Household:

Max $\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{\,1-\sigma_c}}{(1-\sigma_c)} - \xi_N \frac{N_t^{\,1+\sigma_n}}{1+\sigma_n}\right]$

S.t.

$$C_t + k_t - (1-\delta)k_{t-1} + T_t \leq \frac{b_{t-1}}{\Pi_t} + m_t + \Upsilon_t \pi_t^b + w_t N_t + z_t k_t$$

$$b_t = max\left\{\left[\frac{b_{t-1}}{\Pi_t} + m_t + \Upsilon_t \pi_t^b + w_t N_t + z_t k_t - C_t - k_t + (1-\delta)k_{t-1} - T_t\right] .0\right\} + \pi_t^f$$

$\Longrightarrow$

$$\mathcal{L}_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{\left[\frac{C_t^{\,1-\sigma_c}}{(1-\sigma_c)} - \xi_N \frac{N_t^{\,1+\sigma_n}}{1+\sigma_n}\right] + \mu_t \left[\frac{b_{t-1}}{\Pi_t} + m_t + \Upsilon_t \pi_t^b + w_t N_t + z_t k_t - C_t - k_t + (1-\delta)k_{t-1} - T_t\right] + \lambda_t \left[max\left\{\left[\frac{b_{t-1}}{\Pi_t} + m_t + \Upsilon_t \pi_t^b + w_t N_t + z_t k_t - C_t - k_t + (1-\delta)k_{t-1} - T_t\right] .0\right\} + \pi_t^f - b_t\right]\right\}$$

FOC:

$\frac{\partial \mathcal{L}}{\partial C_t} = 0$

etc

The question that arises is that:

$$\frac{\partial \mathcal{L}}{\partial C_t} = \beta^t C_t^{\,-\sigma_c} - \beta^t \mu_t + \beta^t \lambda_t (????)$$

According to the $max(\dots)$ How we take derivative with respect to C_t ?