Money and risk in a DSGE framework: a bayesian application to the Eurozone

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Abstract

We present and test a model of the Eurozone, with a special emphasis on the role of risk aversion and money. The model follows the New Keynesian DSGE framework, money being introduced in the utility function with a non-separability assumption. Money is also introduced in the Taylor rule. By using Bayesian estimation techniques, we shed light on the determinants of output, inflation, money, interest rate, flexible-price output and flexible-price real money balance dynamics. The role of money is investigated further. Its impact on output depends on the degree of risk aversion. Money plays a minor role in the estimated model. Yet, a higher level of risk aversion would imply that money had significant quantitative effects on business cycle fluctuations.

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1 Introduction

Standard New Keynesian literature analyses monetary policy practically without reference to monetary aggregates. In this now traditional framework, monetary aggregates do not explicitly appear as an explanatory factor neither in the output gap and inflation dynamics nor in interest rate determination. Inflation is explained by the expected inflation rate and the output gap. In turn, the output gap depends mainly on its expectations and the real rate of interest (Clarida et al., 1999; Woodford, 2003; Galí and Gertler, 2007; Galí, 2008). Finally, the interest rate is established via a traditional Taylor rule in function of the inflation gap and the output gap.

In this framework, monetary policy impacts aggregate demand, and thus inflation and output, through changes in the real interest rate. An increase in the interest rate reduces output, which decreases the output gap, thus decreases inflation until a new equilibrium is reached. The money stock and money demand do not explicitly appear. The central bank sets the nominal interest rate so as to satisfy the demand for money (Woodford, 2003; Ireland, 2004).

The money transmission mechanism may also emphasize the connections between real money balances and risk aversion. First, there may exist a real balance effect on aggregate demand resulting from a change in prices. Second, as individuals re-allocate their portfolio of assets, the behavior of real money balances induces relative price adjustments on financial and real assets. In the process, aggregate demand changes, and thus output. By affecting aggregate demand, real money balances become part of the transmission mechanism. Hence, interest rate alone is not sufficient to explain the impact of monetary policy and the role played by financial markets (Meltzer, 1995, 1999; Brunner and Meltzer, 1968).

This monetarist transmission process may also imply a specific role to real money balances when dealing with risk aversion. When risk aversion increases, individuals may desire to hold more money balances to face the implied uncertainty and to optimize their consumption through time. Friedman alluded to this process as far back as 1956 (Friedman, 1956). If this hypothesis holds, risk aversion may influence the impact of real money balances on relative prices, financial assets and real assets, affecting aggregate demand and output.

Other considerations as to the role of money are worth mentioning. In a New Keynesian framework, the expected inflation rate or the output gap may "hide" the role of monetary aggregates, for example on inflation determination. Nelson (2008) shows that standard New Keynesian models are built on the strange assumption that central banks can control the long-term interest rate, while this variable is actually determined by a Fisher equation in which expected inflation depends on monetary developments. Reynard (2007) found that in the U.S. and the Euro area, monetary developments provide qualitative and quantitative information as to inflation. Assenmacher-Wesche and Gerlach (2007) confirm that money growth contains information about inflation pressures and may play an informational role as to the state of different non observed (or difficult to observe) variables influencing inflation or output.

How is money generally introduced in New Keynesian DSGE models ? The standard way is to resort to money-in-the-utility (MIU) function, whereby real money balances are supposed to affect the marginal utility of consumption. Kremer et al. (2003) seem to support this non-separability assumption for Germany, and imply that real money balances contribute to the determination of output and inflation dynamics. A recent contribution introduces the role of money with adjustment costs for holding real balances, and shows that real money balances contribute to explain expected future variations of the natural interest rate in the U.S. and the Eurozone (Andrés et al., 2009). Nelson (2002) finds that money is a significant determinant of aggregate demand, both in the U.S. and in the U.K. However, the empirical work undertaken by Ireland (2004), Andrés et al. (2006), and Jones and Stracca (2008) suggests that there is little evidence as to the role of money in the cases of the United States, the Euro area, and the UK.

Our paper differs in its empirical conclusion, resulting in a stronger role to money, at least in the Eurozone, when risk aversion is high enough. It differs also somewhat in its theoretical set up. As in the standard way, we resort to money-in-the-utility function (MIU) with a non-separability assumption between consumption and money. Yet, in our framework, we specify *all* the micro-parameters. This specification permits extracting characteristics and implications of this type of model that cannot be extracted if only aggregated parameters are used. We will see, for example, that the coefficient of relative risk aversion plays a significant role in explaining the role of money. We test the model and estimate the risk aversion parameter over the sample period. As risk aversion can be very high in short periods of time, but cannot be estimated over such short periods, we test the model again by calibrating a higher risk aversion parameter (twice the previously estimated value). This strategy allows us to compare the impact of both levels of risk aversion on the dynamics of the variables.

Our model differs also in its inflation and output dynamics. Standard New Keynesian DSGE models give an important role to endogenous inertia on both output (consumption habits) and inflation (price indexation). In fact, both dynamics may have a stronger forward-looking component than an inertial component. This appears to be the case, at least in the Euro area if not clearly in the U.S. (Galí et al., 2001). These inertial components may hide part of the role of money. Hence, our choice to remain as simple as possible on that respect in order to try to unveil a possible role for money balances.

Finally, Backus et al. (1992) have shown that capital appears to play a rather minor role in the business cycle. To simplify the analysis and focus on the role of money, we therefore do not include a capital accumulation process in the model, as in Galí (2008).

We differ from existing theoretical (and empirical) analyses by specifying the flexible price counterparts of output and real money balances. This imposes a more elaborate theoretical structure, which provides an improvement on the literature and enriches the model.

We also differ from the empirical analyses of the Eurozone by using Bayesian techniques in a New Keynesian DSGE framework, like in Smets and Wouters (2007), while introducing money in the model. Current literature attempts to introduce money only by aggregating model parameters, therefore leaving aside relevant information. Here we estimate all micro-parameters of the model under average (estimated) and high risk aversion. This is an important innovation and leads to interesting implications.

In order to assess further the role of money we also incorporate and estimate different Taylor rules (without and with different money variables) and analyse their impact on the dynamics of the model with the two levels of risk aversion.

In the process we unveil transmission mechanisms generally neglected in traditional New Keynesian analyses. Given a high enough risk aversion, the framework highlights in particular the non-negligible role of money in explaining output variations.

The dynamic analysis of the model sheds light on the change in the role of money through time in explaining fluctuations in output. It shows that the impact of money is stronger in the short than in the long run.

Section 2 of the paper describes the theoretical set up. In Section 3, the model is calibrated and estimated with Bayesian techniques and by using Euro area data. Variance decompositions are analysed in this section, with an emphasis on the impact of the coefficient of relative risk aversion. Section 4 presents alternative introductions of money in the Taylor rule. Section 5 concludes and the Appendix presents additional theoretical and empirical results.

2 The model

The model consists of households that supply labor, purchase goods for consumption, hold money and bonds, and firms that hire labor and produce and sell differentiated products in monopolistically competitive goods markets. Each firm sets the price of the good it produces, but not all firms reset their price during each period. Households and firms behave optimally: households maximize the expected present value of utility, and firms maximize profits. There is also a central bank that controls the nominal rate of interest. This model is inspired by Galí (2008), Walsh (2017) and Smets and Wouters (2003).

2.1 Households

We assume a representative infinitely-lived household, seeking to maximize

$$E_t\left[\sum_{k=0}^{\infty}\beta^k U_{t+k}\right] \tag{1}$$

where U_t is the period utility function and $\beta < 1$ is the discount factor.

We assume the existence of a continuum of goods represented by the interval $[0, 1]$. The household decides how to allocate its consumption expenditures among the different goods. This requires that the consumption index C_t be maximized for any given level of expenditures. Furthermore, and conditional on such optimal behavior, the period budget constraint takes the form

$$P_tC_t + M_t + Q_tB_t \leq B_{t-1} + W_tN_t + M_{t-1} \tag{2}$$

for $t = 0, 1, 2...$, where W_t is the nominal wage, P_t is an aggregate price index (see Appendix A), N_t is hours of work (or the measure of household members employed), B_t is the quantity of one-period nominally riskless discount bonds purchased in period t and maturing in period $t+1$ (each bond pays one unit of money at maturity and its price is Q_t where $i_t = -\log Q_t$ is the short term nominal rate) and M_t is the quantity of money holdings at time t. The above sequence of period budget constraints is supplemented with a solvency condition, such as $\forall t \lim_{n\longrightarrow\infty} E_t[B_n] \geq 0$.

In the literature, utility functions are usually time-separable. To introduce an explicit role for money balances, we drop the assumption that household preferences are time-separable across consumption and real money balances. Preferences are measured with a CES utility function including real money balances. Under the assumption of a period utility given by

$$U_t = \frac{1}{1-\sigma}\left((1-b)\,C_t^{1-\nu} + be^{\varepsilon_t^m}\,(M_t/P_t)^{1-\nu}\right)^{\frac{1-\sigma}{1-\nu}} - \frac{\chi}{1+\eta}N_t^{1+\eta} \tag{3}$$

consumption, labor, money and bond holdings are chosen to maximize Eq. 1 subject to Eq. 2 and the solvency condition. This CES utility function depends positively on the consumption of goods, C_t, positively on real money balances, M_t/P_t, and negatively on labour N_t. σ is the coefficient of relative risk aversion of households (or the inverse of the intertemporal elasticity of substitution), ν is the inverse of the elasticity of money holdings with respect to the interest rate, and can be seen as a *non separability* parameter, and η is the inverse of the elasticity of work effort with respect to the real wage.

It must be noticed that ν must be lower than σ. If $\nu = \sigma$, Eq. 3 becomes a standard separable utility function whereby the influence of real money balances on output, inflation and flexible-price output disappears. This case has been studied in the literature. In our model, the difference between the risk aversion parameter and the separability parameter, $\sigma - \nu$, plays a significant role.

The utility function also contains a structural money demand shock, ε_t^m. b and χ are positive scale parameters.

As described in Appendix A, this setting leads to the following conditions, which, in addition to the budget constraint, must hold in equilibrium. The resulting log-linear version of the first order condition corresponding to the demand for contingent bonds implies that

$$\begin{aligned}\hat{c}_t \;=\;& E_t\left[\hat{c}_{t+1}\right] - \left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right)/\left(\nu - a_1\left(\nu-\sigma\right)\right) \\ & - \frac{\left(1-a_1\right)\left(\nu-\sigma\right)}{\nu - a_1\left(\nu-\sigma\right)}E_t\left[\Delta\hat{m}_{t+1} - \hat{\pi}_{t+1}\right] + \xi_{t,c}\end{aligned} \tag{4}$$

where $\xi_{t,c} = -\frac{(1-a_1)(\nu-\sigma)}{(1-\nu)(\nu-a_1(\nu-\sigma))}E_t\left[\Delta\varepsilon_{t+1}^m\right]$ and by using the steady state of the first order conditions $a_1^{-1} = 1 + \left(\frac{b}{1-b}\right)^{\frac{1}{\nu}}(1-\beta)^{\frac{\nu-1}{\nu}}$. The lowercase $(\hat{\ })$ denotes the log-linearized (around the steady state) form of the original aggregated variables.

The demand for cash that follows from the household's optimization problem is given by

$$-\,\nu\left(\hat{m}_t - \hat{p}_t\right) + \nu\hat{c}_t + \varepsilon_t^m = a_2\hat{\imath}_t \tag{5}$$

with $a_2 = \frac{1}{\exp(1/\beta)-1}$ and where real cash holdings depend positively on consumption with an elasticity equal to unity and negatively on the nominal interest rate. In what follows we will take the nominal interest rate as the central bank's policy instrument. In the literature, due to the assumption that consumption and real money balances are additively separable in the utility function, cash holdings do not enter any of the other structural equations: accordingly, the above equation becomes recursive to the rest of the system of equations.

The first order condition corresponding to the optimal consumption-leisure arbitrage implies that

$$\eta \hat{n}_t + (\nu - a_1 (\nu - \sigma)) \hat{c}_t - (\nu - \sigma)(1 - a_1)(\hat{m}_t - \hat{p}_t) + \xi_{t,m} = \hat{w}_t - \hat{p}_t \quad (6)$$

where $\xi_{t,m} = -\frac{(\nu-\sigma)(1-a_1)}{1-\nu}\varepsilon_t^m$.

Finally, these equations represent the Euler condition for the optimal intratemporal allocation of consumption (Eq. 4), the intertemporal optimality condition setting the marginal rate of substitution between money and consumption equal to the opportunity cost of holding money (Eq. 5), and the intratemporal optimality condition setting the marginal rate of substitution between leisure and consumption equal to the real wage (Eq. 6).

2.2 Firms

We assume a continuum of firms indexed by $i \in [0, 1]$. Each firm produces a differentiated good but uses an identical technology with the following production function,

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (7)$$

where $A_t = \exp(\varepsilon_t^a)$ is the level of technology, assumed to be common to all firms and to evolve exogenously over time, and α is the measure of decreasing returns.

All firms face an identical isoelastic demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given. As in the standard Calvo (1983) model, our generalization features monopolistic competition and staggered price setting. At any time t, only a fraction $1 - \theta$ of firms, with $0 < \theta < 1$, can reset their prices optimally, while the remaining firms index their prices to lagged inflation.

2.3 Central bank

The central bank is assumed to set its nominal interest rate according to a generalized smoothed Taylor rule such as:

$$\hat{\imath}_t = (1 - \lambda_i)\left(\lambda_\pi (\hat{\pi}_t - \pi_c) + \lambda_x \left(\hat{y}_t - \hat{y}_t^f\right) + \lambda_m \widetilde{M}_{t,k}\right) + \lambda_i \hat{\imath}_{t-1} + \varepsilon_t^i \quad (8)$$

where λ_π, λ_x and λ_m are policy coefficients reflecting the weight on inflation, on the output gap and on a money variable; the parameter $0 < \lambda_i < 1$ captures the degree of interest rate smoothing; ε_t^i is an exogenous *ad hoc* shock accounting for fluctuations of the nominal interest rate. π_c is an inflation target and $\widetilde{M}_{t,k}$ is a money variable: when $k = 0$, money does not enter the Taylor rule; $k = 1$ to 3 corresponds respectively to the real money gap (difference between real money balances and its flexible-price counterpart), the nominal money growth and the real money growth.

2.4 DSGE model

Solving the model (Appendix A) leads to six micro-founded equations and six dependent variables: inflation, nominal interest rate, output, flexible-price output, real money balances and its flexible-price counterpart.

Flexible-price output and flexible-price real money balances are completely determined by shocks. Flexible-price output is mainly driven by technology shocks (whereas fluctuations in the output gap can be attributed to supply and demand shocks). The flexible-price real money balances are mainly driven by money shocks and flexible-price output.

$$\hat{y}_t^f = \upsilon_a^y \varepsilon_t^a + \upsilon_m^y \widehat{mp}_t^f - \upsilon_c^y + \upsilon_{sm}^y \varepsilon_t^m \tag{9}$$

$$\widehat{mp}_t^f = \upsilon_{y+1}^m E_t \left[\hat{y}_{t+1}^f\right] + \upsilon_y^m \hat{y}_t^f + \frac{1}{\nu}\varepsilon_t^m \tag{10}$$

$$\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa_{x,t}\left(\hat{y}_t - \hat{y}_t^f\right) + \kappa_{m,t}\left(\widehat{mp}_t - \widehat{mp}_t^f\right) \tag{11}$$

$$\begin{aligned} \hat{y}_t \quad = \quad & E_t\left[\hat{y}_{t+1}\right] - \kappa_r\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) \\ & + \kappa_{mp} E_t\left[\Delta \widehat{mp}_{t+1}\right] + \kappa_{sm} E_t\left[\Delta \varepsilon_{t+1}^m\right] \end{aligned} \tag{12}$$

$$\widehat{mp}_t = \hat{y}_t - \kappa_i \hat{\imath}_t + \frac{1}{\nu}\varepsilon_t^m \tag{13}$$

$$\hat{\imath}_t = (1-\lambda_i)\left(\lambda_\pi\left(\hat{\pi}_t - \pi_c\right) + \lambda_x\left(\hat{y}_t - \hat{y}_t^f\right) + \lambda_m \widetilde{M}_{t,k}\right) + \lambda_i \hat{\imath}_{t-1} + \varepsilon_t^i \tag{14}$$

where
$\upsilon_a^y = \frac{1+\eta}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$
$\upsilon_m^y = \frac{(1-\alpha)(\nu-\sigma)(1-a_1)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha}$
$\upsilon_c^y = \frac{(1-\alpha)}{(\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha} \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$

$$
\begin{aligned}
&\upsilon_{sm}^{y} = \frac{(\nu-\sigma)(1-a_1)(1-\alpha)}{((\nu-(\nu-\sigma)a_1)(1-\alpha)+\eta+\alpha)(1-\nu)}\\
&\upsilon_{y+1}^{m} = -\frac{a_2}{\nu}\left(\nu-(\nu-\sigma)\,a_1\right)\\
&\upsilon_{y}^{m} = 1+\frac{a_2}{\nu}\left(\nu-(\nu-\sigma)\,a_1\right)\\
&\kappa_{x,t} = \left(\nu-(\nu-\sigma)\,a_1+\frac{\eta+\alpha}{1-\alpha}\right)\frac{(1-\alpha)\left(\frac{1}{\theta}-\beta\right)(1-\theta)\left(1+(\varepsilon-1)\varepsilon_t^p\right)}{1+\left(\alpha+\varepsilon_t^p\right)(\varepsilon-1)}\\
&\kappa_{m,t} = (\sigma-\nu)\,(1-a_1)\,\frac{(1-\alpha)\left(\frac{1}{\theta}-\beta\right)(1-\theta)\left(1+(\varepsilon-1)\varepsilon_t^p\right)}{1+\left(\alpha+\varepsilon_t^p\right)(\varepsilon-1)}\\
&\kappa_r = \frac{1}{\nu-a_1(\nu-\sigma)}\\
&\kappa_{mp} = \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)}\\
&\kappa_{sm} = -\frac{(1-a_1)(\nu-\sigma)}{(\nu-a_1(\nu-\sigma))(1-\nu)}\\
&\kappa_i = a_2/\nu\\
&\text{with } a_1 = \frac{1}{1+(b/(1-b))^{1/\nu}(1-\beta)^{(\nu-1)/\nu}} \text{ and } a_2 = \frac{1}{\exp(1/\beta)-1}.
\end{aligned}
$$

The structural money shock and the markup shock[1], ε_t^m and ε_t^p, the exogenous component of the interest rate, ε_t^i, and of the technology, ε_t^a, are assumed to follow a first-order autoregressive process with an *i.i.d.*-normal error term such as $\varepsilon_t^k = \rho_k \varepsilon_{t-1}^k + \omega_{k,t}$ where $\varepsilon_{k,t} \sim N(0;\sigma_k)$ for $k = \{m, p, i, a\}$.

As can be seen, σ and ν influence all macro-parameters. This influence highlights the fact that separability and risk aversion are prominent factors involved in output, inflation, real money balances and nominal interest rate dynamics, as well as in flexible-price output and flexible-price real money balances. Moreover, as far as money is concerned, it is the three macro-parameters, υ_m^y, κ_m and κ_{mp}, that are essential to highlight its possible role in the dynamics of the model: these coefficients determine the *weight* of money in Eq. 9, Eq. 11 and Eq. 12.

3 Empirical results

As in Smets and Wouters (2003) and An and Schorfheide (2007), we apply Bayesian techniques to estimate our DSGE model. Contrary to Ireland (2004) or Andrés et al. (2006), we did not opt to estimate our model by using the maximum of likelihood because such computation hardly converges toward a global maximum. First, we estimate the risk aversion level and all parameters over the sample period. Second, we re-estimate all parameters but with a constant risk aversion level calibrated to approximately twice its estimated value. We also test four specifications of the Taylor rule under these two alternatives risk aversion levels.

[1] The markup shock is introduced and explained in Appendix A.

3.1 Euro area data

In our model of the Eurozone, $\hat{\pi}_t$ is the log-linearized detrended inflation rate measured as the yearly log difference of detrended GDP Deflator from one quarter to the same quarter of the previous year; $\hat{y}_t$ is the log-linearized detrended output per capita measured as the difference between the log of the real GDP per capita and its trend; and $\hat{\imath}_t$ is the short-term (3-month) detrended nominal interest rate. These Data are extracted from the Euro Area Wide Model database (AWM) of Fagan et al. (2001). $\widehat{mp}_t$ is the log-linearized detrended real money balances per capita measured as the difference between the real money per capita and its trend, where real money per capita is measured as the log difference between the money stock per capita and the GDP Deflator. We use the $M3$ monetary aggregate from the Eurostat database. $\hat{y}_t^f$, the flexible-price output, and $\widehat{mp}_t^f$, the flexible-price real money balances, are completely determined by structural shocks. To make output and real money balances stationary, we use first log differences, as in Adolfson et al. (2008).

3.2 Calibration

Following standard conventions, we calibrate beta distributions for parameters that fall between zero and one, inverted gamma distributions for parameters that need to be constrained to be greater than zero, and normal distributions in other cases.

The calibration of σ is inspired by Rabanal and Rubio-Ramírez (2005) and by Casares (2007). They choose, respectively, a risk aversion parameter of 2.5 and 1.5. In line with these values, we consider that $\sigma = 2$ corresponds to a standard risk aversion while values above that level imply higher and higher risk aversion, hence our choice of $\sigma = 4$ to represent a high level of risk aversion, twice the standard value. Excepted for risk aversion, we adopt the same priors in the two models.

As in Smets and Wouters (2003), the standard errors of the innovations are assumed to follow inverse gamma distributions and we choose a beta distribution for shock persistence parameters (as well as for the backward component of the Taylor rule) that should be lesser than one.

The calibration of α, β, θ, η, and ε comes from Smets and Wouters (2003, 2007), Casares (2007) and Galí (2008). The smoothed Taylor rule (λ_i, λ_π, λ_x and λ_m) is calibrated following Gerlach-Kristen (2003), Andrés et al. (2009) and Barthélemy et al. (2011), analogue priors as those used by Smets and Wouters (2003) for the monetary policy parameters. In order to observe the behavior of the central bank, we assign a higher standard error

(0.50) and a Normal prior law for the Taylor rule's coefficients except for the smoothing parameter, which is restricted to be positive and below one (Beta distribution). The inflation target, π_c, is calibrated to 2% and estimated. υ (the non-separability parameter) must be greater than one. κ_i (Eq. 13) must be greater than one as far as this parameter depends on the elasticity of substitution of money demand with respect to the cost of holding money balances, as explained in Söderström (2005); while still informative, this prior distribution is dispersed enough to allow for a wide range of possible and realistic values to be considered (i.e. $\sigma > \upsilon > 1$).

Our prior distributions are not dispersed to focus on the role of risk aversion. The calibration of the shock persistence parameters and the standard errors of the innovations follows Smets and Wouters (2003) and Fève et al. (2010). All the standard errors of shocks are assumed to be distributed according to inverted Gamma distributions, with prior means of 0.02. The latter ensures that these parameters have a positive support. The autoregressive parameters are all assumed to follow Beta distributions. All these distributions are centered around 0.75, except for the autoregressive parameter of the monetary policy shock, which is centered around 0.50, as in Smets and Wouters (2003). We take a common standard error of 0.1 for the shock persistence parameters, as in Smets and Wouters (2003).

3.3 Results

As already said, we calibrate first the level of risk aversion to its standard value, $\sigma = 2$, and we estimate it. This model version is considered as a benchmark specification. In the second version, we set $\sigma = 4$, about twice this estimated value. This set-up is motivated by Holden and Subrahmanyam (1996). They show that acquisition of short-term information is encouraged by high risk aversion level, and that the latter can cause all potentially informed investors in the economy to concentrate exclusively on the short-term instead of the long-term. Risk aversion is generally low in the medium and long run while it could be very high in short periods. As we can't estimate risk aversion in the short run, we choose to estimate our model also by setting $\sigma = 4$, i.e. a high risk aversion level.

In this section, we present only results with a Taylor rule incorporating the real money gap ($\widetilde{M}_{t,1} = \widehat{mp}_t - \widehat{mp}_t^f$), the most significant money variable as shown in Section 4. The model is estimated with 117 observations from 1980 (Q4) to 2009 (Q4) in order to avoid high volatility periods before 1980 and to take into consideration the core of the global crisis. The estimation of the implied posterior distribution of the parameters under the two configurations of risk (Table 1) is done using the Metropolis-Hastings algorithm (10 distinct chains, each of 50000 draws; see Smets and Wouters (2007) and Adolfson et al. (2007)). Average acceptation rates per chain for the benchmark model (σ estimated) are included in the interval $[0.256; 0.261]$ and for the high risk aversion model ($\sigma = 4$) are included in the interval $[0.248; 0.252]$. The literature has settled on a value of this acceptance rate around 0.25.

Priors and posteriors distributions are presented in Appendix B. To assess the model validation, we insure convergence of the proposed distribution to the target distribution (Appendix C)

Table 1: Bayesian estimation of structural parameters

	Priors			Posteriors					
				σ estimated			$\sigma = 4$		
	Law	Mean	Std.	5%	Mean	95%	5%	Mean	95%
α	beta	0.33	0.05	0.282	0.378	0.473	0.384	0.484	0.589
θ	beta	0.66	0.05	0.657	0.710	0.764	0.673	0.726	0.777
υ	normal	1.25	0.05	1.380	1.447	1.518	1.491	1.528	1.568
σ	normal	2.00	0.50	1.771	2.157	2.545			
b	beta	0.25	0.10	0.085	0.252	0.410	0.084	0.246	0.399
η	normal	1.00	0.10	0.895	1.053	1.218	0.957	1.120	1.281
ε	normal	6.00	0.10	5.807	5.978	6.143	5.815	5.979	6.141
λ_i	beta	0.50	0.10	0.449	0.573	0.700	0.502	0.614	0.726
λ_π	normal	3.00	0.50	2.856	3.494	4.104	2.874	3.491	4.145
λ_x	normal	1.50	0.50	1.133	1.872	2.614	1.175	1.923	2.632
λ_m	normal	1.50	0.50	0.320	1.011	1.681	0.276	0.964	1.635
π_c	normal	2.00	0.10	1.733	1.903	2.071	1.739	1.908	2.071
ρ_a	beta	0.75	0.10	0.987	0.992	0.997	0.991	0.994	0.998
ρ_p	beta	0.75	0.10	0.960	0.973	0.987	0.958	0.972	0.986
ρ_i	beta	0.50	0.10	0.377	0.460	0.540	0.490	0.560	0.631
ρ_m	beta	0.75	0.10	0.952	0.971	0.991	0.974	0.984	0.995
σ_a	invgamma	0.02	2.00	0.011	0.013	0.016	0.015	0.019	0.022
σ_i	invgamma	0.02	2.00	0.013	0.018	0.023	0.009	0.012	0.015
σ_p	invgamma	0.02	2.00	0.003	0.004	0.006	0.003	0.004	0.006
σ_m	invgamma	0.02	2.00	0.023	0.026	0.029	0.024	0.027	0.030

3.4 Variance decompositions

In this section we analyse the forecast error variance of each variable following exogenous shocks, in two different ways. The analysis is conducted first via an unconditional variance decomposition (long term), and second via a conditional variance decomposition (short term and over time).

3.4.1 Long term analysis

Table 2: Unconditional variance decomposition (percent)

	estimated σ				$\sigma = 4$			
	ε_t^p	ε_t^i	ε_t^m	ε_t^a	ε_t^p	ε_t^i	ε_t^m	ε_t^a
$\hat{y}_t$	1.65	1.09	3.07	94.18	0.83	0.28	10.38	88.51
$\hat{\pi}_t$	97.66	2.14	0.09	0.12	97.64	1.79	0.24	0.33
$\hat{\imath}_t$	78.53	19.64	0.64	1.19	74.41	20.67	1.86	3.07
$\widehat{mp}_t$	1.85	0.91	52.49	44.75	0.83	0.26	60.87	38.04
$\hat{y}_t^f$	0.00	0.00	3.06	96.94	0.00	0.00	10.23	89.77
$\widehat{mp}_t^f$	0.00	0.00	54.42	45.58	0.00	0.00	62.04	37.96

The unconditional variance decomposition (Table 2) shows that, whatever the risk aversion level, the variance of output essentially results from the technology shock, the remaining from the other shocks. If money plays some role, this role is rather minor (an impact of 3.07%) under an estimated standard risk aversion.

Yet, as Table 2 shows, the money shock contribution to the business cycle depends on the value of agents' risk aversion. The estimation with the higher risk aversion ($\sigma = 4$) gives interesting information as to the role of money, and more generally as to the role of each shock in the long run.

These results indicate that a higher coefficient of relative risk aversion increases significantly the impact of money on output. Yet it does not really change the impact of money on inflation dynamics, essentially explained by the markup shock whatever the level of risk aversion. The very small role of the money shock on inflation dynamics is a consequence of the low value of $\kappa_{m,t}$ in Eq. 11, whatever the level of risk aversion, even though $\kappa_{m,t}$ increases with σ. By comparison, the value of κ_{mp} in Eq. 12 is significantly higher, and increases no less significantly with σ (see Table 5 in Appendix D).

If more than 88% of the variance of output is still explained by the technology shock with the high risk calibration ($\sigma = 4$), the role of the interest rate shock and the role of the markup shock decrease whereas the impact of the money shock increases from about 3% to 10.4%, i.e. is multiplied by a factor of 3.4. Similarly, the impact of shocks on flexible-price output also depends on the risk aversion level. The role of the money shock increases with the risk level from about 3% to 10.2%.

Although money enters the Taylor rule, it does not have a significant role in the dynamics of the interest rate, whatever the level of risk aversion.

Furthermore, following an increase in the risk aversion level, the dynamics of real money balances and its flexible-price counterpart are to a lesser extent explained by the technology shock. Unsurprisingly, the variance of these variables are mainly explained by the money shock.

3.4.2 Short term and through time analysis

The analysis through time (conditional variance) of the different shocks on output (Figure 1) shows that the impact of the money shock decreases with the time horizon, as for the interest rate shock[2]. Under high risk aversion, the role of money in the first periods remains around 22%, i.e. twice the value in the long term (10.38%).

As far as inflation variance is concerned, the markup shock not only dominates the process but its impact does not change through time in both risk configurations.

In the short term, as shown in Table 3, the monetary policy shock explains around 83% of the nominal interest rate variance whereas the markup shock explain less than 17% for the two configurations of risk. For longer terms, there is an inversion: whatever the risk aversion level, the interest rate variance is dominated by the price-markup shock and the monetary policy shock explains around 20% of the interest rate variance. Although money is introduced in the Taylor rule, the money shock has a minor impact on the nominal interest rate variance at any time horizon.

[2] The conditional variances decompositions figures for the other variables are not shown here but are available upon request.

Figure 1: Conditional forecast error variance decomposition of Output

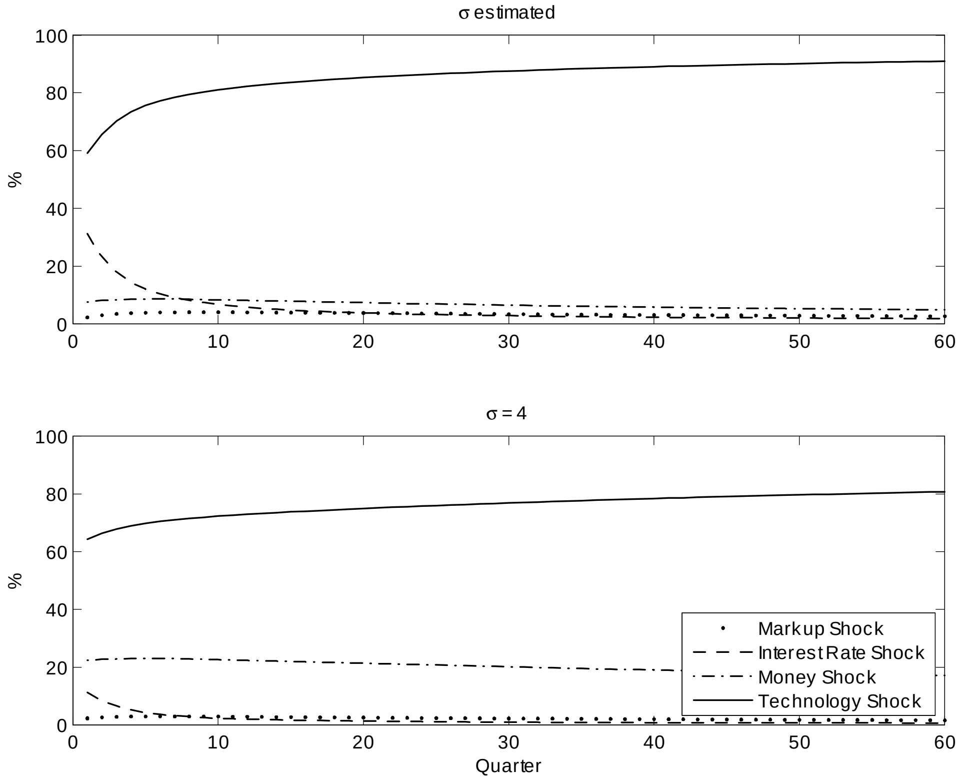


Table 3: First period variance decomposition (percent)

	estimated σ				$\sigma = 4$			
	ε_t^p	ε_t^i	ε_t^m	ε_t^a	ε_t^p	ε_t^i	ε_t^m	ε_t^a
$\hat{y}_t$	2.16	31.17	7.50	59.16	2.23	11.19	22.38	64.20
$\hat{\pi}_t$	77.72	22.16	0.08	0.03	83.73	16.08	0.13	0.06
$\hat{\imath}_t$	16.35	83.44	0.14	0.07	16.66	82.99	0.23	0.13
$\widehat{mp}_t$	1.28	13.76	69.46	15.49	1.09	5.46	77.25	16.20
$\hat{y}_t^f$	0.00	0.00	10.56	89.44	0.00	0.00	24.89	75.11
$\widehat{mp}_t^f$	0.00	0.00	81.72	18.28	0.00	0.00	82.62	17.38

The role of monetary policy on real money balances is different in the short term: the monetary policy shock explains almost 14% of the variance of real money balances in the short term under the standard risk aversion (and around 5% under high risk aversion), whereas, under the two configurations of risk, it has a very small role at longer horizons. Similarly, the technology shock explains around 16% of the real money balances variance in both configurations of risk, whereas at longer horizons it explains around 45% of the real money variance under the estimated risk aversion (and around 38% under the high risk aversion).

It is interesting to notice that the same type of analysis applies to the flexible-price output variance decomposition. In the short term as well as in the long term, technology is the main explanatory factor. The role of money increases with the relative risk aversion coefficient in the short term (from a weight of less than 11% under standard calibration to close to 25% under high risk aversion calibration), as in the long term, whereas the monetary policy and the price-markup shocks play no role in the flexible-price output and the flexible-price real money balances dynamics.

3.5 Interpretation

The estimates of the macro-parameters (aggregated structural parameters) for estimated and high risk aversions are reported in Appendix D (Table 5). These estimates suggest that a change in risk aversion implies significant variations in the value of several macro-parameters, notably υ_m^y, κ_m and κ_{mp} - respectively the *weight* of money in the flexible-price output, inflation and output equations. Moreover, the *weight* of the money shock on output dynamics, κ_{sm}, and on flexible-price output, υ_{sm}^y, increases with risk aversion, thus reinforcing the role of money in the dynamics of the model. It is also worth mentioning that the smoothing parameter in the Taylor rule equation, λ_i, increases with risk aversion. This may reflect the idea that the central bank strives for financial stability in crisis periods.

The comparison between the variance decompositions (Table 2 and 3) of the two model versions illustrates the fact that the role of the money shock on output and flexible-price output depends crucially on the degree of agents' risk aversion, increases accordingly, and becomes significant when risk aversion is high, whatever the time horizon. This result highlights the role of real money to smooth consumption through time, especially when risk aversion reaches certain levels.

Impulse response functions for the two configurations of risk (Appendix E) highlights the role of risk aversion on the dynamics of several of the model's variables. These results also demonstrate the predominant role of the risk aversion level on the impact of the money shock on output, inflation, and real and nominal interest rates. The higher the risk aversion level, the greater the reactions to the shocks.

4 Money in the Taylor rule

To evaluate further the role of money we analyse different specifications of the Taylor rule ($\widetilde{M}_{t,k}$ for $k = \{0,1,2,3\}$, as exposed in Section 2.3), first without money, then with money introduced in three different ways. We thus test both models with four types of Taylor rules:

- With no money ($\widetilde{M}_{t,0} = 0$);
- With a real money gap ($\widetilde{M}_{t,1} = \widehat{mp}_t - \widehat{mp}_t^f$);
- With a nominal money growth ($\widetilde{M}_{t,2} = \widehat{m}_t - \widehat{m}_{t-1}$);
- With a real money growth ($\widetilde{M}_{t,3} = \widehat{mp}_t - \widehat{mp}_{t-1}$).

Table 4: Alternative ECB's Taylor rules

	estimated σ				$\sigma = 4$			
	$\widetilde{M}_{t,0}$	$\widetilde{M}_{t,1}$	$\widetilde{M}_{t,2}$	$\widetilde{M}_{t,3}$	$\widetilde{M}_{t,0}$	$\widetilde{M}_{t,1}$	$\widetilde{M}_{t,2}$	$\widetilde{M}_{t,3}$
λ_i	0.527	0.573	0.561	0.547	0.545	0.614	0.546	0.5469
$(1-\lambda_i)\,\lambda_\pi$	1.594	1.491	1.463	1.537	1.579	1.345	1.585	1.575
$(1-\lambda_i)\,\lambda_x$	1.066	0.799	1.018	1.042	1.034	0.741	1.038	1.039
$(1-\lambda_i)\,\lambda_m$		0.431	0.136*	0.084*		0.371	-0.012*	-0.018*
ST_m^y (%)	7.05	7.50	2.23	3.66	22.61	22.38	23.20	23.28
LT_m^y (%)	2.75	3.07	2.24	2.36	9.56	10.38	9.29	9.15
LMD	-629.8	**-618.2**	-634.9	-635.3	-639.8	**-626.5**	-646.1	-646.1

* estimations are not significant in terms of student tests (t<1.645)

As shown in Table 4, all the coefficients of the inflation and the output gap variables, as well as the interest rate smoothing coefficients are significant (Student tests superior to 1.96) whatever the risk. Yet, the money coefficient is significant only with the money gap variable ($\widetilde{M}_{t,1}$).

Furthermore, the log marginal density (LMD) of the data measured through a Laplace approximation indicates that the Taylor rule including this real money gap performs better than the others, followed by the no-money ($\widetilde{M}_{t,0}$) case.

These results suggest that if money has to be introduced in the ECB monetary policy reaction function, it should rather be a real money gap variable than a money growth variable (contrary to what Andrés et al. (2009) and Barthélemy et al. (2011) found, whereas Fourçans (2004, 2007) didn't find such a role for money growth).

Either way, it is interesting to notice that whatever the formulation of the Taylor rule, the estimated parameters of the whole model are quite similar. This is true with both levels of risk aversion.

The impact of a money shock on output, as shown through the short term (ST_m^y, in the first period) and the long term (LT_m^y) variance decomposition of output with respect to a money shock, are also rather similar whatever the Taylor rule (Table 3). The impact of money increases with the risk aversion coefficient, and is stronger in the short run than in the long run, especially when risk aversion is high.

5 Conclusion

We built and empirically tested a model of the Eurozone, with two levels of risk aversion and with a particular emphasis on the role of money. The model follows the New Keynesian DSGE framework, with money in the utility function whereby real money balances affect the marginal utility of consumption.

By using Bayesian estimation techniques, we shed light not only on the determinants of output and inflation dynamics but in addition on the interest rate, real money balances, flexible-price output and flexible-price real money balances variances. We further investigated how the results are affected when intertemporal risk aversion changes, especially as far as money is concerned.

Money plays a minor role in the estimated model with a moderate risk aversion. Most of the variance of output is explained by the technology shock, the rest by a combination of markup, interest rate and money shocks, a result in line with current literature (Ireland, 2004; Andrés et al., 2006, 2009). However, another calibration with a higher risk aversion (twice the estimated value) implies that money plays a non-negligible role in explaining output and flexible-price output fluctuations. We also found that the short term impact is significantly stronger than the long run one. These results differ from existing literature using New Keynesian DSGE frameworks with money, insofar as it neglects the impact of a high enough risk factor.

On the other hand, the explicit money variable does not appear to have a notable direct role in explaining inflation variability. The overwhelming explanatory factor is the price-markup whatever the level of risk aversion.

Another outcome concerns monetary policy. The higher the risk aversion, the stronger the smoothing of the interest rate. This reflects probably the central bankers' objective not to agitate markets.

Our results suggest that a nominal or real money growth variable does not improve the estimated ECB monetary policy rule. Yet, a real money gap variable (the difference between the real money balances and its flexible-price counterpart) significantly improves the estimated Taylor rule. This being said, the introduction, or not, of a money variable in the ECB monetary policy reaction function does not really appear to change significantly the impact of money on output and inflation dynamics.

All in all, one may infer from our analysis that by changing economic agents' perception of risks, the last financial crisis may have increased the role of real money balances in the transmission mechanism and in output changes.

A Solving the model

- **Price dynamics**

Let's assume a set of firms not reoptimizing their posted price in period t. Using the definition of the aggregate price level and the fact that all firms resetting prices choose an identical price P_t^*, leads to $P_t = \left[\theta P_{t-1}^{1-\Lambda_t} + (1-\theta)\left(P_t^*\right)^{1-\Lambda_t}\right]^{\frac{1}{1-\Lambda_t}}$, where $\Lambda_t = 1 + \frac{1}{\frac{1}{\varepsilon-1}+\varepsilon_t^p}$ is the elasticity of substitution between consumption goods in period t, and $\frac{\Lambda_t}{\Lambda_t-1}$ is the markup of prices over marginal costs (time varying). Dividing both sides by P_{t-1} and log-linearizing around $P_t^* = P_{t-1}$ yields

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{15}$$

In this setup, we don't assume inertial dynamics of prices. Inflation results from the fact that firms reoptimizing in any given period their price plans, choose a price that differs from the economy's average price in the previous period.

- **Price setting**

A firm reoptimizing in period t chooses the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. This problem is solved and leads to a first-order Taylor expansion around the zero inflation steady state:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\right] \tag{16}$$

where $\widehat{mc}_{t+k|t} = mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and $\mu = \log\left(\frac{\varepsilon}{\varepsilon-1}\right)$ is the log of the desired gross markup.

- **Equilibrium**

Market clearing in the goods market requires $Y_t(i) = C_t(i)$ for all $i \in [0,1]$ and all t. Aggregate output is defined as $Y_t = \left(\int_0^1 Y_t(i)^{1-\frac{1}{\Lambda_t}} di\right)^{\frac{\Lambda_t}{\Lambda_t - 1}}$; it follows that $Y_t = C_t$ must hold for all t. One can combine the above goods market clearing condition with the consumer's Euler equation (Eq. 4) to yield the equilibrium condition

$$\begin{aligned}\hat{y}_t &= E_t[\hat{y}_{t+1}] - \frac{1}{\nu - a_1(\nu-\sigma)}(\hat{\imath}_t - E_t[\hat{\pi}_{t+1}]) \\ &\quad + \frac{(\sigma-\nu)(1-a_1)}{\nu - a_1(\nu-\sigma)}(E_t[\Delta\hat{m}_{t+1}] - E_t[\hat{\pi}_{t+1}]) + \xi_{t,c}\end{aligned} \tag{17}$$

Market clearing in the labor market requires $N_t = \int_0^1 N_t(i)\,di$. By using the production function (Eq. 7) and taking logs, one can write the following approximate relation between aggregate output, employment and technology as

$$y_t = \varepsilon_t^a + (1-\alpha)\,n_t \tag{18}$$

An expression is derived for an individual firm's marginal cost in terms of the economy's average real marginal cost:

$$mc_t = (\hat{w}_t - \hat{p}_t) - \widehat{mpn}_t \tag{19}$$

$$= (\hat{w}_t - \hat{p}_t) - \frac{1}{1-\alpha}(\varepsilon_t^a - \alpha\hat{y}_t) \tag{20}$$

for all t, where $\widehat{mpn}_t$ defines the economy's average marginal product of labor. As $mc_{t+k|t} = (\hat{w}_{t+k} - \hat{p}_{t+k}) - mpn_{t+k|t}$ we have

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\Lambda_t}{1-\alpha}(p_t^* - p_{t+k}) \tag{21}$$

where the second equality follows from the demand schedule combined with the market clearing condition $c_t = y_t$. Substituting Eq. 21 into Eq. 16 yields

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}\Theta_{t+k}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k}\right] + \sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\pi_{t+k}\right] \quad (22)$$

where $\Theta_t = \frac{1-\alpha}{1-\alpha+\alpha\Lambda_t} \leq 1$ is time varying in order to take into account the markup shock.

Finally, Eq. 15 and Eq. 22 yield the inflation equation

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \lambda_{mc_t}\widehat{mc}_t \quad (23)$$

where β, $\lambda_{mc_t} = \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta_t$. λ_{mc_t} is strictly decreasing in the index of price stickiness θ, in the measure of decreasing returns α, and in the demand elasticity Λ_t.

Next, a relation is derived between the economy's real marginal cost and a measure of aggregate economic activity. From Eq. 6 and Eq. 18, the average real marginal cost can be expressed as

$$\begin{aligned} mc_t &= \left(\nu - (\nu-\sigma)a_1 + \frac{\eta+\alpha}{1-\alpha}\right)\hat{y}_t - \varepsilon_t^a\left(\frac{1+\eta}{1-\alpha}\right) \\ &\quad + (\sigma-\nu)(1-a_1)(\hat{m}_t - \hat{p}_t) + \xi_{t,m} \end{aligned} \quad (24)$$

Under flexible prices the real marginal cost is constant and equal to $mc = -\mu$. Defining the natural level of output, denoted by y_t^f, as the equilibrium level of output under flexible prices leads to

$$\begin{aligned} mc &= \left(\nu - (\nu-\sigma)a_1 + \frac{\eta+\alpha}{1-\alpha}\right)\hat{y}_t^f - \varepsilon_t^a\left(\frac{1+\eta}{1-\alpha}\right) \\ &\quad + (\sigma-\nu)(1-a_1)\widehat{mp}_t^f + \xi_{t,m} \end{aligned} \quad (25)$$

where $\widehat{mp}_t^f = \hat{m}_t^f - \hat{p}_t^f$, thus implying

$$\hat{y}_t^f = \upsilon_a^y\varepsilon_t^a + \upsilon_m^y\widehat{mp}_t^f - \upsilon_c^y + \upsilon_{sm}^y\varepsilon_t^M \quad (26)$$

where

$$
\begin{aligned}
\upsilon_a^y &= \frac{1+\eta}{(\nu-(\nu-\sigma)\,a_1)\,(1-\alpha)+\eta+\alpha} \\
\upsilon_m^y &= \frac{(1-\alpha)\,(\nu-\sigma)\,(1-a_1)}{(\nu-(\nu-\sigma)\,a_1)\,(1-\alpha)+\eta+\alpha} \\
\upsilon_c^y &= \frac{(1-\alpha)}{(\nu-(\nu-\sigma)\,a_1)\,(1-\alpha)+\eta+\alpha}\log\left(\frac{\varepsilon}{\varepsilon-1}\right) \\
\upsilon_{sm}^y &= \frac{(\nu-\sigma)\,(1-a_1)\,(1-\alpha)}{((\nu-(\nu-\sigma)\,a_1)\,(1-\alpha)+\eta+\alpha)\,(1-\nu)}
\end{aligned}
$$

We deduce from Eq. 17 that $\hat{\imath}_t^f = (\nu-(\nu-\sigma)\,a_1)\,E_t\left[\Delta\hat{y}_{t+1}^f\right]$ and by using Eq. 5 we obtain the following equation of real money balances under flexible prices

$$
\widehat{mp}_t^f = \upsilon_{y+1}^m E_t\left[\hat{y}_{t+1}^f\right] + \upsilon_y^m \hat{y}_t^f + \frac{1}{\nu}\varepsilon_t^M \tag{27}
$$

where $\upsilon_{y+1}^m = -\frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu}$ and $\upsilon_y^m = 1+\frac{a_2(\nu-(\nu-\sigma)a_1)}{\nu}$

Subtracting Eq. 25 from Eq. 24 yields

$$
\widehat{mc}_t = \phi_x\left(\hat{y}_t-\hat{y}_t^f\right) + \phi_m\left(\widehat{mp}_t-\widehat{mp}_t^f\right) \tag{28}
$$

where $\widehat{mp}_t = \hat{m}_t - \hat{p}_t$ is the log linearized real money balances around its steady state, $\widehat{mp}_t^f$ is its flexible-price counterpart, $\phi_x = \nu-(\nu-\sigma)\,a_1+\frac{\eta+\alpha}{1-\alpha}$ and $\phi_m = (\sigma-\nu)\,(1-a_1)$.

By combining Eq. 28 with Eq. 23 we obtain

$$
\hat{\pi}_t = \beta E_t\left[\hat{\pi}_{t+1}\right] + \kappa_{x,t}\left(\hat{y}_t-\hat{y}_t^f\right) + \kappa_{m,t}\left(\widehat{mp}_t-\widehat{mp}_t^f\right) \tag{29}
$$

where $\hat{y}_t - \hat{y}_t^f$ is the *output gap*, $\widehat{mp}_t - \widehat{mp}_t^f$ is the *real money balances gap*,

$$
\kappa_{x,t} = \left(\nu-(\nu-\sigma)\,a_1+\frac{\eta+\alpha}{1-\alpha}\right)\frac{(1-\alpha)\left(\frac{1}{\theta}-\beta\right)(1-\theta)\left(1+(\varepsilon-1)\,\varepsilon_t^p\right)}{1+\left(\alpha+\varepsilon_t^p\right)(\varepsilon-1)}
$$

and

$$
\kappa_{m,t} = (\sigma-\nu)\,(1-a_1)\,\frac{(1-\alpha)\left(\frac{1}{\theta}-\beta\right)(1-\theta)\left(1+(\varepsilon-1)\,\varepsilon_t^p\right)}{1+\left(\alpha+\varepsilon_t^p\right)(\varepsilon-1)}
$$

Then Eq. 29 is our first equation relating inflation to its one period ahead forecast, the output gap and real money balances.

The second key equation describing the equilibrium of the model is obtained by rewriting Eq. 17 so as to determine output

$$\hat{y}_t = E_t\left[\hat{y}_{t+1}\right] - \kappa_r\left(\hat{\imath}_t - E_t\left[\hat{\pi}_{t+1}\right]\right) + \kappa_{mp}E_t\left[\Delta\widehat{mp}_{t+1}\right] + \xi_{t,c} \qquad (30)$$

where $\kappa_r = \frac{1}{\nu-(\nu-\sigma)a_1}$, $\kappa_{mp} = \frac{(\sigma-\nu)(1-a_1)}{\nu-a_1(\nu-\sigma)}$ and $\xi_{t,c} = \kappa_{sm}E_t\left[\Delta\varepsilon^M_{t+1}\right]$ where $\kappa_{sp} = -\frac{1}{\nu-a_1(\nu-\sigma)}$ and $\kappa_{sm} = -\frac{(1-a_1)(\nu-\sigma)}{\nu-a_1(\nu-\sigma)}\frac{1}{1-\nu}$. Eq. 30 is thus a dynamic IS equation including the real money balances.

The third key equation describes the real money stock. From Eq. 5 we obtain

$$\widehat{mp}_t = \hat{y}_t - \kappa_i\hat{\imath}_t + \frac{1}{\nu}\varepsilon^m_t \qquad (31)$$

where $\kappa_i = a_2/\nu$.

B Priors and posteriors

The vertical line of Figures 2 and 3 denotes the posterior mode, the grey line the prior distribution, and the black line the posterior distribution.

Figure 2: Priors and posteriors (σ estimated)

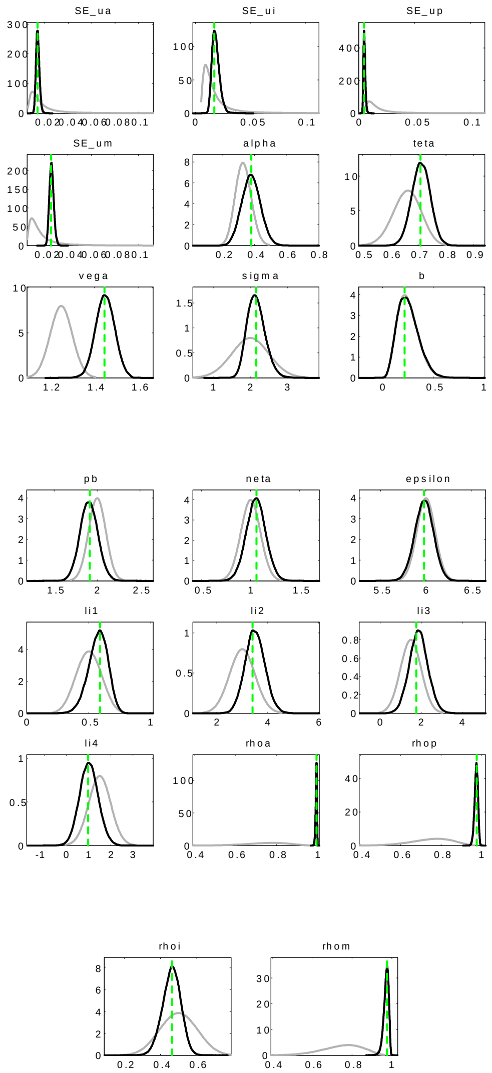
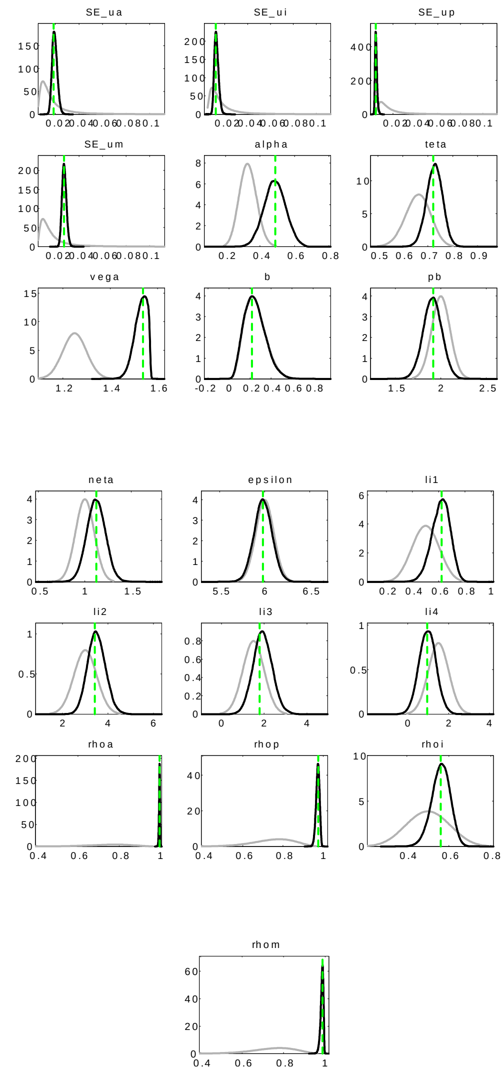


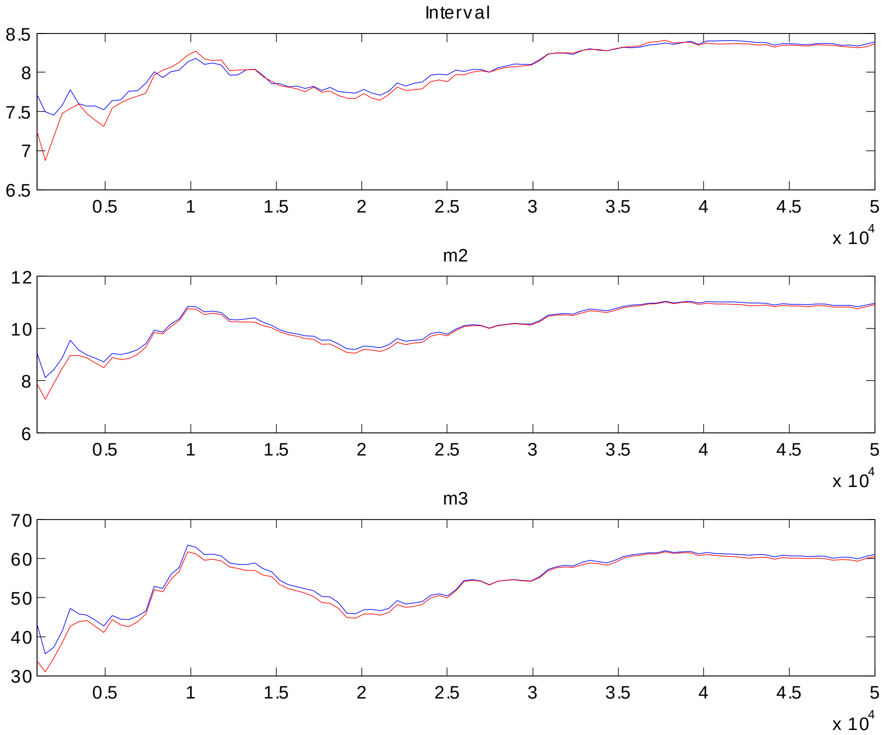
Figure 3: Priors and posteriors ($\sigma = 4$)



C Model validation

The diagnosis concerning the numerical maximization of the posterior kernel indicates that the optimization procedure leads to a robust maximum for the posterior kernel. The convergence of the proposed distribution to the target distribution is thus satisfied.

Figure 4: Multivariate MH convergence diagnosis (σ estimated)

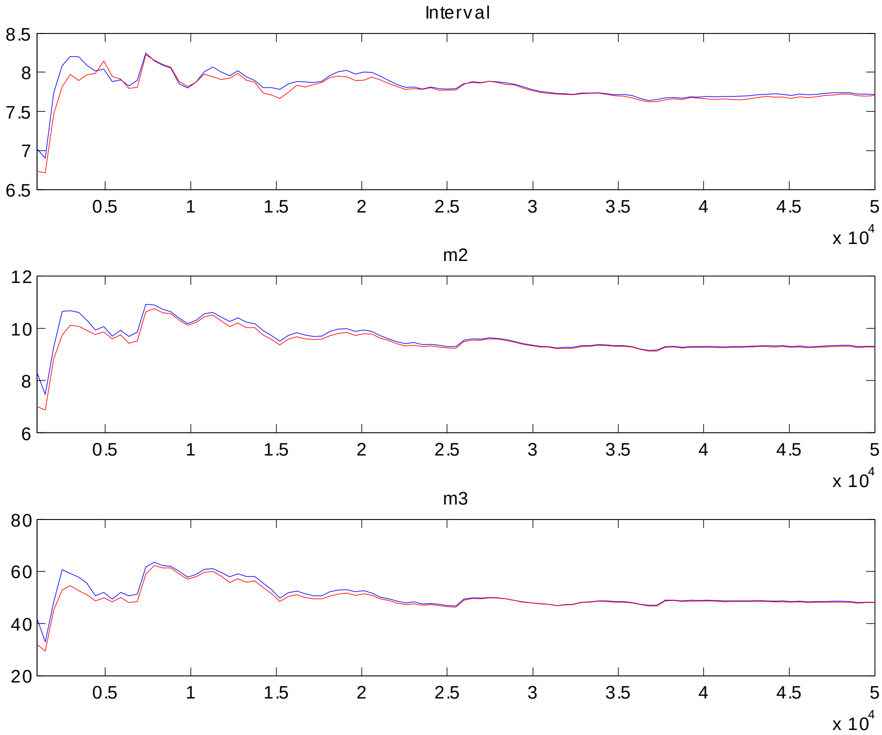


A diagnosis of the overall convergence for the Metropolis-Hastings sampling algorithm is provided in Figure 4 and Figure 5.

Each graph represents specific convergence measures with two distinct lines that show the results within (red line) and between (blue line) chains (Geweke, 1999). Those measures are related to the analysis of the parameters mean (interval), variance (m2) and third moment (m3). For each of the three measures, convergence requires that both lines become relatively horizontal and converge to each other in both models.

From Figure 4, it can be inferred that the model with standard risk aversion needs more chain to stabilize m3 (third moment), in comparison with the case where risk aversion is high (Figure 5).

Figure 5: Multivariate MH convergence diagnosis ($\sigma = 4$)



Diagnosis for each individual parameter (not included but it can be provided upon request) indicates that most of the parameters do not exhibit convergence problems.

Moreover, a BVAR identification analysis (Ratto, 2008) suggests that all parameter values are stable.

The estimates of the innovation component of each structural shock, respectively for the estimated σ and the calibrated $\sigma = 4$, respect the i.i.d. properties and are centered around zero. This reinforces the statistical validity of the estimated model (the corresponding figures can be provided by the authors).

D Macro-parameters

Table 5: Aggregated structural parameters

	σ est.	$\sigma = 4$
υ_a^y	0,8166	0,6849
υ_m^y	-0,1027	-0,3508
υ_c^y	0,0452	0,0304
υ_{sm}^y	0,2294	0,6644
υ_{y+1}^m	-0,6889	-1,0841
υ_y^m	1,6889	2,0841
$\kappa_{x,t}$	0,1057	0,0960
$\kappa_{m,t}$	0,0108	0,0337
κ_r	0,5741	0,3457
κ_{mp}	0,2386	0,7286
κ_{sm}	-0,5328	-1,3799
κ_i	0,3956	0,3748
λ_i	0,5732	0,6146
$(1-\lambda_i)\,\lambda_\pi$	1,4914	1,3455
$(1-\lambda_i)\,\lambda_x$	0,7991	0,7412
$(1-\lambda_i)\,\lambda_m$	0,4314	0,3717

E Impulse response functions

The thin solid line of Figure 6 represents the impulse response functions of the model with estimated risk aversion while the dashed line represents the impulse response functions of the model with high risk aversion ($\sigma = 4$).

After a markup shock, the inflation rate and the nominal interest rise, then gradually decrease toward the steady state. The output and the output gap decrease then increase to their steady state values.

After an interest rate shock, inflation, output and the output gap fall. The real rate of interest rises. Real money growth displays an overshooting/undershooting process in the first periods, then rapidly returns to its steady state value.

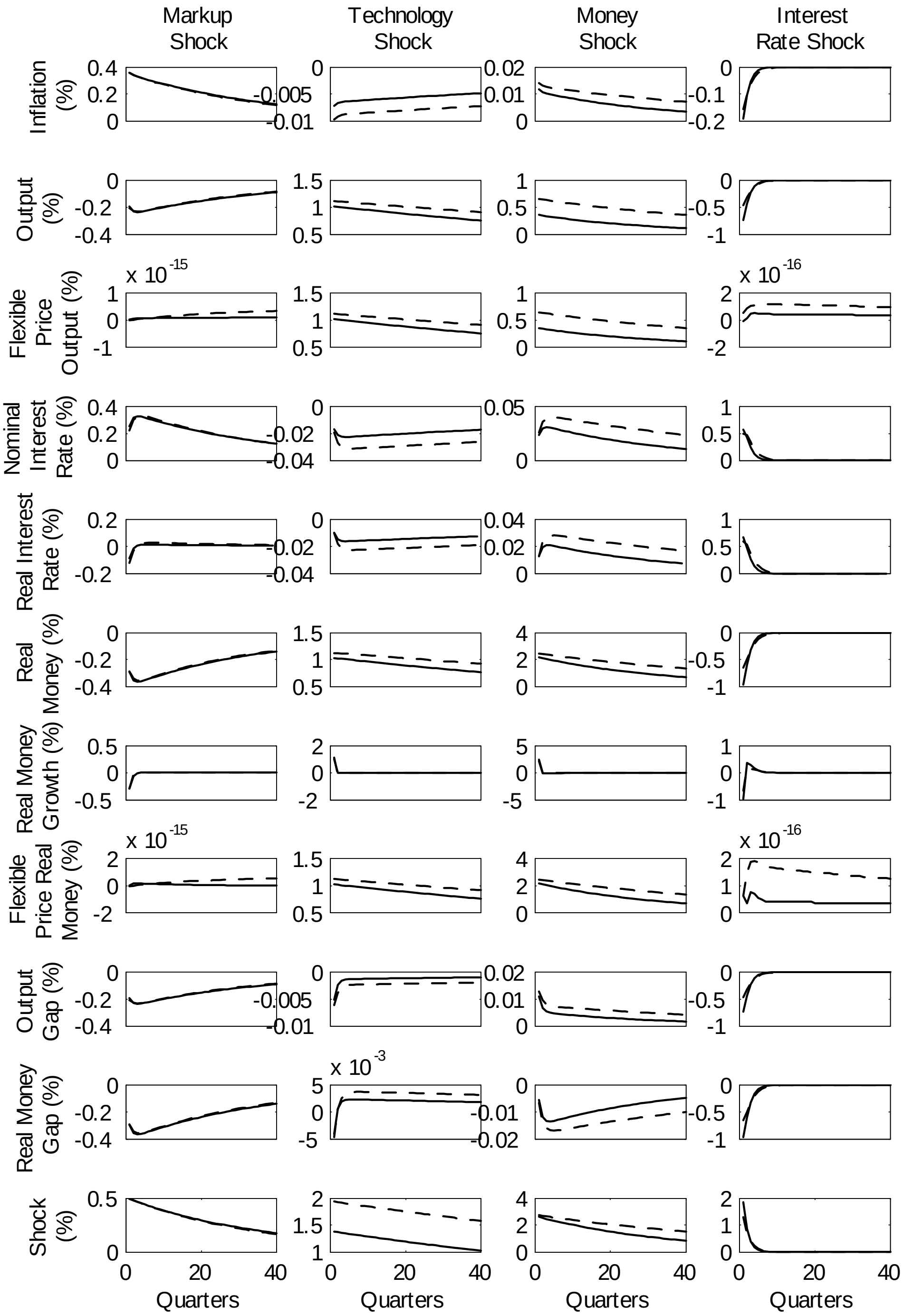
After a technology shock, the output gap, the nominal and real interest rate, and the inflation decrease whereas output as well as real money balances and real money growth rise.

After a money shock, the nominal and the real rate of interest, the output and the output gap rise. Inflation increases a bit then decreases through time to its steady state value.

The flexible-price output and the flexible-price real money balances increase after a technology shock and after a money shock.

All these results are in line with the DSGE literature, especially with Smets and Wouters (2003) and Galí (2008).

Figure 6: Impulse response functions with both risk configurations



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