Lifetime utility maximisation for households:

$$\max \quad U=\sum_{t=0}^{\infty}\beta^t u_t = E\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}-1}{1-\sigma}-\frac{N_{f,t}^{1+\chi}}{1+\chi}-\frac{N_{m,t}^{1+\chi}}{1+\chi}\right]$$

The budget constraint:

$$\begin{aligned} &C_{1,t}+I_{f1,t}+I_{m1,t}+p_{a1,t}(\tfrac{\psi_b}{2}B_{1,t+1}^2+B_{1,t+1}) \\ &=w_{f1,t}N_{f1,t}+u_{f1,t}K_{f1,t}+w_{m1,t}N_{m1,t}+u_{m1,t}K_{m1,t}+p_{a1,t}(1+r_{b,t})B_{1,t} \end{aligned}$$

C_t is consumption, $I_{1,t}$ is investment, $N_{1,t}$ is labour $K_{1,t}$ is capital, $B_{1,t}$ is foreign bonds, $r_{b,t}$ is bond rates, $w_{1,t}$ is wages, $u_{1,t}$ is capital rates, $p_{a1,t}$ is price of a tradable good. The subscripts *f* and *m* denote different sectors whose capital and labour are considered heterogeneous in the model.

Capital movement equation:

$$K_{f1,t+1}=(1-\delta)K_{f1,t}+I_{f1,t}-\tfrac{\psi_k}{2}(\tfrac{I_{f1,t}}{K_{f1,t}}-\delta)^2K_{f1,t}$$

$$K_{m1,t+1}=(1-\delta)K_{m1,t}+I_{m1,t}-\tfrac{\psi_k}{2}(\tfrac{I_{m1,t}}{K_{m1,t}}-\delta)^2K_{m1,t}$$

First-order conditions on C_t 、 $N_{1,t}$ 、 $K_{1,t+1}$ and $B_{1,t+1}$:

$$C_{1,t}^{-\sigma}=\lambda_{1,t}$$

$$N_{f1,t}^{\chi}=\lambda_{1,t}w_{f1,t}$$

$$N_{m1,t}^{\chi}=\lambda_{1,t}w_{m1,t}$$

$$\lambda_{1,t}=\lambda_{1,t+1}\beta\left[u_{f1,t+1}+(1-\delta)+\tfrac{\psi_k}{2}(\tfrac{I_{f1,t+1}}{K_{f1,t+1}}-\delta)(\tfrac{I_{f1,t+1}}{K_{f1,t+1}}+\delta)\right]$$

$$\lambda_{1,t}=\lambda_{1,t+1}\beta\left[u_{m1,t+1}+(1-\delta)+\tfrac{\psi_k}{2}(\tfrac{I_{m1,t+1}}{K_{m1,t+1}}-\delta)(\tfrac{I_{m1,t+1}}{K_{m1,t+1}}+\delta)\right]$$

$$\lambda_{1,t}(1+\psi_b B_{1,t+1})=\lambda_{1,t+1}\beta(1+r_{b,t+1})$$