provided below:

Propensity to consume of young-, middle-aged workers and retirees:

$$\frac{1}{\chi_t^y} = 1 + \beta^\sigma (\Omega_{t+1}^{my} R_t)^{\sigma-1} \frac{1}{\chi_{t+1}^y} \tag{A.40}$$

$$\frac{1}{\chi_t^m} = 1 + \beta^\sigma (\Omega_{t+1}^{rm} R_t)^{\sigma-1} \frac{1}{\chi_{t+1}^m} \tag{A.41}$$

$$\frac{1}{\chi_t^r} = 1 + s_{t+1} \beta^\sigma R_t^{\sigma-1} \frac{1}{\chi_{t+1}^r} \tag{A.42}$$

Ω_t^{rm} and Ω_t^{my} are defined as:

$$\Omega_t^{my} = p_t^y + (1 - p_t^y)(\frac{\chi_t^y}{\chi_t^m})^{\frac{1}{\sigma-1}} \tag{A.43}$$

$$\Omega_t^{rm} = p_t^m + (1 - p_t^m)(\frac{\chi_t^m}{\chi_t^r})^{\frac{1}{\sigma-1}} \tag{A.44}$$

Aggregate human capital of young- and middle-aged workers:

$$\bar{H}_t^y = \frac{\alpha}{1 + \frac{MY_t}{\xi^y}} + \frac{p_{t+1}^y}{\Omega_{t+1}^{my} R_t} \bar{H}_{t+1}^y \frac{g_{t+1}}{n_{t+1}} + (1 - \frac{p_{t+1}^y}{\Omega_{t+1}^{my}}) \frac{\bar{H}_{t+1}^m}{R_t} \frac{g_{t+1}}{n_{t+1}} MY_{t+1}^{-1} \tag{A.45}$$

$$\bar{H}_t^m = \frac{\alpha}{\frac{\xi^y}{MY_t} + 1} + \frac{p_{t+1}^m}{\Omega_{t+1}^{rm} R_t} \bar{H}_{t+1}^m \frac{g_{t+1}}{n_{t+1}^m} \tag{A.46}$$

where I used that $\frac{W_t}{Y_t} = \frac{\alpha}{L_t}$ and $L_t = \xi^y N_t^y + N_t^m$.

Aggregate consumption levels of young-, middle-aged workers and retirees are the following:

$$\bar{C}_t^y = \chi_t^y \left(\frac{R_{t-1} \bar{A}_{t-1}^y}{g_t} + \bar{H}_t^y \right) \tag{A.47}$$

$$\bar{C}_t^m = \chi_t^m \left(\frac{R_{t-1} \bar{A}_{t-1}^m}{g_t} + \bar{H}_t^m \right) \tag{A.48}$$

$$\bar{C}_t^r = \chi_t^r \left(\frac{R_{t-1} \bar{A}_{t-1}^r}{g_t} \right) \tag{A.49}$$

Aggregate consumption is given by:

$$\bar{C}_t = \bar{C}_t^y + \bar{C}_t^m + \bar{C}_t^r = \chi_t^y \left(\frac{R_{t-1} \bar{A}_{t-1}^y}{g_t} + \bar{H}_t^y \right) + \chi_t^m \left(\frac{R_{t-1} \bar{A}_{t-1}^m}{g_t} + \bar{H}_t^m \right) + \chi_t^r \left(\frac{R_{t-1} \bar{A}_{t-1}^r}{g_t} \right) \tag{A.50}$$

Asset market clearing implies:

$$\bar{A}_t = \bar{A}_t^y + \bar{A}_t^m + \bar{A}_t^r = \bar{K}_t \tag{A.51}$$

where:

$$\bar{A}_t^y = p_{t+1}^y \left(\frac{R_{t-1}\bar{A}_{t-1}^y}{g_t} + \frac{\alpha}{1 + \frac{MY_t}{\xi^y}} - \bar{C}_t^y \right) \tag{A.52}$$

$$\bar{A}_t^m = p_{t+1}^m \left(\frac{R_{t-1}\bar{A}_{t-1}^m}{g_t} + \frac{\alpha}{\frac{\xi^y}{MY_t} + 1} - \bar{C}_t^m \right) + (\frac{1 - p_{t+1}^y}{p_{t+1}^y})\bar{A}_t^y \tag{A.53}$$

$$\bar{A}_t^r = (1 - \chi_t^r)\frac{R_{t-1}\bar{A}_{t-1}^r}{g_t} + \frac{(1 - p_{t+1}^m)}{p_{t+1}^m} \left(\bar{A}_t^m - (\frac{1 - p_{t+1}^y}{p_{t+1}^y})\bar{A}_t^y \right) \tag{A.54}$$

Capital stock evolves as follows:

$$\bar{K}_t = 1 - \bar{C}_t + (1 - \delta)\frac{\bar{K}_{t-1}}{g_t} \tag{A.55}$$

and interest rate is determined by:

$$R_t = (1 - \alpha)\frac{g_t}{\bar{K}_{t-1}} + (1 - \delta) \tag{A.56}$$

The growth rate g_t:

$$g_{t+1} = \frac{Y_{t+1}}{Y_t} = (\frac{\bar{K}_t}{\bar{K}_{t-1}}g_t)^{1-\alpha}(\frac{Z_{t+1}}{Z_t})^\alpha(\frac{L_{t+1}}{L_t})^\alpha = (\frac{\bar{K}_t}{\bar{K}_{t-1}}g_t)^{1-\alpha}(1 + z)^\alpha n_t^\alpha(\frac{\xi^y + MY_{t+1}}{\xi^y + MY_t})^\alpha \tag{A.57}$$

Population dynamics:

The laws of motion for MY_t and RM_t:

$$MY_t = \frac{(1 - p_t^y)}{n_t} + \frac{p_t^m}{n_t}MY_{t-1} \tag{A.58}$$

$$RM_t = \frac{1 - p_t^m}{n_t^m} + \frac{s_t}{n_t^m}RM_{t-1} \tag{A.59}$$

where $n_t = \frac{N_t^y}{N_{t-1}^y}$, $n_t^m = \frac{N_t^m}{N_{t-1}^m} = \frac{1-p_t^y}{MY_{t-1}} + p_t^m$ and $n_t^r = \frac{N_t^r}{N_{t-1}^r} = \frac{1-p_t^m}{RM_{t-1}} + s_t$

Steady-state equations:

Demographics : A demographic balanced growth path implies: $n = n^m = n^r$.

$$n = \frac{1-p^y}{MY} + p^m \tag{A.60}$$

$$p^m = 1 + (s - n_m)RM \tag{A.61}$$

$$n^m = \frac{1-p^y}{MY} + p^m \tag{A.62}$$

$$n^r = s + \frac{1-p^m}{RM} \tag{A.63}$$

The remaining steady-state equations can be summarized as follows:

$$\chi^y = 1 - \beta^\sigma (\Omega^{my} R)^{\sigma-1} \tag{A.64}$$

$$\chi^m = 1 - \beta^\sigma (\Omega^{rm} R)^{\sigma-1} \tag{A.65}$$

$$\chi^r = 1 - s\beta^\sigma R^{\sigma-1} \tag{A.66}$$

$$\Omega^{my} = p^y + (1-p^y)(\frac{\chi^y}{\chi^m})^{\frac{1}{\sigma-1}} \tag{A.67}$$

$$\Omega^{rm} = p^m + (1-p^m)(\frac{\chi^m}{\chi^r})^{\frac{1}{\sigma-1}} \tag{A.68}$$

$$\bar{H}^m = \frac{\alpha MY}{(\xi^y + MY)(1 - \frac{p^m}{\Omega^{rm} R}\frac{g}{n^m})} \tag{A.69}$$

$$\bar{H}^y = \left(1 - \frac{p^y}{\Omega^{my} R}\frac{g}{n}\right)^{-1} \left[\frac{\alpha}{1 + \frac{MY}{\xi^y}} + (1 - \frac{p^y}{\Omega^{my}})\frac{\bar{H}^m}{R}\frac{g}{nMY}\right] \tag{A.70}$$

$$\bar{C}^y = \chi^y \left(\frac{R\bar{A}^y}{g} + \bar{H}^y\right) \tag{A.71}$$

$$\bar{C}^m = \chi^m \left(\frac{R\bar{A}^m}{g} + \bar{H}^m\right) \tag{A.72}$$

$$\bar{C}^r = \chi^r \left(\frac{R\bar{A}^r}{g}\right) \tag{A.73}$$

$$\bar{C} = \bar{C}^y + \bar{C}^m + \bar{C}^r \tag{A.74}$$

$$\bar{A}^y = \frac{p^y(\frac{\alpha\xi^y}{\xi^y + MY} - \chi^y \bar{H}^y)}{1 - (1-\chi^y)R\frac{p^y}{g}} \tag{A.75}$$

$$\bar{A}^m = \frac{p^m(\frac{\alpha MY}{\xi^y + MY} - \chi^m \bar{H}^m) + \frac{1-p^y}{p^y}\bar{A}^y}{1 - (1-\chi^m)R\frac{p^m}{g}} \tag{A.76}$$

$$\bar{A}^r = \frac{\frac{1-p^m}{p^m}(\bar{A}^m - \frac{1-p^y}{p^y}\bar{A}^y)}{1 - (1-\chi^r)\frac{R}{g}} \tag{A.77}$$

$$\bar{A} = \bar{A}^y + \bar{A}^m + \bar{A}^r = \bar{K} \tag{A.78}$$

$$\bar{K} = \frac{g(1-\bar{C})}{g + \delta - 1} \; with \; g = (1+z)n \tag{A.79}$$

$$R = (1-\alpha)\frac{g}{\bar{K}} + (1-\delta) \tag{A.80}$$