En el caso de distribución uniforme sobre $[-1,1]$

$$f(\epsilon) = \frac{1}{2} \tag{1}$$

$$I = \int_a^b \epsilon f(\epsilon) d\epsilon \tag{2}$$

$$I = \int_a^b \epsilon \frac{1}{2} d\epsilon = \frac{1}{4}(b^2 - a^2). \tag{3}$$

If ϵ sigue una densidad normal $f(\epsilon; \mu, \sigma^2)$ entonces la integral es

$$I = \int_a^b \epsilon f(\epsilon; \mu, \sigma^2) d\epsilon \tag{4}$$

$$I = \int_a^b \epsilon \times \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\epsilon-\mu}{\sigma})^2} \right) d\epsilon \tag{5}$$

integramos por sustitucion, $z = \frac{\epsilon-\mu}{\sigma}$ de manera que $\epsilon = \mu + \sigma \times z$, y $d\epsilon = \sigma dz$. Reemplazando en (5)

$$I = \int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} (\mu + \sigma \times z) \times \left(\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \right) \sigma dz \tag{6}$$

distribuyendo (6), I se descompone en 2 integrales

$$I_1 = \int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} \mu \times \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \tag{7}$$

$$I_1 = \mu \times \int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz \tag{8}$$

$$I_1 = \mu \left(\Phi\left(\frac{b-\mu}{\sigma}\right) - \Phi\left(\frac{a-\mu}{\sigma}\right) \right) \tag{9}$$

$\Phi(.)$ puede ser calculado con normcdf.

La otra parte de la integral

$$I_2 = \int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} z \times e^{-\frac{1}{2}z^2} dz \tag{10}$$

tambien procedemos por sustitucion, sea $u = z^2$, entonces $du = 2zdz$. Entonces

$$I_2 = \frac{1}{2}\int_{\frac{a-\mu}{\sigma}}^{\frac{b-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} 2ze^{-\frac{1}{2}z^2} dz \tag{11}$$

de manera que

$$I_2 = \frac{1}{2\sqrt{2\pi}}\int_{(\frac{a-\mu}{\sigma})^2}^{(\frac{b-\mu}{\sigma})^2} 2ze^{-\frac{1}{2}z^2} dz \tag{12}$$

$$I_2 = \frac{1}{2\sqrt{2\pi}}\int_{(\frac{a-\mu}{\sigma})^2}^{(\frac{b-\mu}{\sigma})^2} e^{-\frac{u}{2}} dz \tag{13}$$

$$I_2 = \frac{1}{2\sqrt{2\pi}}\Bigg(-2\times e^{-\frac{u}{2}}\Bigg)\Bigg|_{u=(\frac{a-\mu}{\sigma})^2}^{u=(\frac{b-\mu}{\sigma})^2} \tag{14}$$

$$I_2 = \frac{1}{\sqrt{2\pi}}\Bigg(-e^{-\frac{u}{2}}\Bigg)\Bigg|_{u=(\frac{a-\mu}{\sigma})^2}^{u=(\frac{b-\mu}{\sigma})^2} \tag{15}$$

$$I_2 = -\frac{1}{\sqrt{2\pi}}\Bigg(e^{-\frac{u}{2}}\Bigg|_{u=(\frac{a-\mu}{\sigma})^2}^{u=(\frac{b-\mu}{\sigma})^2}\Bigg) \tag{16}$$

entonces

$$I_2 = -\frac{1}{\sqrt{2\pi}}\Big(e^{-\frac{(\frac{b-\mu}{\sigma})^2}{2}} - e^{-\frac{(\frac{a-\mu}{\sigma})^2}{2}}\Big) \tag{17}$$

$$I_2 = \frac{1}{\sqrt{2\pi}}\Big(e^{-\frac{(\frac{a-\mu}{\sigma})^2}{2}} - e^{-\frac{(\frac{b-\mu}{\sigma})^2}{2}}\Big) \tag{18}$$

Por tanto

$$I = I_1 + I_2 = \mu\Bigg(\Phi\Big(\frac{a-\mu}{\sigma}\Big) - \Phi\Big(\frac{b-\mu}{\sigma}\Big)\Bigg) + \frac{1}{\sqrt{2\pi}}\Big(e^{-\frac{(\frac{a-\mu}{\sigma})^2}{2}} - e^{-\frac{(\frac{b-\mu}{\sigma})^2}{2}}\Big) \tag{19}$$

Las otras integrales son directas (el integrando incluye solamente la densidad de probabilidad), C es una constante. En el caso normal

$$\int_a^b Cf(\epsilon)d\epsilon \tag{20}$$

$$C\int_a^b f(\epsilon;\mu,\sigma^2)d\epsilon = C\left(\Phi\Big(\frac{b-\mu}{\sigma}\Big) - \Phi\Big(\frac{a-\mu}{\sigma}\Big)\right) \tag{21}$$

y en el caso uniforme

$$C\int_a^b \frac{1}{2}d\epsilon = C\frac{1}{2}(b-a). \tag{22}$$