

Households of type $i = 1, 2$ maximize logarithmic utility, $u(C_{i,t}) = \ln(C_{i,t})$. Parameters:

Parameter	Description
α	Output elasticity w.r.t. capital
$\beta_{t,t+1}$	Stochastic discount factor
δ	Capital depreciation rate
τ	Tax rate
b	Worker bargaining power
h	Type 1 acceptance rate
μ	Matching efficiency
η	Matching function elasticity
χ	CES efficiency parameter
ϵ	CES substitution elasticity
c	Vacancy cost
d	Job destruction rate
Z	Total factor productivity
$F_{i,0}$	Initial labor force, type i

1. Effective Labor Input (Nested CES)

$$L_t = Z \left[\chi^{\frac{\epsilon-1}{\epsilon}} n_{1,t}^{\frac{\epsilon-1}{\epsilon}} + (1-\chi)^{\frac{\epsilon-1}{\epsilon}} n_{2,t}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \quad (1)$$

2. Aggregate Output

$$Y_t = K_{t-1}^\alpha L_t^{1-\alpha} \quad (2)$$

3. Marginal Products and Capital Return (Pmg_{*i,t*}: marginal product of labor, $i = 1, 2$)

$$\text{Pmg}_{1,t} = Z^{1-\frac{1}{\epsilon}} (1-\alpha) \frac{Y_t}{L_t} \chi \left(\frac{L_t}{n_{1,t}} \right)^{\frac{1}{\epsilon}} \quad (3)$$

$$\text{Pmg}_{2,t} = Z^{1-\frac{1}{\epsilon}} (1-\alpha) \frac{Y_t}{L_t} (1-\chi) \left(\frac{L_t}{n_{2,t}} \right)^{\frac{1}{\epsilon}} \quad (4)$$

$$\alpha \frac{Y_t}{K_{t-1}} = r_t + \delta \quad (5)$$

4. Labor Force

$$F_{1,t} = F_{1,0} \quad (6)$$

$$F_{2,t} = F_{2,0} \quad (7)$$

$$F_t = F_{1,t} + F_{2,t} \quad (8)$$

5. Unemployment ($i = 1, 2$)

$$u_{i,t} = F_{i,t} - n_{i,t} \quad (9)$$

6. Labor Market Tightness

$$\theta_t = \frac{v_t}{u_{1,t} + u_{2,t}} \quad (10)$$

7. Job-Finding Rate

$$f_t = \mu \theta_t^{1-\eta} \quad (11)$$

8. Value of a Filled Job ($i = 1, 2$)

$$J_{i,t} = \text{Pmg}_{i,t} - w_{i,t} + (1-d)\beta_{t,t+1} J_{i,t+1} \quad (12)$$

9. Vacancy-Filling Probability

$$q_t = \mu \theta_t^{-\eta} \quad (13)$$

10. Free Entry Condition

$$c = q_t \beta_{t,t+1} \left[\frac{u_{2,t} J_{2,t+1} + h u_{1,t} J_{1,t+1}}{u_{1,t} + u_{2,t}} \right] \quad (14)$$

11. Nash Bargaining for Wages ($i = 1, 2$)

$$b(1-\tau)J_{i,t} = (1-b)(W_{i,t} - U_{i,t})C_{i,t} \quad (15)$$

12. Value of Employment ($i = 1, 2$)

$$W_{i,t} = \frac{w_{i,t}(1-\tau)}{C_{i,t}} + \beta_{t,t+1} [(1-d)W_{i,t+1} + dU_{i,t+1}] \quad (16)$$

13. Value of Unemployment, Type 1

$$U_{1,t} = \frac{z_{1,t}}{C_{1,t}} + \beta_{t,t+1} [f_t h W_{1,t+1} + (1-h f_t) U_{1,t+1}] \quad (17)$$

14. Value of Unemployment, Type 2

$$U_{2,t} = \frac{z_{2,t}}{C_{2,t}} + \beta_{t,t+1} [f_t W_{2,t+1} + (1-f_t) U_{2,t+1}] \quad (18)$$

15. Unemployment Benefits ($i = 1, 2$)

$$z_{i,t} = 0.612 w_{i,t-1} (1-\tau) \quad (19)$$

16. Employment Dynamics ($i = 1, 2$)

$$n_{i,t} = (1-d)n_{i,t-1} + f_{t-1} \begin{cases} h u_{1,t-1}, & i = 1 \\ u_{2,t-1}, & i = 2 \end{cases} \quad (20)$$

17. **Household Budget Constraint**

$$T_t = \frac{n_{1,t}w_{1,t}\tau + n_{2,t}w_{2,t}\tau - (u_{1,t}z_{1,t} + z_{2,t}u_{2,t})}{F_t} \quad (21)$$

18. **Type 2 Consumption**

$$C_{2,t} = w_{2,t}n_{2,t}(1 - \tau) + z_{2,t}u_{2,t} + T_t F_{2,t} \quad (22)$$

19. **Capital Accumulation**

$$K_t = (1 - \delta)K_{t-1} + I_t \quad (23)$$

20. **Aggregate Consumption**

$$C_t = Y_t - I_t - v_t c \quad (24)$$

21. **Type 1 Consumption**

$$C_{1,t} = C_t - C_{2,t} \quad (25)$$

22. **Euler Equation** (from $u(C_{i,t}) = \ln(C_{i,t})$)

$$\frac{1}{C_{1,t}} = \beta_{t,t+1}(1 + r_{t+1})\frac{1}{C_{1,t+1}} \quad (26)$$