FOC consumption: $\vartheta_t = \beta^t C_t^{-\sigma}$

ss. $\bar{\vartheta} = \beta^t \bar{C}^{-\sigma}$

$\widetilde{\vartheta_t}\bar{\vartheta} = \beta^t \bar{C}^{-\sigma}(-\sigma)\widetilde{c_t}$

$\widetilde{\vartheta_t} = -\sigma\widetilde{c_t}$

FOC labor: $\beta^t N_t^{\eta} = \vartheta_t \frac{W_t}{P_t}$

ss. $\beta^t \bar{N}^{\eta} = \bar{\vartheta}\bar{w}$

$\beta^t \bar{N}^{\eta}\eta\,\widetilde{n_t} = \bar{\vartheta}\bar{w}\left(\widetilde{\vartheta_t} + \widetilde{w_t}\right)$

$\eta\,\widetilde{n_t} = \widetilde{\vartheta_t} + \widetilde{w_t}$

$\widetilde{w_t} = \eta\,\widetilde{n_t} - \widetilde{\vartheta_t}$

FOC bonds: $\frac{\vartheta_t}{P_t} = (1 + i_t)\, E_t\left(\frac{\vartheta_{t+1}}{P_{t+1}}\right)$

ss. $\frac{\bar{\vartheta}}{\bar{P}} = (1 + \bar{\imath})\frac{\bar{\vartheta}}{\bar{P}} = \bar{r}\frac{\bar{\vartheta}}{\bar{P}}$

$\frac{\bar{\vartheta}}{\bar{P}}\left(\widetilde{\vartheta_t} - \widetilde{P_t}\right) = \bar{r}\frac{\bar{\vartheta}}{\bar{P}}\left(\widetilde{r_t} + \widetilde{\vartheta_{t+1}} - \widetilde{P_{t+1}}\right)$

$\widetilde{\vartheta_t} = \widetilde{r_t} + \widetilde{\vartheta_{t+1}} - \widetilde{P_{t+1}} + \widetilde{P_t}$

$\widetilde{\vartheta_t} = \widetilde{r_t} + \widetilde{\vartheta_{t+1}} - lnP_{t+1} + ln\bar{P} + lnP_t - ln\bar{P}$

$\widetilde{\vartheta_t} = \widetilde{r_t} + \widetilde{\vartheta_{t+1}} - (lnP_{t+1} - lnP_t) = \widetilde{\vartheta_t} = \widetilde{r_t} + \widetilde{\vartheta_{t+1}} - \pi_{t+1}$

Energy price: $p_t^M = C_t^{\sigma}\,[(1 - er_t)\,\mu]^{1+e}\,{M_t}^{e}$

ss. $\overline{p^M} = \bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e} - \bar{C}^{\sigma}\,\bar{M}^{e}(\overline{er}\mu)^{1+e}$

$$\widetilde{p_t^M}\,\overline{p^M} = \sigma\widetilde{c_t}\,\bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e} + e\,\widetilde{m_t}\,\bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e} - \bar{C}^{\sigma}\,\bar{M}^{e}(\overline{er}\mu)^{1+e}\sigma\widetilde{c_t} - e\,\widetilde{m_t}\,\bar{C}^{\sigma}\,\bar{M}^{e}(\overline{er}\mu)^{1+e} - \bar{C}^{\sigma}\,\bar{M}^{e}(\overline{er}\mu)^{1+e}(1 + e)\,\widetilde{er_t}$$

$$\widetilde{p_t^M}\overline{p^M} = \bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e}(1 - \overline{er}^{1+e})\,\sigma\,\widetilde{c_t} + \bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e}(1 - \overline{er}^{\,1+e})\,e\,\widetilde{m_t} - \bar{C}^{\sigma}\,\bar{M}^{e}(\overline{er}\,\mu)^{1+e}(1 + e)\widetilde{er_t}$$

$$\widetilde{p_t^M} = \frac{\bar{C}^{\sigma}\,\bar{M}^{e}\mu^{1+e}}{\overline{p^M}}[(1 - \overline{er}^{1+e})(\sigma\widetilde{c_t} + e\,\widetilde{m_t}) - (1 + e)\,\widetilde{er_t}\,\overline{er}^{1+e}]$$

Emissions: $Z_t = (1 - er_t)\mu M_t = \mu M_t - \mu\, er_t M_t$

ss. $\bar{Z} = \mu\bar{M} - \mu\,\overline{er}\,\bar{M}$

$$\widetilde{z_t}\bar{Z} = \mu\,\widetilde{m_t}\,\bar{M} - \mu\,\widetilde{er_t}\overline{er}\bar{M} - \mu\,\overline{er}\bar{M}\,\widetilde{m_t}$$

$$\widetilde{z_t}\bar{Z} = \mu\bar{M}(\widetilde{m_t} - \widetilde{er_t}\overline{er} - \overline{er}\widetilde{m_t})$$

$$\widetilde{z_t} = \frac{\mu\bar{M}}{\bar{Z}}\,(\widetilde{m_t} - \widetilde{er_t}\overline{er} - \overline{er}\widetilde{m_t})$$

Emissions price: $p_t^Z = \delta\ln\left(1 - er_t(j)\right)$

ss. $\overline{p^Z} = \delta\ln(1 - \overline{er})$

$$\widetilde{p_t^Z}\,\overline{p^Z} = -\frac{\delta\,\widetilde{er_t}\,\overline{er}}{1 - \overline{er}}$$

$$\widetilde{p_t^Z} = -\frac{\delta\,\widetilde{er_t}\,\overline{er}}{\overline{p^Z}(1 - \overline{er})}$$

Total cost of emissions: $CE_t(j) = -\delta\mu M_t(j)\left[\ln\left(1 - er_t(j)\right)\left(1 - er_t(j)\right) + er_t(j)\right]$

ss. $\overline{CE} = -\delta\mu\bar{M}\ln(1 - \overline{er}) + \delta\mu\bar{M}\,\overline{er}\ln(1 - \overline{er}) - \delta\mu\bar{M}\,\overline{er}$

$$\widetilde{ce_t}\overline{CE} = -\delta\mu\bar{M}\ln(1 - \overline{er})\left(\widetilde{m_t} - \frac{\overline{er}\,\widetilde{er_t}}{1 - \overline{er}}\right) + \delta\mu\bar{M}\overline{er}\ln(1 - \overline{er})\left(\widetilde{m_t} + \widetilde{er_t} - \frac{\overline{er}\,\widetilde{er_t}}{1 - \overline{er}}\right) - \delta\mu\bar{M}\widetilde{m_t}\,(\widetilde{m_t} + \widetilde{er_t})$$

$$\widetilde{ce_t} = \frac{1}{\overline{CE}}\left\{-\delta\mu\bar{M}\ln(1 - \overline{er})\left(\widetilde{m_t} - \frac{\overline{er}\,\widetilde{er_t}}{1 - \overline{er}}\right) + \delta\mu\bar{M}\overline{er}\ln(1 - \overline{er})\left(\widetilde{m_t} + \widetilde{er_t} - \frac{\overline{er}\,\widetilde{er_t}}{1 - \overline{er}}\right) - \delta\mu\bar{M}\,\overline{er}\,(\widetilde{m_t} + \widetilde{er_t})\right\}$$

Production function: $Y_t(j) = A_t N_t^{\alpha}(j) M_t^{1-\alpha}(j)$

ss. $\bar{Y} = \overline{A\,N}^{\alpha}\bar{M}^{1-\alpha}$

$$\widetilde{y_t}\,\bar{Y} = \widetilde{a_t}\,\bar{A}\bar{N}^{\alpha}\bar{M}^{1-\alpha} + \alpha\,\widetilde{n_t}\,\bar{A}\bar{N}^{\alpha}\bar{M}^{1-\alpha} + (1-\alpha)\widetilde{m_t}\bar{A}\bar{N}^{\alpha}\bar{M}^{1-\alpha}$$

$$\widetilde{y_t}\,\bar{Y} = \bar{A}\bar{N}^{\alpha}\bar{M}^{1-\alpha}[\widetilde{a_t} + \alpha\,\widetilde{n_t} + (1-\alpha)\widetilde{m_t}]$$

$$\widetilde{y_t} = \widetilde{a_t} + \alpha\,\widetilde{n_t} + (1-\alpha)\widetilde{m_t}$$

Marginal productivity of labor: $MPL_t = A_t\,\alpha\,N_t^{\alpha-1}(j)M_t^{1-\alpha}(j)$

ss. $\overline{MPL} = \bar{A}\,\alpha\,\bar{N}^{\alpha-1}\bar{M}^{1-\alpha}$

$$\widetilde{mpl_t}\,\overline{MPL} = \widetilde{a_t}\,\bar{A}\,\alpha\,\bar{N}^{\alpha-1}\bar{M}^{1-\alpha} + (\alpha-1)\widetilde{n_t}\,\bar{A}\,\alpha\,\bar{N}^{\alpha-1}\bar{M}^{1-\alpha} + (1-\alpha)\,\widetilde{m_t}\,\bar{A}\,\alpha\,\bar{N}^{\alpha-1}\bar{M}^{1-\alpha}$$

$$\widetilde{mpl_t}\,\overline{MPL} = \bar{A}\,\alpha\,\bar{N}^{\alpha-1}\bar{M}^{1-\alpha}\,[\widetilde{a_t} + (\alpha-1)\widetilde{n_t} + (1-\alpha)\,\widetilde{m_t}]$$

$$\widetilde{mpl_t} = \widetilde{a_t} + (\alpha-1)\,\widetilde{n_t} + (1-\alpha)\,\widetilde{m_t}$$

Marginal costs: $MC_t = \frac{w_t}{MPL_t}$

ss. $\overline{MC} = \frac{\bar{w}}{\overline{MPL}}$

$$\widetilde{mc_t}\,\overline{MC} = \frac{\bar{w}}{\overline{MPL}}\left(\widetilde{w_t} - \widetilde{mpl_t}\right)$$

$$\widetilde{mc_t} = \widetilde{w_t} - \widetilde{mpl_t}$$

FOC firm to energy:

$$p_t^M + p_t^Z(1-er_t)\mu - \delta\mu\ln(1-er_t) + \delta\mu er_t\ln(1-er_t) - \delta\mu\, er_t = MC_t\, A_t(1-\alpha)N_t^{\alpha}M_t^{-\alpha}$$

$$M_t^{-\alpha} = \frac{1}{MC_t\, A_t(1-\alpha)N_t^{\alpha}}\left[p_t^M + p_t^Z(1-er_t)\mu - \delta\mu\ln(1-er_t) + \delta\mu er_t\ln(1-er_t) - \delta\mu\, er_t\right]$$

ss. $\bar{M}^{-\alpha} = \frac{1}{\overline{MC}\;\bar{A}\;(1-\alpha)\;\bar{N}^{\alpha}}\left[\overline{p^M} + \overline{p^Z}(1-\overline{er})\mu - \delta\mu\ln(1-\overline{er}) + \delta\mu\overline{er}\ln(1-\overline{er}) - \delta\mu\overline{er}\right]$

$$\begin{aligned}-\alpha\widetilde{m_t}\bar{M}^{-\alpha} = \frac{1}{\overline{MC}\;\bar{A}\;(1-\alpha)\bar{N}^{\alpha}}\Bigg[&\overline{p^M}\left(\widetilde{p_t^M} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right) + \overline{p^Z}\mu\left(\widetilde{p_t^Z} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \overline{p^Z}\,\overline{er}\mu\left(\widetilde{p_t^Z} + \widetilde{er_t} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \delta\mu\ln(1-\overline{er})\left(\frac{-\overline{er}\,\widetilde{er_t}}{(1-\overline{er})} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &+ \delta\mu\,\overline{er}\ln(1-\overline{er})\left(\widetilde{er_t} - \frac{\overline{er}\,\widetilde{er_t}}{(1-\overline{er})} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \delta\mu\overline{er}\left(\widetilde{er_t} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\Bigg]\end{aligned}$$

$$\begin{aligned}\widetilde{m_t} = \frac{-\bar{M}^{\alpha}}{\overline{MC}\;\bar{A}\;\alpha(1-\alpha)\bar{N}^{\alpha}}\Bigg[&\overline{p^M}\left(\widetilde{p_t^M} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right) + \overline{p^Z}\mu\left(\widetilde{p_t^Z} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \overline{p^Z}\,\overline{er}\mu\left(\widetilde{p_t^Z} + \widetilde{er_t} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \delta\mu\ln(1-\overline{er})\left(\frac{-\overline{er}\,\widetilde{er_t}}{(1-\overline{er})} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &+ \delta\mu\,\overline{er}\ln(1-\overline{er})\left(\widetilde{er_t} - \frac{\overline{er}\,\widetilde{er_t}}{(1-\overline{er})} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\\ &- \delta\mu\overline{er}\left(\widetilde{er_t} - \widetilde{mc_t} - \widetilde{a_t} - \alpha\widetilde{n_t}\right)\Bigg]\end{aligned}$$

Government constraint: $G_t = tax_t + b_{t+1} - (1+i_t)b_t = tax_t + b_{t+1} - \left(r_t\frac{P_t}{P_{t+1}}\right)b_t$

ss. $\bar{G} = \overline{tax} + \bar{b} - \bar{r}\,\frac{\bar{P}}{\bar{P}}\bar{b}$

$$\widetilde{g_t}\bar{G} = \widetilde{tax_t}\,\overline{tax} + \bar{b}\,\widetilde{b_{t+1}} - \widetilde{r_t}\,\bar{r}\,\bar{b} - \widetilde{p_t}\,\bar{r}\,\bar{b} + \widetilde{p_{t+1}}\bar{r}\bar{b} - \widetilde{b_t}\,\bar{r}\,\bar{b}$$

$$\widetilde{g_t}\bar{G} = \widetilde{tax_t}\,\overline{tax} + \bar{b}\,\widetilde{b_{t+1}} - \bar{r}\,\bar{b}\left(\widetilde{r_t} + \widetilde{p_t} - \widetilde{p_{t+1}} + \widetilde{b_t}\right)$$

$$\widetilde{g_t} = \frac{1}{\bar{G}}\left\{\widetilde{tax_t}\,\overline{tax} + \bar{b}\,\widetilde{b_{t+1}} - \bar{r}\,\bar{b}\left(\widetilde{r_t} - \pi_{t+1} + \widetilde{b_t}\right)\right\}$$

<u>*Tax*</u>: $tax_t = w_t N_t + p_t^M M_t + p_t^Z Z_t$

ss. $\overline{tax} = \bar{w}\bar{N} + \overline{p^M}\bar{M} + \overline{p^Z}\bar{Z}$

$$\widetilde{tax_t}\,\overline{tax} = \bar{w}\bar{N}\widetilde{w_t} + \bar{w}\bar{N}\widetilde{n_t} + \overline{p^M}\bar{M}\,\widetilde{p_t^M} + \overline{p^M}\bar{M}\widetilde{m_t} + \overline{p^Z}\bar{Z}\,\widetilde{p_t^Z} + \overline{p^Z}\bar{Z}\widetilde{z_t}$$

$$\widetilde{tax_t}\,\overline{tax} = \bar{w}\bar{N}\,(\widetilde{w_t} + \widetilde{n_t}) + \overline{p^M}\bar{M}\left(\widetilde{p_t^M} + \widetilde{m_t}\right) + \overline{p^Z}\bar{Z}\left(\widetilde{p_t^Z} + \widetilde{z_t}\right)$$

$$\widetilde{tax_t} = \frac{1}{\overline{tax}}\left[\bar{w}\bar{N}\,(\widetilde{w_t} + \widetilde{n_t}) + \overline{p^M}\bar{M}\left(\widetilde{p_t^M} + \widetilde{m_t}\right) + \overline{p^Z}\bar{Z}\left(\widetilde{p_t^Z} + \widetilde{z_t}\right)\right]$$

<u>*Aggregate resource constraint*</u>: $Y_t = C_t + G_t + CE_t$

ss. $\bar{Y} = \bar{C} + \bar{G} + \overline{CE}$

$$\widetilde{y_t}\bar{Y} = \widetilde{c_t}\bar{C} + \widetilde{g_t}\bar{G} + \widetilde{ce_t}\,\overline{CE}$$

$$\widetilde{y_t} = \frac{1}{\bar{Y}}\left(\widetilde{c_t}\,\bar{C} + \widetilde{g_t}\bar{G} + \widetilde{ce_t}\,\overline{CE}\right)$$

<u>*Flexible price output*</u>:

$$\widetilde{y_t}^f = \frac{1}{\bar{C}(1+\eta) + \alpha(\sigma\bar{Y} - \bar{C})}\left\{\alpha\sigma(\widetilde{g_t}\bar{G} + \widetilde{ce_t}\,\overline{CE}) + \bar{C}(1+\eta)\widetilde{a_t}^f + \bar{C}(1+\eta)(1-\alpha)\,\widetilde{m_t}^f\right\}$$

<u>*Curva di Phillips*</u>: $\pi_t = \tilde{k}\,MC_t + \beta\,E_t(\pi_{t+1})$