

The Neo-Fisher Effect: Econometric Evidence from Empirical and Optimizing Models*

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August 26, 2019

Abstract

This paper econometrically investigates whether permanent monetary tightenings increase inflation in the short run. It estimates an empirical and a New-Keynesian model driven by transitory and permanent monetary and real shocks. Temporary increases in the nominal interest-rate are found to lead, in accordance with conventional wisdom, to a decrease in inflation and output and an increase in real rates. The main novel result of the paper is that permanent increases in the nominal interest rate lead to an immediate increase in inflation and output and a decline in real rates. Permanent monetary shocks explain more than 40 percent of inflation changes. This paper represents the first econometric assessment of the presence of the neo-Fisher effect in the data.

JEL Classification: E52,E58.

Keywords: Neo-Fisher Effect, Inflation, Monetary Policy, SVAR Models, New-Keynesian Models, Bayesian Estimation.

*I am indebted to Stephanie Schmitt-Grohé for many comments and discussions. I also thank for comments Kosuke Aoki, Giovanni DellAriccia, and seminar participants at various institutions. Yoon-Joo Jo and Seungki Hong provided excellent research assistance.

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1 Introduction

According to the neo-Fisher effect, a monetary tightening that is expected to be permanent leads to an increase in inflation in the short run. In this paper, I investigate whether the neo-Fisher effect is present in U.S. data. To this end, I estimate an empirical and an optimizing New-Keynesian model, both driven by transitory and permanent monetary and real shocks. I estimate both models on postwar quarterly data using Bayesian techniques.

I find that both estimated models produce similar dynamics. In accordance with conventional wisdom, they predict that a transitory increase in the nominal interest rate causes a fall in inflation and a contraction in real activity. These effects take place in the context of elevated real interest rates. The main result of the paper is that in response to a permanent increase in the nominal interest rate, inflation increases immediately, reaching its higher long-run level within a year. Furthermore, the adjustment to a permanent increase in the nominal interest rate entails no output loss and is characterized by low real interest rates. In both the empirical and optimizing models permanent monetary shocks are estimated to be the main drivers of inflation, explaining 45 percent of the variance of changes in this variable. By contrast, transitory monetary shocks are estimated to play a modest role in explaining nominal and real variables.

These results have policy, theoretical, and econometric implications. With regard to policy, the results suggest that in countries suffering from below-target inflation and near-zero nominal rates, a credible normalization of nominal rates can achieve reflation swiftly and without a recession. In regard to how the findings inform monetary theory, the fact that the empirical and optimizing models have similar dynamic properties together with the fact that the empirical model is mute about how expectations are formed while the optimizing model assumes rational expectations provides discipline to theories of the transmission of monetary shocks that deviate from the assumption of model-consistent expectations. Finally, with regard to its econometric implications, the results of this paper suggest that distinguishing temporary and permanent monetary disturbances provides a resolution of the well-known

price puzzle, according to which a transitory increase in the nominal interest rate is estimated to cause a short-run increase in inflation.

This paper is related to a number of theoretical and empirical contributions on the effects of interest-rate policy on inflation and aggregate activity. De Michelis and Iacoviello (2016) estimate an SVAR model with permanent monetary shocks to evaluate the Japanese experience with Abenomics. They also study the effect of monetary shocks in the context of a calibrated New Keynesian model. The present paper departs from their work in two important dimensions. First, their SVAR model does not include the short-run policy rate. The inclusion of this variable is key in the present paper, because the short-run comovement of the policy rate with inflation is at the core of the neo-Fisher effect. Second, their theoretical model is not estimated and does not include permanent monetary shocks. By contrast, I allow permanent and transitory monetary shocks to compete in the econometric estimation and, as pointed out above, I find that permanent monetary shocks are the main driver of movements in inflation, while the estimated transitory monetary shock plays a relatively small role. Fève, Matheron, and Sahuc (2010) estimate SVAR and dynamic optimizing models with nonstationary inflation-target shocks to study the role of gradualism in disinflation policy. Specifically, they show, by means of counterfactual experiments, that had the European monetary authority been less gradual in lowering its inflation target during the late 2000s, the eurozone would have suffered a milder slowdown in economic growth. The present paper focuses instead on how the short-run comovement of inflation and the policy rate triggered by a monetary disturbance change depending on whether the impulse is permanent or transitory in nature. King and Watson (2012) find that in estimated New-Keynesian models, postwar U.S. inflation is explained mostly by variations in nonstandard shocks, such as markups. In this paper, I show that once one allows for permanent monetary shocks, almost half of the variance of inflation changes is explained by monetary disturbances. Sims and Zha (2006) estimate a regime-switching model for U.S. monetary policy and find that during the postwar period there were three policy regime switches, but that they were too

small to explain the observed increase in inflation of the 1970s or the later disinflation that started with the Volker chairmanship. The empirical and optimizing models estimated in the present paper attribute much of the movements in inflation in these two episodes to the permanent nominal shock. Cogley and Sargent (2005) use an autoregressive framework to produce estimates of long-run inflationary expectations. The predictions of both models estimated in the present paper are consistent with their estimates. This paper is also related to a body of work that incorporates inflation target shocks in the New-Keynesian model. In this regard, the contribution of the present paper is to allow for a permanent component in this source of inflation dynamics. For example, Ireland (2007) estimates a new-Keynesian model with a time-varying inflation target and shows that, possibly as a consequence of the Fed's attempt to accommodate supply-side shocks, the inflation target increased significantly during the 1960s and 1970s and fell sharply in the early 2000s. Using a similar framework as Ireland's, Milani (2009) shows that movements in the inflation target become less pronounced if one assumes that agents must learn about the level of the inflation target.

This paper is also related to recent theoretical developments on the neo-Fisher effect. Schmitt-Grohé and Uribe (2010, 2014) show that the neo-Fisher effect obtains in the context of standard dynamic optimizing models with flexible prices. Specifically, they show that a credible increase in the nominal interest rate that is expected to be sustained for a prolonged period of time can give rise to an immediate increase in inflation. Schmitt-Grohé and Uribe (2010, 2014) show that this result also obtains in models with nominal rigidity. Cochrane (2017) shows that if the monetary policy regime is passive, a temporary increase in the nominal interest rate can cause an increase in the short-run rate of inflation. This notion of the neo-Fisher effect is different from the one studied in the present paper, which defines the neo-Fisher effect as the short-run response of inflation to a permanent movement in the policy rate. Williamson (2018) considers a model with flexible-price and sticky-price goods and shows that movements in the interest rate generate movements in expected flexible-price inflation of equal size. Cochrane (2014) and Williamson (2016) provide nontechnical

expositions of the neo-Fisher effect. Erceg and Levin (2003) study a calibrated dynamic general equilibrium model with nominal rigidity in which private agents have imperfect information about the permanent and transitory components of monetary-policy shocks. They show that imperfect information of this type can provide an adequate explanation of the observed inflation persistence during disinflation episodes. Aruoba and Schorfheide (2011) estimate a model that combines new Keynesian and monetary frictions. They obtain a path and effects of the permanent component of inflation that is in line with those estimated in this paper.

The remainder of the paper is presented in 6 sections. Section 2 presents evidence consistent with the long-run validity of the Fisher effect. Section 3 presents the proposed empirical model and discusses the econometric estimation. The predictions of the empirical model are presented in section 4. Section 5 presents the New-Keynesian model and discusses its econometric estimation. Section 6 presents the predictions of the estimated New-Keynesian model in regard to the neo-Fisher effect. Section 7 closes the paper with a discussion of actual monetary policy in the ongoing low-inflation era from the perspective of the two estimated models. Data and replication code are available on my website.¹

2 Preliminaries: Evidence on the Fisher Effect

What is the effect of an increase in the nominal interest rate on inflation and output? One can argue on theoretical grounds that the answer to this question depends on (a) whether the increase in the interest rate is expected to be permanent or transitory; and (b) whether the horizon of interest is the short run or the long run. Table 1 suggests an answer that summarizes both the starting point and the contribution of this paper.

A large body of empirical and theoretical studies argue that a transitory positive disturbance in the nominal interest rate causes a transitory increase in the real interest rate, which in turn depresses aggregate demand and inflation, entry (1,2) in the table (see, for exam-

¹<http://www.columbia.edu/~mu2166>.

Table 1: Effect of an Increase in the Nominal Interest Rate on Inflation

	Long	Short
	Run	Run
	Effect	Effect
Transitory shock	0	↓
Permanent shock	↑	↑?

Note. Entry (2,1): The Fisher effect. Entry (2,2) : The Neo-Fisher effect.

ple, Christiano, Eichenbaum, and Evans, 2005, especially figure 1). Similarly, a property of virtually all modern models studied in monetary economics is that a transitory increase in the nominal interest rate has no effect on inflation in the long run, entry (1,1). By contrast, if the increase in the nominal interest rate is permanent, sooner or later, inflation will have to increase by roughly the same magnitude, if the real interest rate, given by the difference between the nominal rate and expected inflation, is not determined by nominal factors in the long run, entry (2,1) in the table. This one-to-one long-run relationship between nominal rates and inflation is known as the Fisher effect. The neo-Fisher effect says that a permanent increase in the nominal interest rate causes an increase in inflation not only in the long run but also in the short run, entry (2,2) in the table. Ascertaining whether the neo-Fisher effect is present in U.S. data is the focus of the present investigation.

Before plunging into an econometric analysis of the neo-Fisher effect, I wish to briefly present evidence consistent with the Fisher effect. The rationale for doing so is that my empirical analysis of the neo-Fisher effect assumes the empirical validity of the Fisher effect, interpreted as a description of the long-run relationship between the nominal interest rate and inflation. The Fisher equation takes the form

$$i_t = R_t + E_t \pi_{t+1}$$

where i_t denotes the nominal interest rate, R_t denotes the real interest rate, π_t denotes the inflation rate, and E_t denotes expectations conditional on information available in period t .

This expression says that the nominal interest rate incorporates two types of compensation to lenders. One is a compensation for the loss of purchasing power of money due to expected inflation during the investment period, and the other is a real compensation for postponing consumption. Assuming that on average expected inflation equals actual inflation, we have that

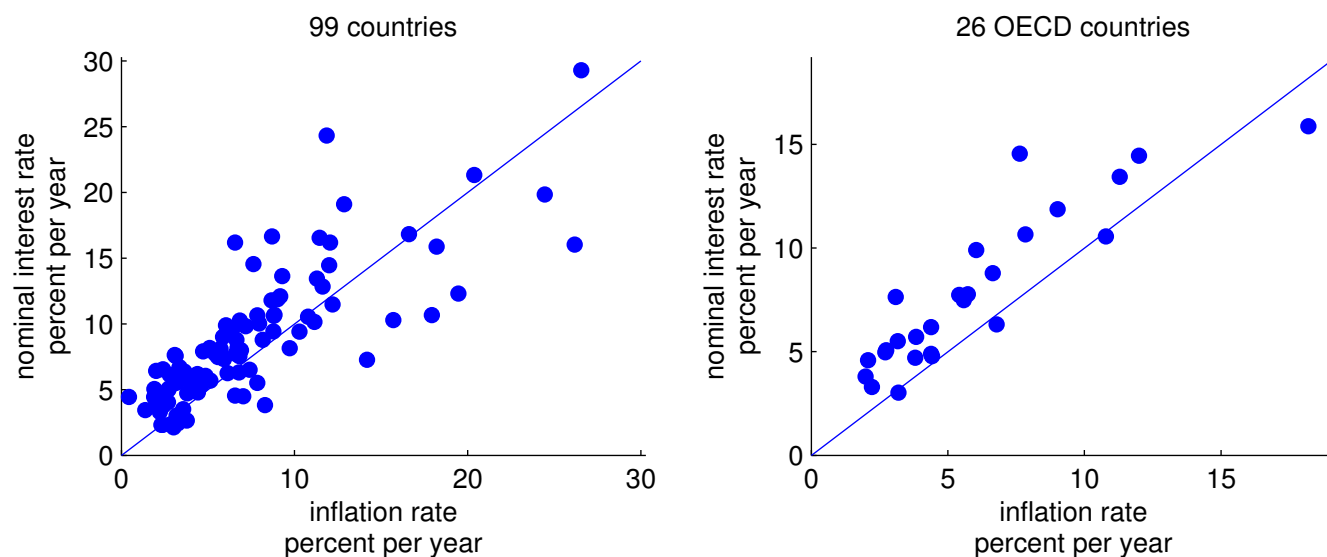
$$i = R + \pi,$$

where variables without a subscript refer to long-run averages. Further assuming that the average real interest rate is determined mostly by non-monetary factors (such as technology, demographics, distortionary taxes, or economic openness), and that long-run variations in inflation are larger than variations in real rates, the above expression delivers a near one-to-one long-run relationship between the nominal interest rate and the rate of inflation.

The left panel of figure 1 displays times averages of inflation and nominal interest rates across 99 countries. Each dot in the graph corresponds to one country. The typical sample covers the period 1989 to 2012. The scatter plot is consistent with the Fisher effect in the sense that increases in the nominal interest rate are roughly associated with one-for-one increases in the rate of inflation. This is also the case for the subsample of OECD countries (right panel), which are on average half as inflationary as the group of non-member countries. Figure 2 presents empirical evidence consistent with the Fisher effect from the time perspective. It plots inflation and the nominal interest rate in the United States over the period 1954:Q4 to 2018:Q2. In spite of the fact that the data have a quarterly frequency, it is possible to discern a positive long-run association between inflation and the nominal rate. This relation becomes even more apparent if one removes the cyclical component of both series as in Nicolini (2017), who separates trend and cycle using the HP filter. The high-inflations of the 1970s and 1980s coincided with high levels of the interest rate. Symmetrically, the relatively low rates of inflation observed since the early 1990s have been accompanied by low nominal rates.

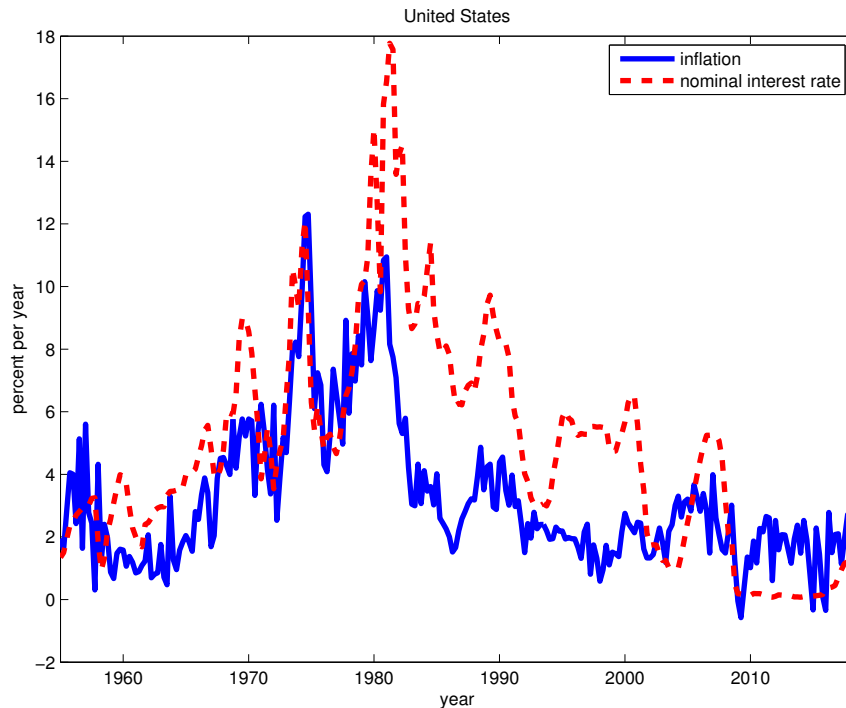
The Fisher effect, however, does not provide a prediction of when inflation should be

Figure 1: Average Inflation and Nominal Interest Rates: Cross-Country Evidence



Notes. Each dot represents one country. For each country, averages are taken over the longest available noninterrupted sample. The average sample covers the period 1989 to 2012. The solid line is the 45-degree line. Source: World Development Indicators (WDI) available at data.worldbank.org/indicator. Inflation is the CPI inflation rate (code FP.CPI.TOTL.ZG). The nominal interest rate is the treasury bill rate. The WDI database provides this time series not directly, but as the difference between the lending interest rate (code FR.INR.LEND) and the risk premium on lending (lending rate minus treasury bill rate, code FR.INR.RISK). Countries for which one or more of these series were missing as well as outliers, defined as countries with average inflation or interest rate above 50 percent, were dropped from the sample.

Figure 2: Inflation and the Nominal Interest Rate in the United States



Notes. Quarterly frequency. Source: See section 3.3.

expected to catch up with a permanent increase in the nominal interest rate. It only states that it must eventually do so. A natural question, therefore, is how quickly does inflation adjust to a permanent increase in the nominal interest rate? The remainder of this paper is devoted to addressing this question.

3 An Empirical Model

The empirical model aims to capture the dynamics of three macroeconomic indicators, namely, the logarithm of real output per capita, denoted y_t , the inflation rate, denoted π_t and expressed in percent per year, and the nominal interest rate, denoted i_t and also expressed in percent per year. I assume that y_t , π_t , and i_t are driven by four exogenous shocks: a nonstationary (or permanent) monetary shock, denoted X_t^m , a stationary (or transitory) monetary shock, denoted z_t^m , a nonstationary nonmonetary shock, denoted X_t^n , and a stationary nonmonetary shock, denoted z_t^n . The focus of my analysis is the short-run ef-

fects of permanent and transitory interest-rate shocks, embodied in the exogenous variables X_t^m and z_t^m . The shocks X_t^n and z_t^n are meant to capture the nonstationary and stationary components of combinations of nonmonetary disturbances of different natures, such as technology shocks, preference shocks, or markup shocks, which my analysis is not intended to individually identify.

I assume that output is cointegrated with X_t^n and that inflation and the nominal interest rate are both cointegrated with X_t^m . Because X_t^m is the permanent component of both inflation and the nominal interest rate, throughout the paper, I refer to it as a permanent monetary shock. The exogenous variable X_t^m admits different interpretations. Under certain conditions, it can be interpreted as an inflation-target shock. For example, I show later that X_t^m fell significantly in the United States in the first half of the 1980s. It is reasonable to interpret this movement in X_t^m as a reduction in the U.S. inflation target. In other contexts, however, movements in X_t^m are less interpretable as changes in the inflation target. For example, the Bank of Japan lowered interest rates to zero in the mid 1990s and continues to keep them at that level to date. At the same time the average Japanese rate of inflation has moved further below its target of 2 percent. Regardless of how X_t^m is interpreted, the focus of the present paper is the short-run behavior of inflation and the nominal interest rate in response to a shock that changes both of them in the long run, independently of whether or not this long-run movement is accompanied by a change in the communicated inflation target.

I define the following vector containing stationary variables

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix} \equiv \begin{bmatrix} y_t - X_t^n \\ \pi_t - X_t^m \\ i_t - X_t^m \end{bmatrix}.$$

The variable \hat{y}_t represents detrended output, and $\hat{\pi}_t$ and \hat{i}_t represent the cyclical components of inflation and the nominal interest rate, respectively. Because inflation and the nominal

interest rate share a common nonstationary component, they are cointegrated. In other words, the Fisher effect holds, in the sense that shocks that cause a permanent change in the nominal interest rate also cause the same permanent change in the inflation rate. But the assumption that π_t and i_t are cointegrated says nothing about the neo-Fisher effect, that is, about the short-run effect on inflation and output of a permanent monetary shock.

I assume that the law of motion of the vector $\begin{bmatrix} \hat{y}_t & \hat{\pi}_t & \hat{i}_t \end{bmatrix}$ takes the autoregressive form²

$$\begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix} = \sum_{i=1}^L B_i \begin{bmatrix} \hat{y}_{t-i} \\ \hat{\pi}_{t-i} \\ \hat{i}_{t-i} \end{bmatrix} + C \begin{bmatrix} \Delta X_t^m \\ z_t^m \\ \Delta X_t^n \\ z_t^n \end{bmatrix} \quad (1)$$

where $\Delta X_t^m \equiv X_t^m - X_{t-1}^m$ and $\Delta X_t^n \equiv X_t^n - X_{t-1}^n$ denote changes in the nonstationary shocks. The objects B_i , for $i = 1, \dots, L$, are 3-by-3 matrices of coefficients, C is a 3-by-4 matrix of coefficients, and L is a scalar denoting the lag length of the autoregressive block containing endogenous variables.

I assume that the driving forces follow univariate AR(1) laws of motion of the form

$$\begin{bmatrix} \Delta X_{t+1}^m \\ z_{t+1}^m \\ \Delta X_{t+1}^n \\ z_{t+1}^n \end{bmatrix} = \rho \begin{bmatrix} \Delta X_t^m \\ z_t^m \\ \Delta X_t^n \\ z_t^n \end{bmatrix} + \psi \begin{bmatrix} \epsilon_{t+1}^1 \\ \epsilon_{t+1}^2 \\ \epsilon_{t+1}^3 \\ \epsilon_{t+1}^4 \end{bmatrix} \quad (2)$$

where ρ and ψ are 4-by-4 diagonal matrices of coefficients, and ϵ_t^i are i.i.d. disturbances distributed $N(0, 1)$.

²The presentation of the model omits intercepts. A detailed exposition is in the appendix.

3.1 Identification Restrictions

Thus far, I have introduced three identification assumptions, namely, that output is cointegrated with X_t^n and that inflation and the interest rate are cointegrated with X_t^m . In addition, to identify the transitory monetary shock, I adopt a methodology pioneered by Uhlig (2005) and impose sign restrictions on the impact effect of these disturbances on endogenous variables. Specifically, I assume that

$$C_{12} \leq 0 \text{ and } C_{22} \leq 0,$$

where C_{ij} denotes the (i, j) element of C . These two conditions restrict transitory exogenous increases in the interest rate to have nonpositive impact effects on output and inflation. Finally, without loss of generality, I introduce the normalizations $C_{32} = C_{14} = 1$.

To establish the identifiability of the estimated parameters of the model I apply the test proposed by Iskrev (2010). This procedure consists in calculating the derivative of the predicted autocovariogram of the observables with respect to the vector of estimated parameters. Identifiability obtains if the matrix of derivatives has rank equal to the length of the vector of estimated parameters. Evaluating the parameters of the model at their posterior mean, I find that the rank condition is satisfied. This means that in a neighborhood of the posterior mean, the predicted covariogram is uniquely determined by the value of the vector of estimated parameters.

3.2 Observables, Priors, and Estimation Method

All variables in the system (1)-(2) are unobservable. To estimate the parameters of the matrices defining this system, I use observable variables for which the model has precise predictions. Specifically, I use observations of output growth, Δy_t , the change in the nominal

interest rate, Δi_t , and the interest-rate-inflation differential,

$$r_t \equiv i_t - \pi_t.$$

These three variables are stationary by the maintained long-run identification assumptions. The following equations link the observables to variables included in the unobservable system (1)-(2):

$$\begin{aligned} \Delta y_t &= \hat{y}_t - \hat{y}_{t-1} + \Delta X_t^n, \\ r_t &= \hat{i}_t - \hat{\pi}_t, \\ \Delta i_t &= \hat{i}_t - \hat{i}_{t-1} + \Delta X_t^m \end{aligned} \tag{3}$$

I assume that Δy_t , r_t , and Δi_t are observed with measurement error. Formally, letting o_t be the vector of variables observed in quarter t , I assume that

$$o_t = \begin{bmatrix} \Delta y_t \\ r_t \\ \Delta i_t \end{bmatrix} + \mu_t \tag{4}$$

where μ_t is a 3-by-1 vector of measurement errors distributed i.i.d. $N(\emptyset, R)$, and R is a diagonal variance-covariance matrix.

To compute the likelihood function, it is convenient to use the state-space representation of the model. Define the vector of endogenous variables $\hat{Y}_t \equiv [\hat{y}_t \ \hat{\pi}_t \ \hat{i}_t]'$ and the vector of

driving forces $u_t \equiv [\Delta X_t^m \ z_t^m \ \Delta X_t^n \ z_t^n]'$. Then the state of the system is given by

$$\xi_t \equiv \begin{bmatrix} \widehat{Y}_t \\ \widehat{Y}_{t-1} \\ \vdots \\ \widehat{Y}_{t-L+1} \\ u_t \end{bmatrix},$$

and the system composed of equations (1)-(4) can be written as follows:

$$\xi_{t+1} = F\xi_t + P\epsilon_{t+1}$$

$$o_t = H'\xi_t + \mu_t,$$

where the matrices F , P , and H are known functions of B_i , $i = 1, \dots, L$, C , ρ , and ψ and are presented in the appendix. This representation allows for the use of the Kalman filter to evaluate the likelihood function, which facilitates estimation.

I estimate the model on quarterly data using Bayesian techniques. I include 4 lags in equation (1) ($L = 4$), which is a lag length commonly adopted in the related literature (e.g., Christiano, Eichenbaum, and Evans, 2005). Table 2 displays the prior distributions of the estimated coefficients. The prior distributions of all elements of B_i , for $i = 1, \dots, L$, are assumed to be normal. In the spirit of the Minnesota prior (MP), I assume a prior parameterization in which at the mean of the prior parameter distribution the elements of \widehat{Y}_t follow univariate autoregressive processes. So when evaluated at their prior mean, only the main diagonal of B_1 takes nonzero values and all other elements of B_i for $i = 1, \dots, L$ are nil. Because the system (1)-(2) is cast in terms of stationary variables, I deviate from the random-walk assumption of the MP and instead impose an autoregressive coefficient of 0.95 in all equations, so that all elements along the main diagonal of B_1 take a prior mean

Table 2: Prior Distributions

Parameter	Distribution	Mean.	Std. Dev.
Main diagonal elements of B_1	Normal	0.95	0.5
All other elements of B_i , $i = 1, \dots, L$	Normal	0	0.25
C_{21}, C_{31}	Normal	-1	1
$-C_{12}, -C_{22}$	Gamma	1	1
All other estimated elements of C	Normal	0	1
ψ_{ii} , $i = 1, 2, 3, 4$	Gamma	1	1
ρ_{ii} , $i = 1, 2, 3$	Beta	0.3	0.2
ρ_{44}	Beta	0.7	0.2
R_{ii}	Uniform $\left[0, \frac{\text{var}(o_t)}{10}\right]$	$\frac{\text{var}(o_t)}{10 \times 2}$	$\frac{\text{var}(o_t)}{10 \times \sqrt{12}}$

of 0.95. I assign a prior standard deviation of 0.5 to the diagonal elements of B_1 , which implies a coefficient of variation close to one half ($0.5/0.95$). As in the MP, I impose lower prior standard deviations on all other elements of the matrices B_i for $i = 1, \dots, L$, and set them to 0.25.

The coefficient C_{21} takes a normal prior distribution with mean -1 and standard deviation 1. The value assigned to the mean of this distribution implies a prior belief that the impact effect of a permanent interest rate shock on inflation, given by $1 + C_{21}$, can be positive or negative with equal probability. I make the same assumption about the impact effect of permanent monetary shocks on the nominal interest rate itself, therefore assigning to C_{31} a normal prior distribution with mean -1 and standard deviation 1. As explained above, $-C_{12}$ and $-C_{22}$ are restricted to be nonnegative. I assume that these objects have Gamma prior distributions with mean and standard deviations equal to one. Thus, identification of the transitory monetary shock is achieved via restrictions of prior distributions. All other parameters of the matrix C , except C_{32} and C_{14} , which are normalized to unity, are assigned a normal prior distribution with mean 0 and standard deviation 1.³

³One might wonder whether a rationale like the one I used to set the prior mean of C_{21} could apply to C_{13} , the parameter governing the impact output effect of a nonstationary nonmonetary shock, X_t^n , which is given by $1 + C_{13}$. To see why a prior mean of 0 for C_{13} might be more reasonable, consider the effect of an innovation in the permanent component of TFP, which is perhaps the most common example of a nonstationary nonmonetary shock in business-cycle analysis. Specifically, consider a model with the Cobb-

The parameters ψ_{ii} , for $i = 1, \dots, 4$, representing the standard deviations of the four exogenous innovations in the AR(1) process (2) are all assigned Gamma prior distributions with mean and standard deviation equal to one. I impose nonnegative serial correlations on the four exogenous shocks ($\rho_{ii} \in (0, 1)$ for $i = 1, \dots, 4$), and adopt Beta prior distributions for these parameters. I assume relatively small means of 0.3 for the prior serial correlations of the two monetary shocks and the nonmonetary nonstationary shock and a relatively high mean of 0.7 for the stationary nonmonetary shock. The small prior mean serial correlations for the monetary shocks reflect the usual assumption in the related literature of serially uncorrelated monetary shocks. The relatively small prior mean serial correlation for the nonstationary nonmonetary shock reflects the fact that the growth rate of the stochastic trend of output is typically estimated to have a small serial correlation. Similarly, the relatively high prior mean of the serial correlation of the stationary nonmonetary shock reflects the fact that typically these shocks (e.g., the stationary component of TFP) are estimated to be persistent. The prior distributions of all serial correlations are assumed to have a standard deviation of 0.2. The variances of all measurement errors are assumed to have a uniform prior distribution with lower bound 0 and upper bound of 10 percent of the sample variance of the corresponding observable indicator.

Finally, to draw from the posterior distribution of the estimated parameters, I apply the Metropolis-Hastings sampler to construct a Monte-Carlo Markov chain (MCMC) of one million draws after burning the initial 100 thousand draws. Posterior means and error bands around the impulse responses shown in later sections are constructed from a random subsample of the MCMC chain of length 100 thousand with replacement.

Douglas production function $y_t = X_t^n + z_t^n + \alpha k_t + (1 - \alpha)h_t$ expressed in logarithms. Consider first a situation in which capital and labor, denoted k_t and h_t , do not respond contemporaneously to changes in X_t^n . In this case, the contemporaneous effect of a unit increase in X_t^n on output is unity, which implies that a prior mean of 1 for $1 + C_{13}$, or equivalently a prior mean of 0 for C_{13} is the most appropriate. Now consider the impact effect of changes in X_t^n on k_t and h_t . It is reasonable to assume that the stock of capital, k_t , is fixed in the short run. The response of h_t depends on substitution and wealth effects. The former tends to cause an increase in employment, and the latter a reduction. Which effect will prevail is not clear, giving credence to a prior of 0 for C_{13} . One could further think about the role of variable input utilization. An increase in X_t^n is likely to cause an increase in utilization, further favoring a prior mean of 0 over one of -1 for C_{13} .

3.3 Data and Unit Root Tests

I estimate the empirical model on quarterly U.S. data spanning the period 1954:Q3 to 2018:Q2. The proxy for y_t is the logarithm of real GDP seasonally adjusted in chained dollars of 2012 minus the logarithm of the civilian noninstitutional population 16 years old or older. The proxy for π_t is the growth rate of the implicit GDP deflator expressed in percent per year. In turn, the implicit GDP deflator is constructed as the ratio of GDP in current dollars and real GDP both seasonally adjusted. The proxy for i_t is the monthly Federal Funds Effective rate converted to quarterly frequency by averaging and expressed in percent per year. The source for nominal and real GDP is the Bureau of Economic Analysis (bea.gov), the source for population is the Bureau of Labor Statistics (bls.gov), and the source for the Federal Funds rate is the Board of Governors of the Federal Reserve System (federalreserve.gov). All series were downloaded in August 2018.

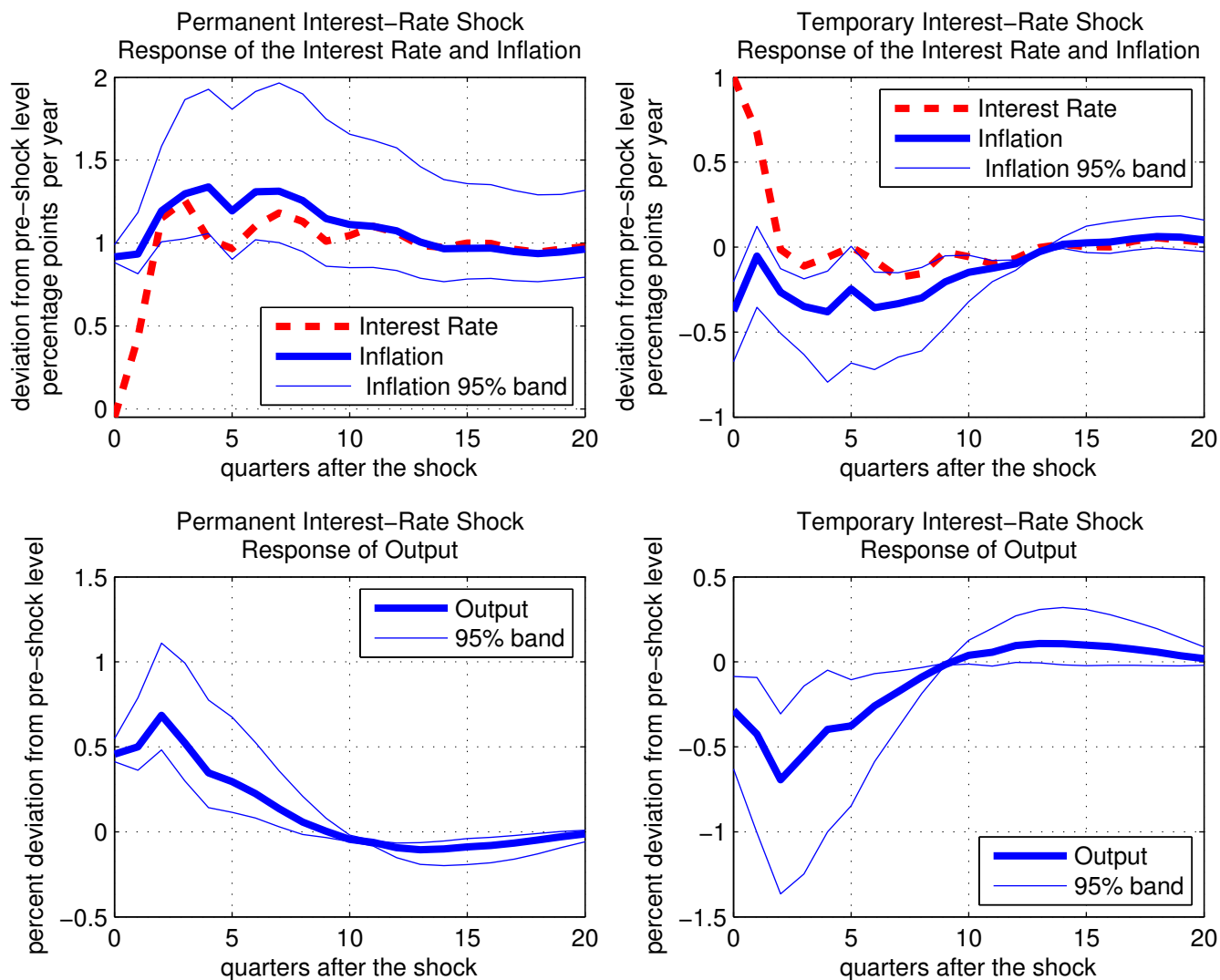
Before presenting the predictions of the empirical model, I briefly report standard unit-root tests based on univariate representations of the data. The augmented Dickey-Fuller (ADF) test, which is a commonly used test of the null hypothesis of a unit root, fails to reject the null hypothesis for y_t , i_t , and π_t , and rejects it for $i_t - \pi_t$ at standard confidence levels of 10 percent or less.⁴ These results are in line with the assumption that the interest rate, inflation, and output, all possess unit roots, and that the interest-inflation differential is stationary.

4 The Neo-Fisher Effect in the Empirical Model

Figure 3 displays mean posterior estimates of the responses of inflation, output, and the nominal interest rate to permanent (left panels) and temporary (right panels) interest-rate

⁴Specifically, for a random variable x_t , the ADF test considers the null hypothesis that $x_t = x_{t-1} + \eta_0 + \sum_{i=1}^I \eta_i \Delta x_{t-i} + \epsilon_t$, where ϵ_t is white noise, against the alternative hypothesis that $x_t = \delta x_{t-1} + \gamma t + \eta_0 + \sum_{i=1}^I \eta_i \Delta x_{t-i} + \epsilon_t$, with $\delta < 1$. For i_t and π_t , I restrict γ to be zero (no time trend), and for $i_t - \pi_t$, I restrict γ and η_0 to be zero (no time trend or drift). I include 4 lags of Δx_t ($I = 4$). The p values for $x_t = y_t, i_t, \pi_t, i_t - \pi_t$ are, respectively, 0.604, 0.131, 0.135, and 0.0351.

Figure 3: Impulse Responses to Permanent and Temporary Interest-Rate Shocks: Empirical Model



Notes. Impulse responses are posterior mean estimates. Asymmetric error bands are computed using the Sims-Zha (1999) method. Replication code: `plot_ir_chain.m` in `empirical_model.zip`.

shocks, along with asymmetric 95-percent error bands constructed using the method proposed by Sims and Zha (1999). The size of the permanent interest-rate shock is set to ensure that on average it leads to a 1 percent increase in the nominal interest rate in the long run, where the average is taken over the posterior distribution of impulse responses. Because inflation is cointegrated with the nominal interest rate, it also is expected to increase by 1 percent in the long run. The main result conveyed by figure 3 is that the adjustment of inflation to its higher long-run level takes place in the short run. In fact, inflation increases by 1 percent on impact and remains around that level thereafter.

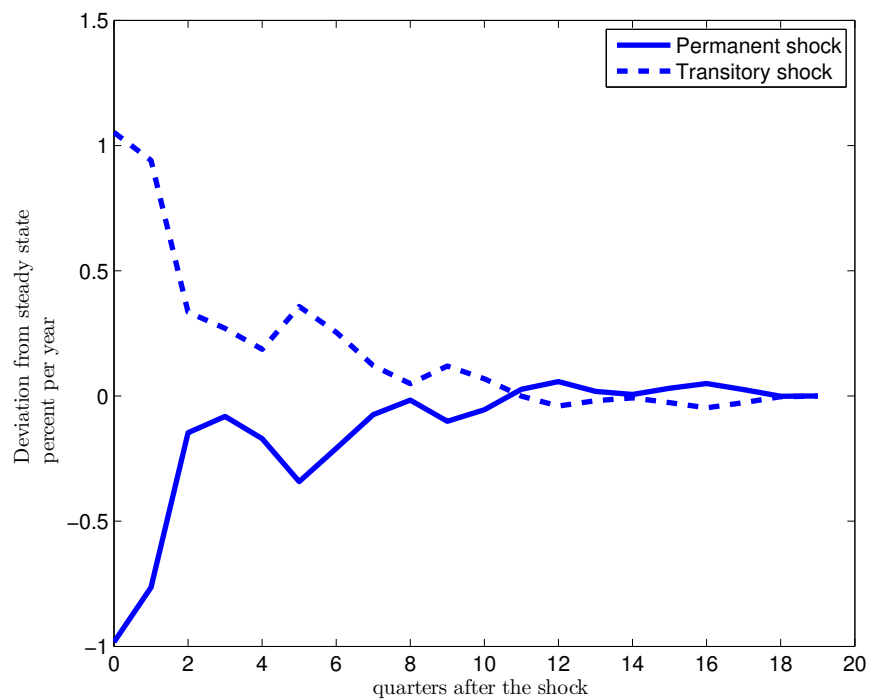
On the real side of the economy, the permanent increase in the nominal interest rate does not cause a contraction in aggregate activity. Indeed, output exhibits a transitory expansion.⁵ This effect could be the consequence of low real interest rates resulting from the swift reflation of the economy following the permanent interest-rate shock. Figure 4 displays with a solid line the response of the real interest rate, defined as $i_t - E_t\pi_{t+1}$, to a permanent interest-rate shock. Because of the faster response of inflation relative to that of the nominal interest rate, the real interest rate falls by almost 1 percent on impact and converges to its steady-state level from below, implying that the entire adjustment to a permanent interest-rate shock takes place in the context of low real interest rates.

The responses of nominal and real variables to a transitory interest-rate shock, shown in the right panels of figure 3 are quite conventional. Both inflation and output fall below trend and remain low for a number of quarters. The real interest rate, whose impulse response is shown with a broken line in figure 4, increases on impact and remains above its long-run value during the transition, which is in line with the contractionary effect of the transitory increase in the interest rate.

Interestingly, the model does not suffer from the price puzzle, which plagues empiri-

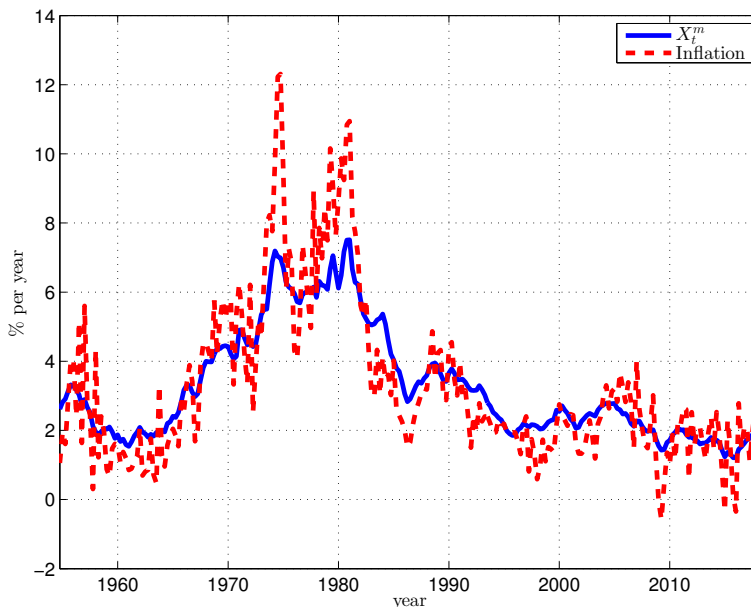
⁵In period 11, the error band narrows to 3 basis points. This is not an uncommon feature of error bands of the type proposed by Sims and Zha (see, for example, the applications in their paper). It is a reflection of little uncertainty about the position of the impulse response in that period. Additional uncertainty may remain about other features of the impulse response in that period, such as its shape. A similar comment applies for the responses of inflation and output to a temporary monetary shock.

Figure 4: Response of the Real Interest Rate to Permanent and Transitory Interest-Rate Shocks: Empirical Model



Notes. Posterior mean estimates. The real interest rate is defined as $i_t - E_t \pi_{t+1}$. Replication code: `real_interest_rate.m` in `empirical_model.zip`.

Figure 5: Inflation and Its Permanent Component: Empirical Model



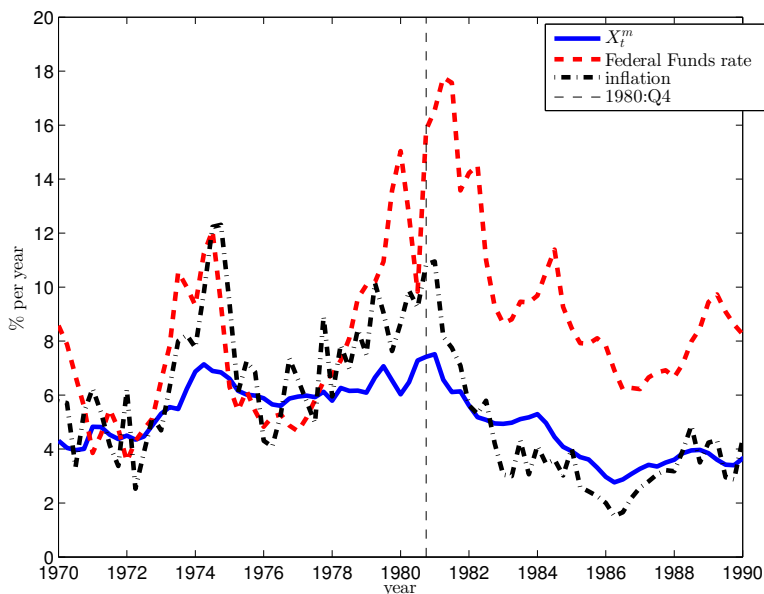
Note. Quarterly frequency. The inferred path of the permanent component of inflation, X_t^m , was computed by Kalman smoothing and evaluating the empirical model at the posterior mean of the estimated parameter vector. The initial value of X_t^m was normalized to make the average value of X_t^m equal to the average rate of inflation over the sample period, 1954:Q4 to 2018:Q2. Replication code: `plot_xm.m` in `empirical_model.zip`.

cal models with only stationary monetary shocks, pointing to the importance of explicitly distinguishing between temporary and permanent shocks.

What does the permanent component of U.S. inflation look like according to the estimated empirical model? Figure 5 displays the actual rate of inflation along with its permanent component, given by the nonstationary monetary shock, X_t^m , over the estimation period (1954:Q4 to 2018:Q2). The path of X_t^m resembles the estimate of long-run inflation expectations reported in much of the related empirical literature, see, for example, Cogley and Sargent (2005) and the references cited therein. This result is reassuring because it shows that the short-run effects of temporary and permanent monetary shocks reported in figure 3 are not based on an estimate of the permanent component of inflation that is at odds with those obtained in the related literature.

Figure 5 reveals a number of features of the low-frequency drivers of postwar inflation in the United States. First, inflationary factors began to build up much earlier than the oil crisis

Figure 6: The Volcker Disinflation



Note. See note to figure 5.

of the early 1970s. Indeed, the period 1963 to 1972, corresponding to the last seven years in office of Fed Chairman William M. Martin and the first three years of Chairman Arthur F. Burns, were characterized by a continuous increase in the permanent component of inflation, from about 2 percent per year to about 5 percent per year. Second, the high inflation rates associated with the oil crises of the mid 1970s were not entirely due to nonmonetary shocks. The Fed itself contributed by maintaining X_t^m at the high level it had reached prior to the oil crisis. Third, the figure indicates that the normalization of rates that began in 2015 and put an end to seven years of near-zero nominal rates triggered by the global financial crisis is interpreted by the empirical model as having a significant permanent component.

It is of interest to zoom in on the Volcker era, which arguably represents the largest disinflation episode in the postwar United States. Figure 6 displays the nominal interest rate, the inflation rate, and the permanent monetary shock X_t^m over the period 1970 to 1990. The vertical broken line indicates 1980:Q4, which according to Goodfriend and King (2005) represents the beginning of the “deliberate disinflation.” The graph suggests that according to the estimated model the Volcker policy was a combination of a large transitory

Table 3: Variance Decomposition: Empirical Model

	Δy_t	$\Delta \pi_t$	Δi_t
Permanent Monetary Shock, ΔX_t^m	9.1	44.6	21.9
Transitory Monetary Shock, z_t^m	2.1	6.2	10.9
Permanent Non-Monetary Shock, ΔX_t^n	49.8	27.9	13.5
Transitory Non-Monetary Shock, z_t^n	39.1	21.4	53.7

Note. Posterior means. The variables Δy_t , $\Delta \pi_t$, and Δi_t denote output growth, the change in inflation, and the change in the nominal interest rate, respectively. Replication code: `table_vardecomp.m` in `empirical_model.zip`.

increase in the policy rate and a gradual decrease in its permanent component. The impulse responses shown in figure 3 suggest that both of these measures are deflationary. This is consistent with the fact that, as shown in figure 6, inflation fell faster than its permanent component. Specifically, at the beginning of the stabilization program, 1980:Q4, inflation was about 3 percentage points above its permanent component, whereas by 1983 it was already below it, in spite of the fact that the permanent component continued to fall. In fact, one of the most remarkable features of the Volcker disinflation is the speed at which inflation fell. This transition toward low inflation was characterized by depressed economic activity, which is consistent with the enormous magnitude of the hike in the transitory component of the interest rate. According to the empirical model, a decrease in the permanent component of the interest rate would have sufficed to bring about low inflation without unemployment.⁶

How important are nonstationary monetary shocks? The relevance of the neo-Fisher effect depends not only on whether it can be identified in actual data, which has been the focus of this section thus far, but also on whether permanent monetary shocks play a significant role in explaining short-run movements in the inflation rate. If nonstationary monetary shocks played a marginal role in explaining cyclical movements in nominal variables, the neo-Fisher effect would just be an interesting curiosity. To shed light on this question, table 3 displays the variance decomposition of the three variables of interest, output growth, the

⁶This statement is, of course, subject to the Lucas critique. However, it is confirmed by the optimizing model I study in section 5.

change in inflation, and the change in the nominal interest rate, predicted by the estimated empirical model. The table shows that the nonstationary monetary shock explains 45 percent of the change in inflation, 22 percent of changes in the nominal interest rate, and 9 percent of the growth rate of output. Thus, the empirical model assigns a significant role to this type of monetary disturbance, especially in explaining movements in nominal variables. In comparison, the stationary monetary shock explains a relatively small fraction of movements in the three macroeconomic indicators included in the model. These results suggest that the neo-Fisher effect emanates from a relevant driver of nominal variables. More generally, in light of the fact that the majority of studies in Monetary Economics limits attention to the study of stationary nominal shocks, the results reported in table 3 call for devoting more attention to understanding the effects of nonstationary monetary disturbances.

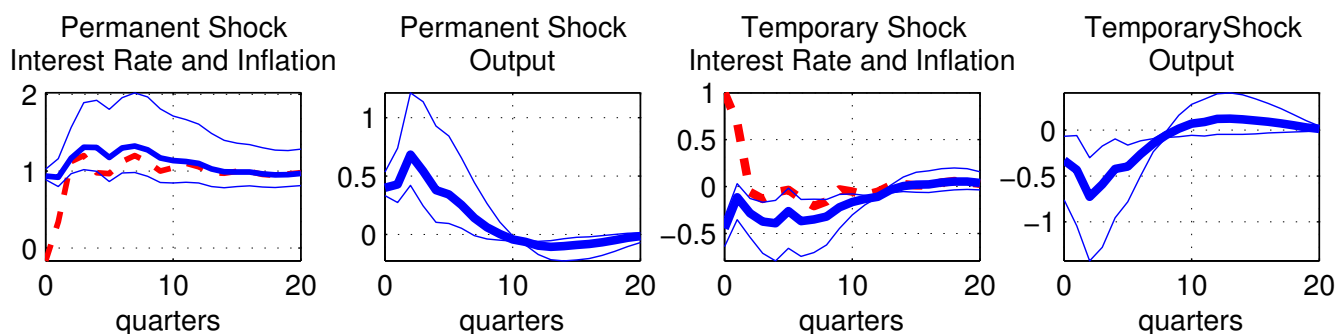
4.1 Robustness

Between 2009 and 2015, the Federal Funds rate was technically nil, and interest-rate policy was said to have hit the zero lower bound (ZLB). The zero lower bound on nominal rates introduces nonlinearities that my linear empirical model cannot capture. Formulating and estimating a nonlinear model is beyond the scope of this paper. As an imperfect alternative, I estimate the linear model truncating the sample in 2008:Q4. The results are shown in the top panels of figure 7. The impulse responses are qualitatively similar to those obtained with the longer sample.

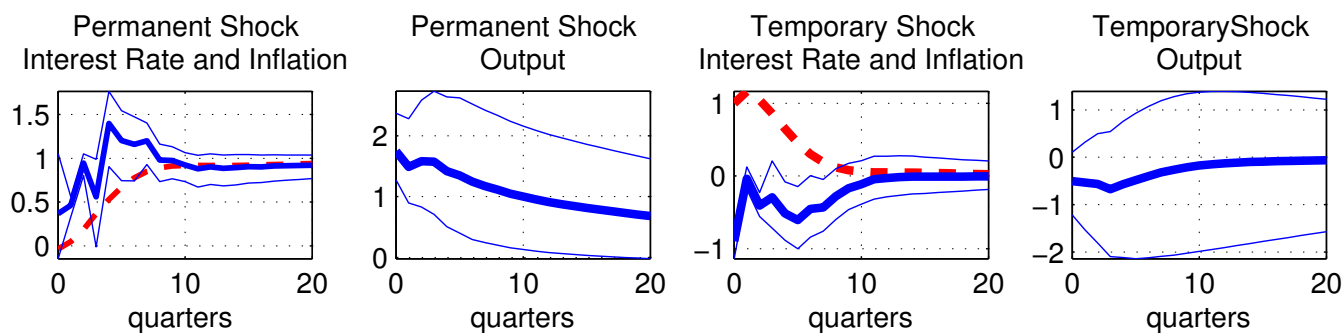
As a second robustness check, I estimate the model on Japanese data from 1955.Q3 to 2016.Q4. I rely on the results of the previous robustness check in deciding not to truncate the zero-rate period that started in 1995. There are two additional benefits of keeping the period 1995-2016. First, this period might provide valuable information on the effect of permanent monetary shocks, as it involves more than two decades of highly stable rates. Second, excluding the period 1995-2016 results in a relatively short sample of slightly over 20 years, which might make it difficult to distinguish the transitory and permanent components

Figure 7: Robustness Checks: Empirical Model

Truncating the Sample at the Beginning of the ZLB Period (2008:Q4)



Estimation on Japanese Data (1955:Q3 to 2016:Q4)



Notes. Thick lines are posterior means. Thick broken lines correspond to the nominal interest rate. Thin lines are 95% asymmetric error bands computed using the Sims-Zha (1999) method.

of monetary disturbances. The estimated impulse responses appear in the bottom panel of figure 7. The figure suggests that the main results obtained using U.S. data carry over to employing Japanese data.

Summarizing, the main result of this section is that the estimated empirical model predicts that a permanent increase in the nominal interest rate causes an immediate increase in inflation and transitional dynamics characterized by low real interest rates, and no output loss. The remainder of this paper is devoted to ascertaining whether these results carry over to optimizing models.

5 A New-Keynesian Model with Permanent and Transitory Monetary Shocks

In this section, I investigate the presence of a neo-Fisher effect in the context of an estimated standard optimizing model in the neo-Keynesian tradition. The model features price stickiness à la Rotemberg and is driven by six shocks: permanent and transitory interest-rate shocks, permanent and transitory productivity shocks, a preference shock, and a labor-supply shock. Monetary policy is described by a Taylor-type interest-rate feedback rule.

I assume that there is a representative household with preferences defined over streams of consumption and labor effort and exhibiting external habit formation in consumption. The lifetime utility function of the household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t e^{\xi_t} \left\{ \frac{[(C_t - \delta \tilde{C}_{t-1})(1 - e^{\theta_t} h_t)^\chi]^{1-\sigma} - 1}{1 - \sigma} \right\}, \quad (5)$$

where C_t denotes consumption in period t , \tilde{C}_t denotes the cross sectional average of consumption, h_t denotes hours worked in period t , ξ_t and θ_t denote exogenous preference shocks, E_t denotes the expectations operator conditional on information available in period t , and

$\beta, \delta \in (0, 1)$ and $\sigma, \chi > 0$ are parameters.

The preference shocks are assumed to follow AR(1) processes of the form

$$\xi_{t+1} = \rho_\xi \xi_t + \sigma_\xi \epsilon_{t+1}^\xi$$

and

$$\theta_{t+1} - \theta = \rho_\theta(\theta_t - \theta) + \sigma_\theta \epsilon_{t+1}^\theta,$$

where ϵ_t^ξ and ϵ_t^θ are i.i.d. innovations distributed $N(0, 1)$, and $\rho_\xi, \rho_\theta \in (-1, 1)$, $\sigma_\xi, \sigma_\theta > 0$, and θ are parameters.

Households are subject to the budget constraint

$$P_t C_t + \frac{B_{t+1}}{1 + I_t} + T_t = B_t + W_t^n h_t + \Phi_t, \quad (6)$$

where P_t denotes the nominal price of consumption, B_{t+1} denotes a one-period nominal discount bond purchased in t and paying the nominal interest rate I_t in $t + 1$, T_t denotes nominal lump-sum taxes, W_t^n denotes the nominal wage rate, and Φ_t denotes nominal profits received from firms.

Households choose processes $\{C_t, h_t, B_{t+1}\}_{t=0}^\infty$ to maximize the utility function (5) subject to the budget constraint (6) and to some borrowing limit that prevents them from engaging in Ponzi schemes. Letting $\beta^t \Lambda_t / P_t$ denote the Lagrange multiplier associated with the budget constraint, the first-order conditions of the household's optimization problem are

$$e^{\xi_t} (C_t - \delta \tilde{C}_{t-1})^{-\sigma} (1 - e^{\theta_t} h_t)^{\chi(1-\sigma)} = \Lambda_t$$

$$\frac{\chi e^{\theta_t} (C_t - \delta \tilde{C}_{t-1})}{1 - e^{\theta_t} h_t} = W_t$$

and

$$\Lambda_t = \beta(1 + I_t) E_t \left[\frac{\Lambda_{t+1}}{1 + \Pi_{t+1}} \right],$$

where $W_t \equiv W_t^n/P_t$ denotes the real wage, and $\Pi_t \equiv P_t/P_{t-1} - 1$ denotes the consumer-price inflation rate.

The consumption good C_t is assumed to be a composite of a continuum of varieties C_{it} indexed by $i \in [0, 1]$. The aggregation technology is assumed to be of the CES form

$$C_t = \left[\int_0^1 C_{it}^{1-1/\eta} di \right]^{\frac{1}{1-1/\eta}},$$

where the parameter $\eta > 0$ denotes the elasticity of substitution across varieties. Given C_t , the household chooses the consumption of varieties C_{it} to minimize total expenditure, $\int_0^1 P_{it} C_{it} di$, subject to the aggregation technology, where P_{it} denotes the nominal price of variety i . This problem delivers the following demand for individual varieties:

$$C_{it} = C_t \left(\frac{P_{it}}{P_t} \right)^{-\eta}, \quad (7)$$

where the price level P_t is given by

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}, \quad (8)$$

and represents the minimum cost of one unit of the composite consumption good.

The firm producing variety i operates in a monopolistically competitive market and faces quadratic price adjustment costs à la Rotemberg. The production technology uses labor and is buffeted by stationary and nonstationary productivity shocks. Specifically, output of variety i is given by

$$Y_{it} = e^{z_t} X_t^n h_{it}^\alpha, \quad (9)$$

where Y_{it} denotes output of variety i in period t , h_{it} denotes labor input used in the production of variety i , and z_t and X_t^n denote the stationary and nonstationary productivity shocks, respectively. The transitory productivity shock and the growth rate of the nonstationary

productivity shock, $g_t \equiv \ln(X_t^n/X_{t-1}^n)$, are assumed to follow AR(1) processes of the form

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon_{t+1}^z$$

and

$$g_t - g = \rho_g (g_{t-1} - g) + \sigma_g \epsilon_t^g,$$

where ϵ_t^z and ϵ_t^g are exogenous disturbances distributed i.i.d. $N(0, 1)$, and $\sigma_z, \sigma_g > 0$, $\rho_z, \rho_g \in (0, 1)$, and g are parameters.

The expected present discounted value of profits of the firm producing variety i is given by

$$E_0 \sum_{t=0}^{\infty} q_t \left[\frac{P_{it}}{P_t} C_{it} - W_t h_{it} - \frac{\phi}{2} X_t^n \left(\frac{P_{it}}{\tilde{X}_t^m P_{it-1}} - 1 \right)^2 \right], \quad (10)$$

where the parameter $\phi > 0$ governs the degree of price stickiness. The variable q_t , defined as

$$q_t \equiv \beta^t \frac{\Lambda_t}{\Lambda_0},$$

denotes a pricing kernel reflecting the assumption that profits belong to the representative household.

Note that the price adjustment cost in the profit equation (10) is multiplied by the nonstationary technological factor X_t^n . This keeps nominal rigidities from vanishing along the balanced growth path. Also, the model features price indexation to a variable \tilde{X}_t^m , which is taken as exogenous by the firm. To ensure that the Fisher effect holds in equilibrium the indexation factor \tilde{X}_t^m must be cointegrated with the permanent components of inflation and the interest rate, which I denote X_t^m and will define more precisely shortly. Failing to incorporate this type of indexation would imply lack of cointegration between inflation and the nominal interest rate. In this case, the model would be unable to capture the empirical evidence on the long-run comovement between inflation and nominal rates presented in section 2. Within the general class just described, however, indexation can take different

forms. Here, to allow for the possibility that in the short run firm-level inflation and X_t^m are decoupled, the evolution of the indexation factor \tilde{X}_t^m is assumed to be of the form

$$\tilde{X}_t^m = (X_t^m)^{\gamma_m} \left(\tilde{X}_{t-1}^m \right)^{1-\gamma_m}, \quad (11)$$

where $\gamma_m \in [0, 1)$ is a smoothing parameter. The smaller γ_m is, the less responsive the indexation factor \tilde{X}_t^m will be to short-run movements in the permanent component of inflation. In subsection 6.1, I generalize this formulation to allow firms to index to past inflation.

The problem of the firm producing variety i is to choose process $\{P_{it}, C_{it}, Y_{it}, h_{it}\}_{t=0}^{\infty}$ to maximize (10) subject to the demand equation (7), to the production technology (9), and to the requirement that demand be satisfied at the price set by the firm,⁷

$$Y_{it} \geq C_{it} \quad (12)$$

Letting $q_t P_{it}/(P_t \mu_t)$ be the Lagrange multiplier on the demand constraint (12), the first-order conditions associated with the firm's profit maximization problem are

$$\mu_t = \frac{P_{it}/P_t}{W_t/(\alpha e^{z_t} X_t^n h_{it}^{\alpha-1})}$$

$$\eta C_{it} \left(\frac{\eta-1}{\eta} - \frac{1}{\mu_t} \right) = -\phi X_t^n \frac{P_t}{\tilde{X}_t^m P_{it-1}} \left(\frac{P_{it}}{\tilde{X}_t^m P_{it-1}} - 1 \right) + \phi E_t \frac{q_{t+1}}{q_t} \tilde{X}_{t+1}^n \frac{P_{it+1} P_t}{\tilde{X}_{t+1}^m P_{it}^2} \left(\frac{P_{it+1}}{\tilde{X}_{t+1}^m P_{it}} - 1 \right)$$

The first optimality condition says that the multiplier μ_t represents the markup of prices over marginal cost. The second optimality condition says that, all other things equal, if the price markup is above its normal level, $\mu_t > \eta/(\eta-1)$, the firm will increase prices at a rate below normal, $P_{it}/P_{it-1} < \tilde{X}_t^m$.

⁷Strictly speaking, the right-hand side of this constraint must include the demand for goods of variety i by all firms for the purpose of generating the units of composite goods devoted to cover the price adjustment costs. However, because price adjustment costs are quadratic in the transitory component of firm-level inflation and because the latter is nil along the deterministic balanced growth path, this source of demand for good i and all of its derivatives with respect to P_{it} are zero in equilibrium up to first order.

I assume that the monetary authority follows a Taylor-type interest-rate feedback rule with policy smoothing, as follows

$$1 + I_t = \left[A \left(\frac{1 + \Pi_t}{X_t^m} \right)^{\alpha_\pi} \left(\frac{Y_t}{X_t^n} \right)^{\alpha_y} X_t^m \right]^{1-\gamma_I} (1 + I_{t-1})^{\gamma_I} e^{z_t^m},$$

where Y_t denotes aggregate output, z_t^m denotes a stationary monetary shock, X_t^m represents a permanent monetary shock, and A , α_π , α_y and $\gamma_I \in [0, 1)$ are parameters.

Let

$$g_t^m \equiv \ln \left(\frac{X_t^m}{X_{t-1}^m} \right)$$

denote the growth rate of the permanent monetary shock. I postulate AR(1) processes for g_t^m and z_t^m ,

$$g_t^m = \rho_{gm} g_{t-1}^m + \sigma_{gm} \epsilon_t^{gm}$$

and

$$z_t^m = \rho_{zm} z_{t-1}^m + \sigma_{zm} \epsilon_t^{zm},$$

where ϵ_t^{gm} and ϵ_t^{zm} are exogenous i.i.d. innovations distributed $N(0, 1)$, and $\rho_{gm}, \rho_{zm} \in (0, 1)$ and $\sigma_{gm}, \sigma_{zm} > 0$ are parameters.

I assume that government consumption is nil at all times. Thus, the fiscal authority faces the budget constraint

$$T_t + \frac{B_{t+1}}{1 + I_t} = B_t.$$

Further, I assume that fiscal policy is Ricardian, in the sense that the government sets T_t to ensure intertemporal solvency independently of the paths of the price level or the nominal interest rate.

Clearing of the labor market requires that the demand for labor by firms equal the household's supply of labor, that is,

$$\int_0^1 h_{it} di = h_t. \tag{13}$$

Because all households are identical, so are individual and aggregate consumption per capita,

$$C_t = \tilde{C}_t.$$

I focus attention on a symmetric equilibrium in which all firms charge the same nominal price and employ the same amount of labor, that is, an equilibrium in which h_{it} and P_{it} are the same for all $i \in [0, 1]$. We then have from equations (7), (8), (9), and (13) that $P_{it} = P_t$, $C_{it} = C_t$, $h_{it} = h_t$, and $Y_{it} = e^{z_t} X_t^n h_t^\alpha$, for all i . Output, measured in units of the final good is then given by $Y_t \equiv \left(\int_0^1 P_{it} Y_{it} di \right) / P_t = e^{z_t} X_t^n h_t^\alpha$. As long as the nominal wage is positive, the firm will choose to satisfy the demand constraint (12) with equality. By virtue of this condition, we have that in equilibrium

$$Y_t = C_t.$$

Finally, I express the model in terms of stationary variables by dividing all variables with stochastic trends by their respective permanent components. Thus, I create the variables $c_t \equiv C_t/X_t^n$, $y_t \equiv Y_t/X_t^n$, $w_t \equiv W_t/X_t^n$, $\lambda_t \equiv \Lambda_t/X_t^{n-\sigma}$, $1 + \pi_t \equiv (1 + \Pi_t)/X_t^m$, $1 + i_t \equiv (1 + I_t)/X_t^m$, and $\tilde{x}_t^m \equiv \tilde{X}_t^m/X_t^m$.

A competitive equilibrium is then a set of process $\{y_t, h_t, \lambda_t, \pi_t, i_t, w_t, mc_t, \tilde{x}_{t+1}^m\}$ satisfying

$$e^{\xi_t} \left(y_t - \delta \frac{y_{t-1}}{e^{g_t}} \right)^{-\sigma} (1 - e^{\theta_t} h_t)^{\chi(1-\sigma)} = \lambda_t$$

$$\frac{\chi e^{\theta_t} \left(y_t - \delta \frac{y_{t-1}}{e^{g_t}} \right)}{1 - e^{\theta_t} h_t} = w_t$$

$$\lambda_t = \beta(1 + i_t) E_t \left[\frac{\lambda_{t+1}}{1 + \pi_{t+1}} e^{-g_{t+1}^m - \sigma g_{t+1}} \right],$$

$$y_t = e^{z_t} h_t^\alpha$$

$$mc_t = \frac{w_t}{\alpha e^{z_t} h_t^{\alpha-1}}$$

$$\frac{1 + \pi_t}{\tilde{x}_t^m} \left(\frac{1 + \pi_t}{\tilde{x}_t^m} - 1 \right) = \beta E_t e^{(1-\sigma)g_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{1 + \pi_{t+1}}{\tilde{x}_{t+1}^m} \left(\frac{1 + \pi_{t+1}}{\tilde{x}_{t+1}^m} - 1 \right) + \frac{1}{\phi(\mu - 1)} (\mu \text{mc}_t - 1) y_t \quad (14)$$

$$1 + i_t = [A(1 + \pi_t)^{\alpha_\pi} y_t^{\alpha_y}]^{1-\gamma_I} \left(\frac{1 + i_{t-1}}{e^{g_t^m}} \right)^{\gamma_I} e^{z_t^m},$$

$$\tilde{x}_t^m = e^{-(1-\gamma_m)g_t^m} \tilde{x}_{t-1}^{m(1-\gamma_m)}$$

where $\text{mc}_t \equiv 1/\mu_t$ and $\mu \equiv \eta/(\eta - 1)$ denote, respectively, the equilibrium real marginal cost and the steady-state product markup. Equation (14) is a Phillips curve and says that all other things equal, current inflation is increasing in the marginal cost. A first-order approximation of the Phillips curve around $\pi_t = \pi = 0$ yields

$$\hat{\pi}_t - \hat{\tilde{x}}_t^m = \tilde{\beta} E_t (\hat{\pi}_{t+1} - \hat{\tilde{x}}_{t+1}^m) + \kappa \widehat{\text{mc}}_t, \quad (15)$$

where $\tilde{\beta} \equiv \beta e^{(1-\sigma)g}$, $\kappa \equiv \frac{(\eta-1)y}{\phi}$, $\hat{\pi}_t \approx \pi_t - \pi$, $\widehat{\text{mc}}_t \approx \ln(\text{mc}_t/\text{mc})$, $\hat{\tilde{x}}_t^m \approx \ln \tilde{x}_t^m$, and $\text{mc} = 1/\mu$. This is a familiar expression of a linear Phillips curve, except that it is cast in terms of deviations of the cyclical component of inflation, $\hat{\pi}_t$ from the cyclical component of the indexation factor, $\hat{\tilde{x}}_t^m$.

5.1 Data, Priors, and Estimation

As in much of the DSGE literature, I estimate a subset of the parameters of the model and calibrate the remaining ones using standard values in business-cycle analysis. The set of estimated parameters includes those that play a central role in determining the model's implied short-run dynamics.

Table 4 displays the values assigned to each calibrated parameter. I set the subjective discount factor, β , equal to 0.9982, which implies a growth-adjusted discount factor, $\beta e^{-\sigma g}$ equal to 0.99; the reciprocal of the intertemporal elasticity of substitution, σ , to 2; the intratemporal elasticity of substitution across varieties of intermediate goods, η , to 6, which

Table 4: Calibrated Parameters in the New Keynesian Model

Parameter	Value	Description
β	0.9982	subjective discount factor
σ	2	inverse of intertemp. elast. subst.
η	6	intra-temporal elast. of subst.
α	0.75	labor semielast. of output
g	0.004131	mean output growth rate
θ	0.4055	preference parameter
χ	0.625	preference parameter

Note. The time unit is one quarter.

implies a steady-state markup of price over marginal cost of 20 percent (Galí, 2008); the labor semi elasticity of the production function, α , to 0.75; the unconditional mean of per capita output growth, g , to 0.004131 (1.65 percent per year), which matches the average growth rate of real GDP per capita in the United States over the estimation period (1954:Q4 to 2018:Q2); and the parameters θ and χ to ensure, given all other parameter values, that in the steady state households allocate one third of their time to work, $h = 1/3$ and a unit Frisch elasticity of labor supply (Galí, 2008), $(1 - e^{\theta h})/(e^{\theta h}) = 1$.

I estimate the remaining parameters of the model using the same observables as in the estimation of the empirical model of section 3, namely, per-capita output growth, the interest-rate-inflation differential, and the change in the nominal interest rate. The data sources are as described in subsection 3.3. As in the case of the empirical model, the econometric estimation employs Bayesian techniques. Table 5 displays means, standard deviations of the prior distributions of the estimated parameters. I impose loose priors on all estimated parameters. I assume a Gamma prior distribution with a mean of 50 and a standard deviation of 20 for the parameter ϕ governing the degree of price stickiness. The assumed mean prior value of ϕ ensures that given the calibrated values of η , α , and θ , the value of κ in the linearized Phillips curve (15) is 0.043, as in Galí (2008). The speed of adjustment of the indexation factor, dictated by $\gamma_m \in (0, 1)$, takes a uniform distribution with support $[0, 1]$. The parameters α_π and α_y have Gamma prior distributions with means equal to 1.5 and 0.125 (two commonly

Table 5: Prior and Posterior Parameter Distributions: New-Keynesian Model

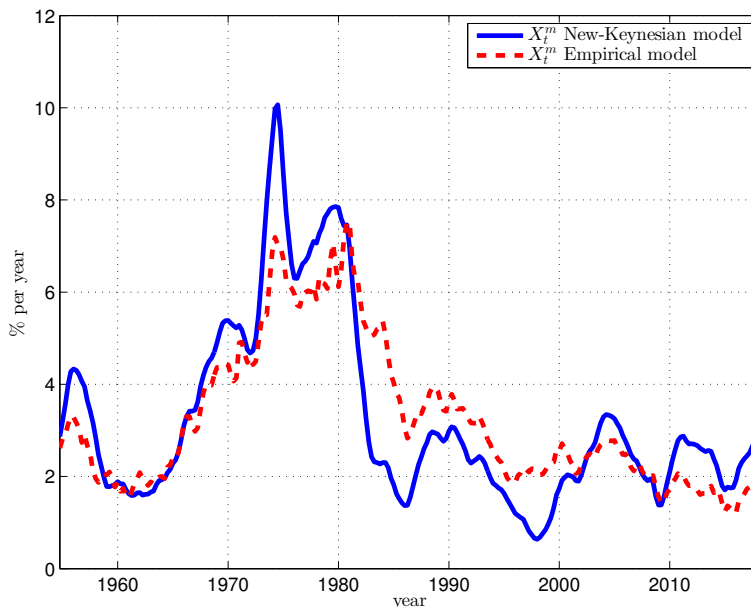
Parameter	Prior Distribution			Posterior Distribution			
	Distribution	Mean	Std	Mean	Std	5%	95%
ϕ	Gamma	50	20	159	31.3	111	214
α_π	Gamma	1.5	0.25	1.83	0.31	1.35	2.37
α_y	Gamma	0.125	0.1	0.687	0.2	0.386	1.03
γ_m	Uniform	0.5	0.289	0.464	0.195	0.201	0.851
γ_I	Uniform	0.5	0.289	0.579	0.108	0.366	0.722
δ	Uniform	0.5	0.289	0.294	0.0508	0.21	0.378
ρ_ξ	Beta	0.7	0.2	0.902	0.0259	0.856	0.941
ρ_θ	Beta	0.7	0.2	0.673	0.201	0.305	0.954
ρ_z	Beta	0.7	0.2	0.667	0.206	0.289	0.954
ρ_g	Beta	0.3	0.2	0.403	0.0915	0.236	0.538
ρ_{gm}	Beta	0.3	0.2	0.331	0.176	0.0553	0.625
ρ_{zm}	Beta	0.3	0.2	0.195	0.126	0.0346	0.432
σ_ξ	Gamma	0.01	0.01	0.0251	0.00393	0.0199	0.0325
σ_θ	Gamma	0.01	0.01	0.00164	0.0013	0.000119	0.00417
σ_z	Gamma	0.01	0.01	0.00124	0.001	9.22e-05	0.00318
σ_g	Gamma	0.01	0.01	0.00626	0.000841	0.00492	0.00769
σ_{gm}	Gamma	0.0025	0.0025	0.00103	0.00032	0.000567	0.0016
σ_{zm}	Gamma	0.0025	0.0025	0.00155	0.000271	0.00107	0.00189
R_{11}	Gamma	3.78e-06	2.18e-06	4.3e-06	2.43e-06	1.17e-06	8.95e-06
R_{22}	Gamma	2.08e-06	1.2e-06	4.43e-06	4.99e-07	3.66e-06	5.29e-06
R_{33}	Gamma	2.36e-07	1.36e-07	2.24e-07	1.25e-07	6.29e-08	4.65e-07

Note. The time unit is one quarter. Growth rates and log-deviations from trend are expressed in per one (1 percent is denoted 0.01). Replication code: `table_param.m.m` in `optimizing_model.zip`.

used values) and standard deviations equal to 0.25 and 0.1, respectively. The parameter γ_I measuring the smoothness of monetary policy has a uniform prior distribution with support $[0, 1]$. The degree of external habit formation, given by $\delta \in [0, 1)$, adopts a uniform prior distribution with support $[0, 1]$. Echoing the priors imposed in the empirical model, I assume that the serial correlations of the permanent nonstationary shock and of the two monetary shocks (ρ_g , ρ_{gm} , and ρ_{zm} , respectively) have Beta distributions with mean equal to 0.3 and standard deviation equal to 0.2, and that the serial correlations of all stationary nonmonetary shocks (ρ_ξ , ρ_θ , and ρ_z) have Beta distributions with mean 0.7 and standard deviation 0.2. Similarly, In line with the empirical model, I assume that the standard deviations of the two monetary shocks, σ_{gm} and σ_{zm} , take Gamma distributions with mean and standard deviation equal to 1/400, that the standard deviation of the permanent nonmonetary shock, σ_g , has a Gamma distribution with mean and standard deviation equal to 1/100, and that the standard deviations of all stationary nonmonetary shocks (σ_ξ , σ_θ , and σ_z) have Gamma distributions with mean and standard deviation equal to 1/100. Finally, as in the empirical model, I introduce measurement error and assume that their variances have uniform distributions with an upper bound equal to 10 percent of the variance. of the observables.

The last four columns of table 5 displays means, standard deviaitons, and 5 and 95 percent intervals of the estimated posterior distributions, based on a Random Walk Metropolis Hastings MCMC chain of length one million after discarding 100 thousand burn-in draws. Most parameters are estimated with significant uncertainty, a feature that is common in estimates of small-scale New Keynesian models (Ireland, 2007). Nonetheless, the data speaks with a strong voice on the parameters ϕ and δ , governing price stickiness and habit formation, which are key determinants of the propagation of nominal and real shocks. Furthermore, it is reassuring, given the focus of this paper, that the estimated path of the nonstationary monetary shock, the latent variable X_t^m , resembles its counterpart in the empirical model. This is shown in Figure 8, which displays the inferred paths of X_t^m from the New-Keynesian and empirical models. Overall, the nonstationary monetary shock implied by the optimizing

Figure 8: Inflation and Its Permanent Component: New Keynesian Model



Note. Quarterly frequency. The inferred path of the permanent component of inflation, X_t^m , was computed by Kalman smoothing and evaluating the model at the posterior mean of the estimated parameter vector. The initial value of X_t^m was normalized to make the average value of X_t^m equal to the average rate of inflation over the sample period, 1954:Q4 to 2018:Q2. Replication code: `plot_xm.m` in `optimizing_model.zip`.

model tracks the one stemming from the empirical model quite well. The sample correlation of the two series is 0.86. The path of X_t^m implied by the New-Keynesian model is more volatile than the one coming from the empirical model, perhaps due to the fact that the empirical model has a richer lag structure.

An additional measure of coherence between the predictions of the estimated empirical and New-Keynesian models is a comparison of their implied variance decompositions. Table 6 displays this information for the New Keynesian model. Comparing tables 3 and 6 shows that both the empirical and New Keynesian models predict that the permanent monetary shock explains more than 40 percent of changes in the rate of inflation. Thus, in both models the nonstationary component of monetary disturbances plays a significant role in explaining movements in nominal variables. Also, in both models the stationary monetary shock accounts for a relatively small share of movements in the rate of inflation. Another similarity between the empirical and optimizing models is that in both the nonstationary

Table 6: Variance Decomposition: New Keynesian Model

	Δy_t	$\Delta \pi_t$	Δi_t
Permanent Monetary Shock, g_t^m	1.7	42.8	9.3
Transitory Monetary Shock, z_t^m	3.0	2.1	35.7
Permanent Productivity Shock, g_t	84.7	2.2	4.8
Transitory Productivity Shock, z_t	0.4	5.1	2.1
Preference Shock, ξ_t	9.7	42.8	46.0
Labor-Supply Shock, θ_t	0.4	5.1	2.0

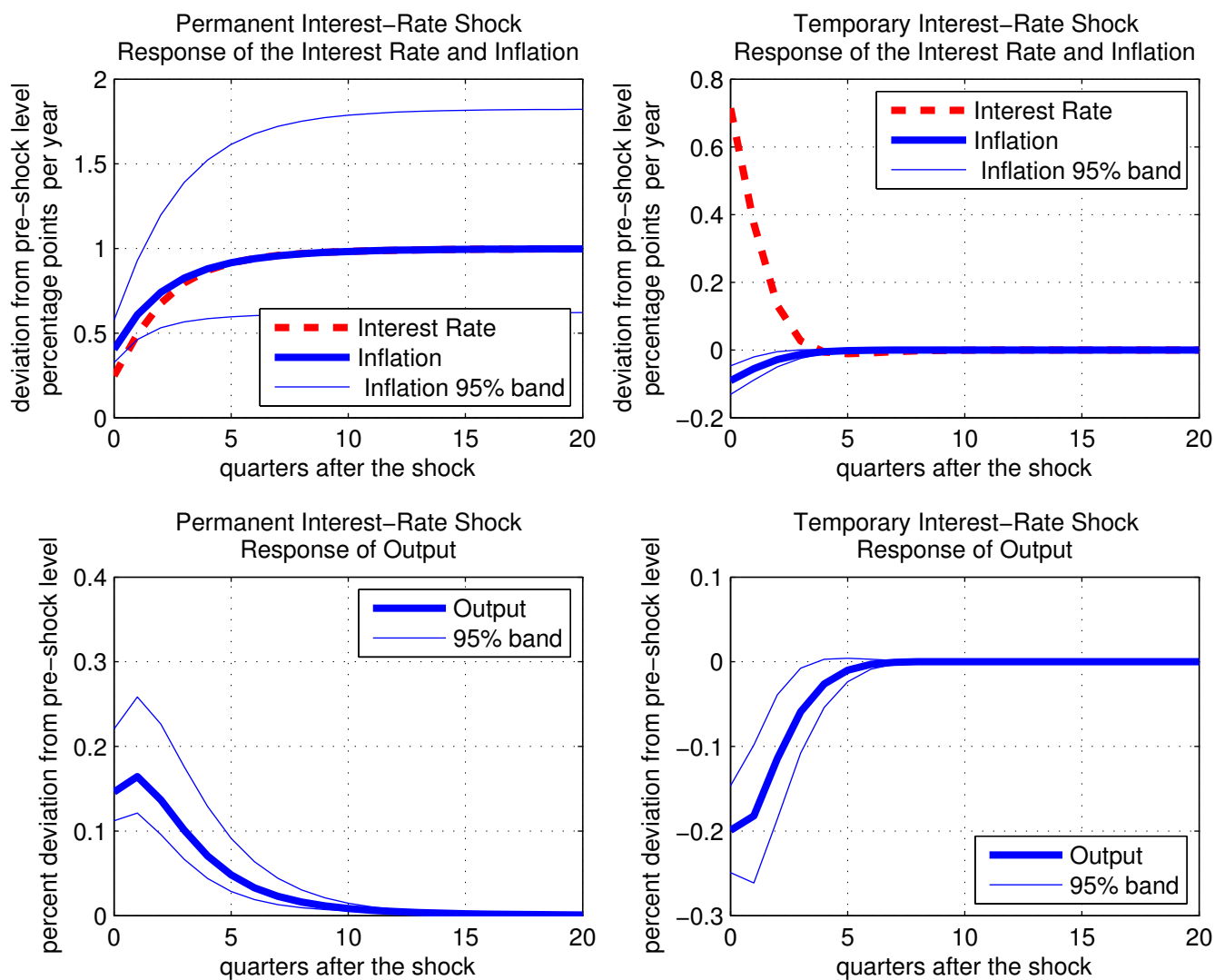
Note. Posterior means. The variables Δy_t , $\Delta \pi_t$, and Δi_t denote output growth, the change in inflation, and the change in the nominal interest rate, respectively. Replication code: `table_vardecomp.m` in `optimizing_model.zip`.

nonmonetary shock explains a significant fraction of variations in output growth, although in the New Keynesian model this role is estimated to be much larger. Finally, in both models nonmonetary stationary shocks explain almost half of changes in the nominal interest rate. Thanks to its more structural nature, the optimizing model is able to say more about which specific nonmonetary stationary sources of uncertainty are the most relevant. Specifically, while the empirical model encapsulates all stationary nonmonetary disturbances in a single shock, z_t^n , the New Keynesian model can distinguish finer categories and suggests that it is a demand shock, namely, the preference shock ξ_t , that has the largest impact on changes in the nominal interest rate.

6 The Neo-Fisher Effect in the New-Keynesian Model

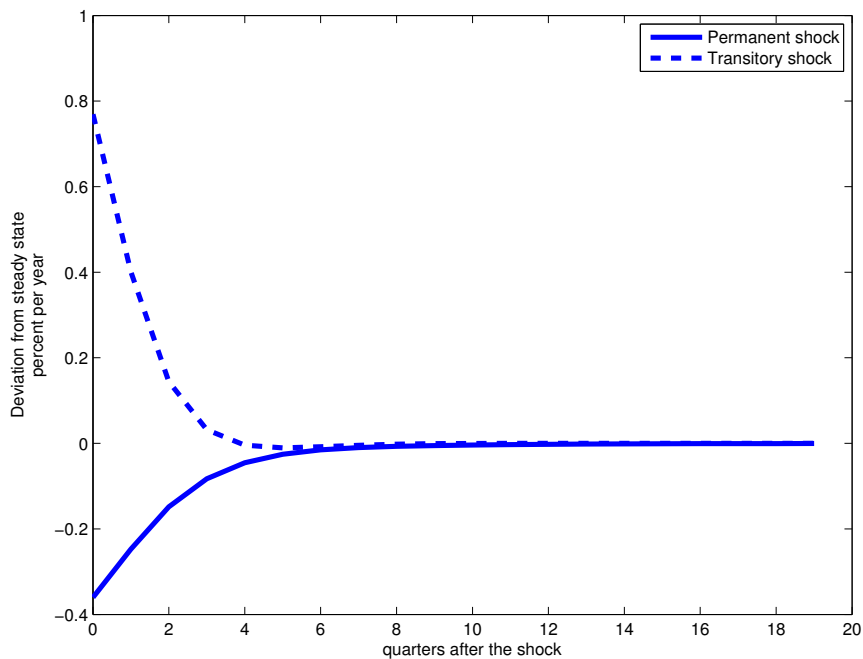
Figure 9 displays the impulse responses of inflation, the policy rate, and output to permanent and transitory monetary shocks implied by the estimated New-Keynesian model. The main message conveyed by the figure is that qualitatively the responses implied by the New-Keynesian model concur with those implied by the empirical model of sections 3 and 4. An increase in the nominal interest rate that is understood to be permanent by private agents (left panels of the figure) causes an increase in inflation in the short run, without loss of aggregate activity. By contrast, an increase in the nominal interest rate that is interpreted

Figure 9: Impulse Responses to Permanent and Temporary Interest-Rate Shocks: New-Keynesian Model



Notes. Impulse responses are posterior mean estimates. Asymmetric error bands are computed using the Sims-Zha (1999) method. Replication code: `plot_ir_chain.m` in `optimizing_model.zip`.

Figure 10: Response of the Real Interest Rate to Permanent and Transitory Interest-Rate Shocks: New-Keynesian Model



Notes. Posterior mean estimates. The real interest rate is defined as $i_t - E_t\pi_{t+1}$. Replication code: `real_interest_rate.m` in `optimizing_model.zip`.

to be transitory (right panels of the figure) causes a fall in inflation and a contraction in aggregate activity.

Why does a permanent increase in the nominal interest rate cause no loss in aggregate activity while a temporary tightening does? In the discussion of the predictions of the empirical model, I suggested that the answer could be in the behavior of the real interest rate. This conjecture is supported by the dynamics of the optimizing model. Figure 10 shows that in response to a permanent increase in the nominal interest rate inflation not only begins to increase immediately, but does so at a rate faster than the nominal interest rate. As a result, the real interest rate falls. By contrast, a temporary increase in the nominal interest rate causes a fall in inflation and an increase in the real interest rate. A natural question is why inflation moves faster than the interest rate in the short run when the monetary shock is expected to be permanent. The answer has to do with the presence of nominal rigidities and with the way the central bank conducts monetary policy. In response to a permanent

monetary shock that increases the nominal interest rate by one percent in the long run, the central bank raises the short-run policy rate quickly but gradually. At the same time, firms know that, by the Fisher effect, the price level will increase by one percent in the long run, and that they too will have to increase their own price in the same proportion in the long run, to avoid making losses. Since firms face quadratic costs of adjusting prices, they find it optimal to begin increasing the price immediately. Since all firms do the same, inflation itself begins to increase as soon as the shock is announced.

6.1 Robustness

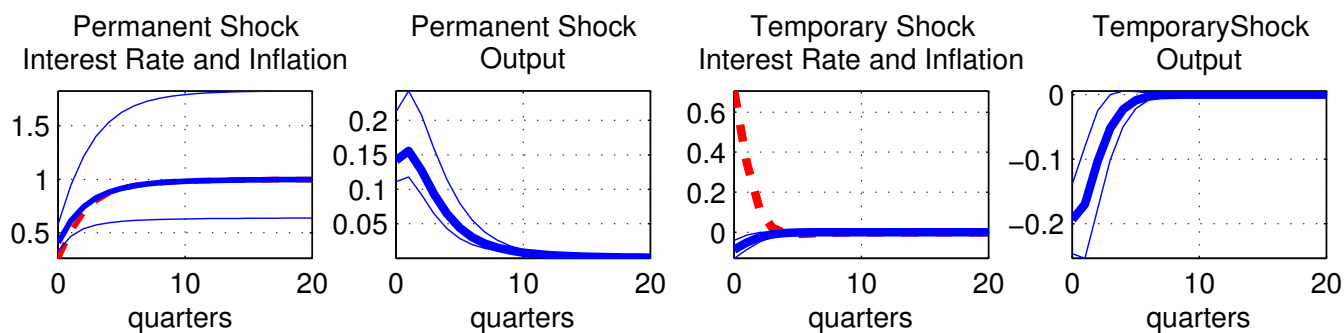
Figure 11 displays three tests. The top two panels are the counterparts of the robustness checks applied on the empirical model (see figure 7). They show that the predictions of the theoretical model are qualitatively stable to truncating the sample in 2008:Q4 to exclude the period in which U.S. monetary policy could have been limited by the zero lower bound on interest rate, and to estimating the model on Japanese data. The bottom panel considers a variant of the New-Keynesian model in which firms can index prices to past aggregate inflation. Specifically, this version assume that the indexation factor \tilde{X}_t^m takes the form

$$\tilde{X}_t^m = [X_t^{m\gamma_{mm}}(1 + \Pi_{t-1})^{1-\gamma_{mm}}]^{\gamma_m} (\tilde{X}_{t-1}^m)^{1-\gamma_m}$$

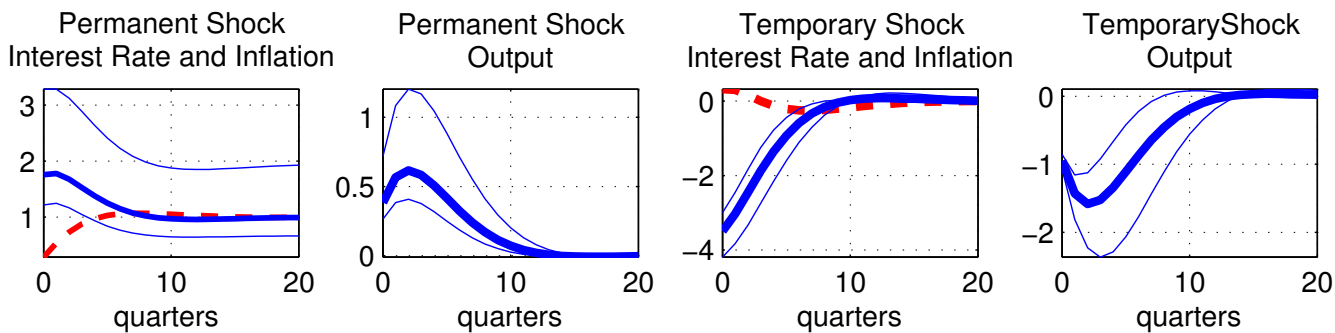
with the new parameter $\gamma_{mm} \in [0, 1]$. This formulation nests the baseline formula given by (11). Reestimating the model on the baseline dataset (i.e., U.S. data over the period 1954:Q4 to 2018:Q2) yields a posterior mean value of γ_{mm} of 0.061 and a posterior standard deviation of 0.058, suggesting that indexation to past inflation is relevant. The bottom panel of figure 11 shows that this version of the model produces predictions that are qualitatively in line with the baseline ones.

Figure 11: Robustness Checks: Optimizing Model

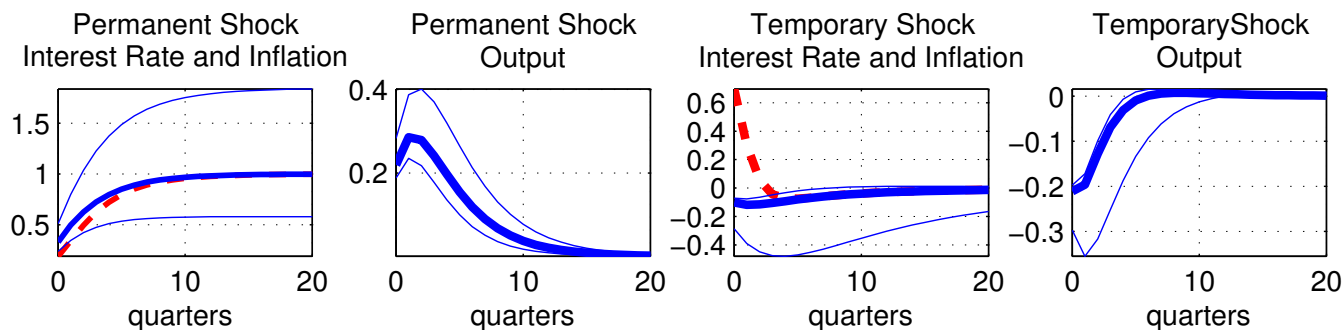
Truncating the Sample at the Beginning of the ZLB Period (2008:Q4)



Estimation on Japanese Data (1955:Q3 to 2016:Q4)



Allowing for Indexation to Past Inflation



Notes. Thick lines are posterior means. Thick broken lines correspond to the nominal interest rate. Thin lines are 95% asymmetric error bands computed using the Sims-Zha (1999) method.

7 Conclusion

Discussions of how monetary policy can lift an economy out of chronic below-target inflation are almost always based on the logic of how transitory interest-rate shocks affect real and nominal variables. Nowadays, there is little theoretical or empirical controversy around how this type of monetary shock transmits to the rest of the economy: An increase in the nominal interest rate causes an increase in the real interest rate, which puts downward pressure on both aggregate activity and price growth. Within this logic, a central bank trying to reflate a low-inflation economy will tend to set interest rates as low as possible. Soon enough these economies find themselves with zero or negative nominal rates and with the low-inflation problem not going away. After some time, the Fisher effect kicks in, and the situation perpetuates. The monetary authority keeps the interest rate low because inflation is still below target (the temporary-interest-rate-shock logic) and inflation is low because the interest rate has been low for a long period of time (the Fisher effect).

In this paper I argue, based on econometric evidence drawn from an empirical and an optimizing model that a gradual and permanent increase in the nominal interest rate causes a fast adjustment of inflation to a permanently higher level, low real interest rates, and no output loss. These findings are consistent with the prediction, sometimes referred to as neo-Fisherian, that a credible announcement of a gradual return of the nominal interest rate to normal levels can achieve a swift reflation of the economy with sustained levels of economic activity.

Appendix: Detailed Exposition of the Empirical Model

Let Y_t be a vector collecting these three variables,

$$Y_t \equiv \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix},$$

where y_t denotes the logarithm of real output per capita, π_t denotes the inflation rate expressed in percent per year, and i_t denotes the nominal interest rate expressed in percent per year. Let \tilde{Y}_t

$$\tilde{Y}_t \equiv \begin{bmatrix} (y_t - X_t^n) \times 100 \\ \pi_t - X_t^m \\ i_t - X_t^m \end{bmatrix},$$

where X_t^m is a permanent monetary shock, z_t^m is a transitory monetary shock, X_t^n is a nonstationary nonmonetary shock, and z_t^n is a stationary nonmonetary shock. Let \hat{Y}_t denote the deviation of \tilde{Y}_t from its unconditional mean, that is,

$$\hat{Y}_t \equiv \begin{bmatrix} \hat{y}_t \\ \hat{\pi}_t \\ \hat{i}_t \end{bmatrix} \equiv \tilde{Y}_t - E\tilde{Y}_t,$$

where E denotes the unconditional expectations operator.

The law of motion of \hat{Y}_t takes the autoregressive form

$$\hat{Y}_t = \sum_{i=1}^L B_i \hat{Y}_{t-i} + C u_t \tag{16}$$

where

$$u_t \equiv \begin{bmatrix} x_t^m \\ z_t^m \\ x_t^n \\ z_t^n \end{bmatrix},$$

$$x_t^m \equiv \Delta X_t^m - \Delta X^m$$

and

$$x_t^n \equiv (\Delta X_t^n - \Delta X^n) \times 100.$$

with Δ denoting the time-difference operator, $\Delta X^m \equiv E\Delta X_t^m$, and $\Delta X^n \equiv E\Delta X_t^n$. The variables x_t^m and x_t^n denote demeaned changes in the nonstationary shocks. The objects B_i , for $i = 1, \dots, L$, are 3-by-3 matrices of coefficients, C is a 3-by-4 matrix of coefficients, and L is a scalar denoting the lag length of the empirical model. The vector u_t is assumed to follow an AR(1) law of motion of the form

$$u_{t+1} = \rho u_t + \psi \epsilon_{t+1}, \tag{17}$$

where ρ and ψ are 4-by-4 diagonal matrices of coefficients, and ϵ_t is a 4-by-1 i.i.d. disturbance distributed $N(\emptyset, I)$.

The observable variables used in the estimation of the empirical model are output growth expressed in percent per quarter, the change in the nominal interest rate, and the interest-rate-inflation differential, defined as

$$r_t \equiv i_t - \pi_t.$$

The following equations link the observables to variables included in the unobservable system

(16)-(17):

$$\begin{aligned}
100 \times \Delta y_t &= 100 \times \Delta X^n + \hat{y}_t - \hat{y}_{t-1} + x_t^n \\
r_t &= r + \hat{i}_t - \hat{\pi}_t \\
\Delta i_t &= \Delta X^m + \hat{i}_t - \hat{i}_{t-1} + x_t^m
\end{aligned} \tag{18}$$

where $r \equiv Er_t$ represents the unconditional mean of the interest-rate-inflation differential. The variables Δy_t , r_t , and Δi_t are assumed to be observed with measurement error. Let o_t be the vector of variables observed in quarter t . Then

$$o_t = \begin{bmatrix} \Delta y_t \times 100 \\ r_t \\ \Delta i_t \end{bmatrix} + \mu_t \tag{19}$$

where μ_t is a 3-by-1 vector of measurement errors distributed i.i.d. $N(\emptyset, R)$, and R is a diagonal variance-covariance matrix.

The state-space representation of the system composed of equations (16), (17), (18), and (19) can be written as follows:

$$\begin{aligned}
\xi_{t+1} &= F\xi_t + P\epsilon_{t+1} \\
o_t &= A' + H'\xi_t + \mu_t,
\end{aligned}$$

where

$$\xi_t \equiv \begin{bmatrix} \hat{Y}_t \\ \hat{Y}_{t-1} \\ \vdots \\ \hat{Y}_{t-L+1} \\ u_t \end{bmatrix},$$

The matrices F , P , A , and H are known functions of B_i , $i = 1, \dots, L$, C , ρ , ψ , ΔX^n , ΔX^m ,

and r . Specifically, let

$$B \equiv [B_1 \cdots B_L],$$

and let I_j denote an identity matrix of order j , \emptyset_j denote a square matrix of order j with all entries equal to zero, and $\emptyset_{i,j}$ denote a matrix of order i by j with all entries equal to zero. Also let L , S , and V denote, respectively, the number of lags, the number of shocks, and the number of endogenous variables included in the empirical model. Then, for $L \geq 2$ we have

$$F = \begin{bmatrix} B & C\rho \\ [I_{V(L-1)} \emptyset_{V(L-1),V}] & \emptyset_{V(L-1),S} \\ \emptyset_{S,VL} & \rho \end{bmatrix}, P = \begin{bmatrix} C\psi \\ \emptyset_{V(L-1),S} \\ \psi \end{bmatrix};$$

$$A' = \begin{bmatrix} 100 \times \Delta X^n \\ r \\ \Delta X^m \end{bmatrix}, \text{ and } H' = \begin{bmatrix} M_\xi & \emptyset_{V,V(L-2)} & M_u \end{bmatrix},$$

where, in the specification considered in the body of the paper ($S = 4$, $V = 3$, and a particular ordering of the endogenous and exogenous variables in the vectors \hat{Y}_t and u_t), the matrices M_ξ and M_u take the form

$$M_\xi = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \text{ and } M_u = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

The case $L = 1$ is a special case of $L = 2$ in which $B_2 = \emptyset_V$.

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