Exchange Rate Flexibility under the Zero Lower Bound
David Cook and Michael B. Devereux

Online Appendix - Not for Publication.

Appendix A

Equilibrium with Fully Flexible Prices

We first define the equilibrium of the model in a fully flexible price world equilibrium, where $\kappa = 0$ in each country. In that case, $P_{Ht}(i) = P_{Ht}$, $P_{Ft}(i) = P_{Ft}$, and $V_t = V_t^* = 1$. In addition (given the presence of optimal subsidies) we have $P_{Ht} = A^{-1}W_t$ and $P_{Ft}^* = A^{-1}W_t^*$. Letting a bar denote values in a flexible price world equilibrium, we may describe the equilibrium by the equations:

$$U_C(\bar{C}_t, \zeta_t)A = \bar{T}_t^{1-v/2}V'(\bar{N}_t), \qquad U_C^*(\bar{C}_t^*, \zeta_t^*)A^* = \bar{T}_t^{-(1-v/2)}V'(\bar{N}_t^*) \tag{1}$$

$$U_C(\bar{C}_t, \zeta_t)\bar{T}_t^{v-1} = U_C^*(\bar{C}_t^*, \zeta_t^*), \tag{2}$$

$$A\bar{N}_t = \frac{v}{2}\bar{T}_t^{1-v/2}\bar{C}_t + (1 - \frac{v}{2})\bar{T}_t^{v/2}\bar{C}_t^*, \tag{3}$$

$$A^*\bar{N}_t^* = \frac{v}{2}\bar{T}_t^{-(1-v/2)}\bar{C}_t^* + (1 - \frac{v}{2})\bar{T}_t^{-v/2}\bar{C}_t^* \tag{4}$$

This implicitly describes the efficient world equilibrium for consumption, output (or employment), and the terms of trade.

We analyze equations (1)-(4) by taking a linear approximation around the globally efficient steady state. For a given variable X, define $\bar{x}$ to be the log difference of the global efficient value from the non-stochastic steady state, except for ε_t (as defined in the text), and π_{Ht} and r_t, which refer respectively to the *level* of the inflation rate and nominal interest rate. Since the model is symmetric, we have $\overline{T} = 1$ in a steady state. Then we may express the linear approximation of (1)-(4) as:

$$\sigma\bar{c}_t - \varepsilon_t + \phi\bar{y}_t + (1 - \frac{v}{2})\bar{\tau}_t = 0 \tag{5}$$

$$\sigma\bar{c}_t^* - \varepsilon_t^* + \phi\bar{y}_t^* - (1 - \frac{v}{2})\bar{\tau}_t = 0 \tag{6}$$

$$\bar{y}_t = (\frac{v}{2}\bar{c}_t + (1 - \frac{v}{2})\bar{c}_t^*) + v(1 - \frac{v}{2})\bar{\tau}_t, \tag{7}$$

$$\bar{y}_t^* = (\frac{v}{2}\bar{c}_t^* + (1-\frac{v}{2})\bar{c}_t) - v(1-\frac{v}{2})\bar{\tau}_t, \tag{8}$$

$$\sigma\bar{c}_t - (\varepsilon_t - \varepsilon_t^*) - \sigma\bar{c}_t^* - (v-1)\bar{\tau}_t = 0, \tag{9}$$

We may solve the system (5)-(9) to obtain the first order solutions for consumption, output and the terms of trade in response to savings shocks. We can write home and foreign consumption responses to savings shocks as:

$$\bar{c}_t = \frac{1}{\phi+\sigma}\varepsilon_t^W + \left(\frac{1+\phi v(2-v)}{\sigma+\phi D}\right)\varepsilon_t^R$$

$$\bar{c}_t^* = \frac{1}{\phi+\sigma}\varepsilon_t^W - \left(\frac{1+\phi v(2-v)}{\sigma+\phi D}\right)\varepsilon_t^R$$

A savings shock reduces the efficient flexible price world consumption level, but the impact on individual country consumption depends on the source of the shock, and the degree of home bias in preferences.

The impact of savings shocks on flexible price output levels are likewise written as:

$$\bar{y}_t = \frac{1}{\phi+\sigma}\varepsilon_t^W + \left[\left(\frac{(v-1)}{\sigma+\phi D}\right)\right]\varepsilon_t^R$$

$$\bar{y}_t^* = \frac{1}{\phi+\sigma}\varepsilon_t^W - \left[\left(\frac{(v-1)}{\sigma+\phi D}\right)\right]\varepsilon_t^R$$

A world savings shock reduces equilibrium output in both countries. When there is home bias in preferences, so that $v > 1$, a relative home savings shock tends to reduce home output and raise foreign output.

Demand shocks also affect the flexible price efficient response of the terms of trade. We can show that:

$$\frac{\bar{\tau}_t}{2} = -\frac{\phi(v-1)}{\sigma+\phi D}\varepsilon_t^R \tag{10}$$

In response to a relative home country savings shock, relative home output falls, but the terms of trade deteriorates, in a fully flexible price equilibrium.

If monetary authorities could adjust nominal interest rates freely to respond to demand shocks, then the flexible price efficient global equilibrium could be sustained. We denote the interest rate that would sustain the flexible price efficient outcome as the 'natural interest rate'. Let ρ be the steady state value for the natural interest rate. Then a log linear approximation of (**??**) leads to the expressions for the flexible price equilibrium nominal interest rate in the home country as:

$$\bar{r}_t = \rho + \sigma E_t(\bar{c}_{t+1} - \bar{c}_t) - E_t(\varepsilon_{t+1} - \varepsilon_t) + E_t\pi_{Ht+1} + (1 - \frac{v}{2})E_t(\bar{\tau}_{t+1} - \bar{\tau}_t) \qquad (11)$$

Assume that an efficient monetary policy is to keep the domestic rate of inflation equal to zero. In addition, for now, assume that demand shocks follow an AR(1) process so that $\varepsilon_{t+1} = \mu\varepsilon_t + u_t$ and $\varepsilon^*_{t+1} = \mu\varepsilon^*_t + u^*_t$, where u_t and u^*_t are mean-zero and i.i.d., then the value of $\widetilde{r}_t$ when the right hand side is driven by demand shocks alone can be derived as:

$$\bar{r}_t = \rho + \left(\frac{\phi}{\phi + \sigma}\varepsilon^W_t + \left(\frac{\phi(v-1)}{\sigma + \phi D}\right)\varepsilon^R_t\right)(1-\mu) \qquad (12)$$

In similar manner, the foreign efficient nominal interest rate is:

$$\bar{r}_t^* = \rho + \left(\frac{\phi}{\phi + \sigma}\varepsilon^W_t - \left(\frac{\phi(v-1)}{\sigma + \phi D}\right)\varepsilon^R_t\right)(1-\mu) \qquad (13)$$

Natural interest rates respond to both aggregate and relative savings shocks. An aggregate savings shock raises global marginal utility and raises natural interest rates. A relative savings shock affects natural interest rates in separate ways in the two countries, but this depends upon the degree of home bias in preferences. With identical preferences across countries, the natural interest rate is independent of purely relative demand shocks, and is equalized across countries.

This discussion has direct bearing on the degree to which the zero bound constraint will bind across countries in response to time preference shocks (negative demand shocks) emanating from one country. In general, these shocks will have both aggregate and relative components. If there are full security markets and identical preferences, we see that natural interest rates are always equated across countries. But when $v \geq 1$ a home country shock has a smaller impact on the foreign natural interest rate, so the home country may be constrained by the zero lower bound, but the foreign country will not be so constrained.

Optimal Policy: Discretion

We describe optimal policy as in Cook and Devereux (2013). Policy makers are assumed to cooperate in the sense that policies are chosen to minimize a global loss function. Issues of non-cooperative policy and strategic interaction between policymakers lie outside the scope of the analysis followed here. Our approach is similar to the optimal cooperative monetary policy in Engel 2010. (see Cook and Devereux 2013 and Benigno and Benigno 2010 for a further discussion of non-cooperative policy making).

Cooperative policy under discretion (and multiple currencies) may be characterized by the solution to the following optimization problem:

$$\begin{aligned}
& \max_{\widehat{y}_t^R, \widetilde{y}_t^W, \pi_t^W, \pi_t^R, r_t^W, r_t^R} L_t \\
= \ & V_t + \lambda_{1t}\left[\pi_t^W - k(\phi+\sigma)\widetilde{n}_t^W - \beta E_t \pi_{t+1}^W\right] \\
& + \lambda_{2t}\left[\pi_t^R - k(\phi+\sigma_D)\widetilde{y}_t^R - \beta E_t \pi_{t+1}^R\right] \\
& + \psi_{1t}\left[\sigma E_t(\widehat{y}_{t+1}^W - \widehat{y}_t^W) - E_t\left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right)\right] \\
& + \psi_{2t}\left[\sigma_D E_t(\widehat{y}_{t+1}^R - \widehat{y}_t^R) - E_t\left(r_t^R - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R\right)\right] \\
& + \gamma_{1t}\left(r_t^W + r_t^R\right) + \gamma_{2t}\left(r_t^W - r_t^R\right)
\end{aligned}$$

The policy optimum involves the choice of the output gaps, government spending gaps, inflation rates and the foreign interest rate to maximize this Lagrangian. The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final two constraints are the non-negativity constraint on the two policy interest rates. The mechanics of optimal policy require policymakers to adjust interest rates so as to implement the optimal solution, so long as the interest rates are feasible.

The first order conditions of the maximization are:

$$-(\sigma_D + \phi)\widehat{y}_t^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_2 \tag{14}$$

$$-(\sigma + \phi)\widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} \tag{15}$$

$$k\lambda_{1t} = \theta \pi_t^W \tag{16}$$

$$k\lambda_{2t} = \theta \pi_t^R \tag{17}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{18}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{19}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{20}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{21}$$

These equations, in conjunction with (10)-(11) and (13)-(14), give the conditions determining average and relative output gaps; $\widehat{y}_t^R, \widehat{y}_t^W$, inflation rates; π_t^R, π_t^W, fiscal gaps; $\widehat{cg}_t^R, \widehat{cg}_t^W$, the Lagrange multipliers; $\lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}$, and the value of either $r_t^W + r_t^R$, or γ_{1t} and $r_t^W - r_t^R$, or γ_{2t}, under optimal policy.

In the single currency area, cooperative policy is described in a similar manner, but involves only the choice of r_t^W subject to a non-negativity constraint.

Optimal Policy: Commitment

Cooperative policy under commitment with multiple currencies is characterized by:

$$\begin{aligned}
&\max_{\widehat{y}_t^R, \widetilde{y}_t^W, \pi_t^W, \pi_t^R, r_t^W, r_t^R} L_0 \\
= \;& V_0 + \sum_{t=0}^{\infty} \lambda_{1t} \left[\pi_t^W - k(\phi+\sigma)\widetilde{n}_t^W - \beta E_t \pi_{t+1}^W\right] \\
& + \sum_{t=0}^{\infty} \lambda_{2t} \left[\pi_t^R - k(\phi+\sigma_D)\widetilde{y}_t^R - \beta E_t \pi_{t+1}^R\right] \\
& + \sum_{t=0}^{\infty} \psi_{1t} \left[\sigma E_t(\widehat{y}_{t+1}^W - \widehat{y}_t^W) - E_t\left(r_t^W - \widetilde{r}_t^W - \pi_{t+1}^W\right)\right] \\
& + \sum_{t=0}^{\infty} \psi_{2t} \left[\sigma_D E_t(\widehat{y}_{t+1}^R - \widehat{y}_t^R) - E_t\left(r_t^R - \frac{\widetilde{r}_t^R}{2} - \pi_{t+1}^R\right)\right] \\
& + \sum_{t=0}^{\infty} \gamma_{1t}\left(r_t^W + r_t^R\right) + \sum_{t=0}^{\infty} \gamma_{2t}\left(r_t^W - r_t^R\right)
\end{aligned}$$

where V_0 is defined in (37) of the text.

The policy optimum involves the choice of the output gaps, inflation rates and the foreign interest rate to maximize this Lagrangian. The first two constraints are the inflation equations in average and relative terms. The second two constraints are the average and relative 'IS' equations. The final two constraints are the non-negativity constraint on the two policy interest rates.

The first order conditions of the maximization are, for time $t = 0$:

$$-(\sigma_D + \phi)\widehat{y}_0^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_{2t} \tag{22}$$

$$-(\sigma + \phi)\widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} \tag{23}$$

$$k\lambda_{1t} = \theta \pi_t^W \tag{24}$$

$$k\lambda_{2t} = \theta \pi_t^R \tag{25}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{26}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{27}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{28}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{29}$$

and for time $t > 0$, the conditions are:

$$-(\sigma_D + \phi)\widehat{y}_0^R = \lambda_{2t} k(\phi + \sigma_D) + \sigma_D \psi_{2t} - \sigma_D \psi_{2t-1} \tag{30}$$

$$-(\sigma + \phi)\widehat{y}_t^W = \lambda_{1t} k(\phi + \sigma) + \sigma \psi_{1t} - \sigma \psi_{1t-1} \tag{31}$$

$$k(\lambda_{1t} - \beta\lambda_{1t-1} - \psi_{1t-1}) = \theta \pi_t^W \tag{32}$$

$$k(\lambda_{2t} - \beta\lambda_{2t-1} - \psi_{2t-1}) = \theta \pi_t^R \tag{33}$$

$$\psi_{2t} = \gamma_{1t} + \gamma_{2t} \tag{34}$$

$$\psi_{2t} = \gamma_{1t} - \gamma_{2t} \tag{35}$$

$$\gamma_{1t} \geq 0, \quad (r_t^W + r_t^R) \geq 0, \quad \gamma_{1t}\ (r_t^W + r_t^R) = 0 \tag{36}$$

$$\gamma_{2t} \geq 0, \quad (r_t^W - r_t^R) \geq 0, \quad \gamma_{2t}\ (r_t^W - r_t^R) = 0 \tag{37}$$

These equations, in conjunction with (**??**)-(**??**) and (**??**)-(**??**) for each period t, give the conditions determining average and relative output gaps; $\widehat{y}_t^R, \widehat{y}_t^W$, inflation rates; π_t^R, π_t^W, fiscal gaps; $\widehat{cg}_t^R, \widehat{cg}_t^W$, the Lagrange multipliers; $\lambda_{1t}, \lambda_{2t}, \psi_{1t}, \psi_{2t}$, and the value of either $r_t^W + r_t^R$, or γ_{1t} and $r_t^W - r_t^R$, or γ_{2t}, under an optimal policy with commitment.

Again, under the single currency area, the optimal policy problem is simply a subset of the above problem, where r_t^W is chosen subject to a non-negativity constraint.

Appendix B. Adjustment with Fiscal Policy

Farhi et. al.. (2012) have described how a mix of tax and subsidies can achieve "Fiscal Devaluation" in a small economy, exactly replicating the effects of a nominal exchange rate devaluation. Therefore, if fiscal policy is sufficiently flexible, it can completely eliminate the loss of monetary autonomy implied by a fixed exchange rate regime. More generally, it has been established by Correa et. al.. (2012) that a combination of state-contingent taxes and subsidies can undo the effects of the zero bound, and fully replicate the flexible price equilibrium in standard New Keynesian models. A similar set of results applies to our model. We show below that a combination of

VAT adjustment and payroll tax changes can be used to ensure price stability and zero output gaps, achieving the fully optimal flexible price equilibrium. But when monetary policy is constrained by the zero bound, fiscal adjustment will be required even in a situation of flexible exchange rates. So we need to identify the set of optimal fiscal instruments both in the single currency model as well as the model with flexible exchange rates. The main result we show is that the tax-subsidy mix is the same in both cases. We can express the extended model in terms of world averages and world relatives as follows:

$$\pi_t^W = k((\phi+\sigma)\widehat{y}_t^W + t_{VAT,t}^W + t_{WAGE,t}^W - \varepsilon_t^W) + \beta E_t \pi_{t+1}^W \quad (38)$$

$$\sigma E_t(\widehat{y}_{t+1}^W - \widehat{y}_t^W) = E_t(\varepsilon_{t+1}^W - t_{VAT,t+1}^W) - (\varepsilon_t^W - t_{VAT,t}^W) + E_t\left(r_t^W - E_t\pi_{t+1}^W - \rho\right) \quad (39)$$

$$\pi_t^R = k((\phi+\sigma_D)\widehat{y}_t^R - \zeta(\varepsilon_t^R - t_{VAT}^R) + t_{WAGE}^R) + \beta E_t \pi_{t+1}^R \quad (40)$$

$$\sigma_D E_t(\widehat{y}_{t+1}^R - \widehat{y}_t^R) = \zeta(E_t(\varepsilon_{t+1}^R - t_{VAT,t+1}^R) - (\varepsilon_t^R - t_{VAT,t}^R)) + E_t\left(r_t^R - \pi_{t+1}^R\right) \quad (41)$$

Here, $t_{VAT,t}^W$ ($t_{VAT,t}^R$) represents the world average (world relative) VAT tax at time t, assuming taxes are zero in steady state. Likewise $t_{WAGE,t}^W$ ($t_{WAGE,t}^R$) represent the world average (world relative) payroll tax. An increase in world average VAT tax raises consumer prices, shifting back labor supply and pushing up marginal costs for firms, thus reducing world output. A payroll tax also increases marginal costs and reduces output. At the same time, an expected rise in the VAT rate $E_t t_{VAT,t+1}^W - t_{VAT,t}^W$ will reduce the expected real interest rate, inclusive of taxes, and reduce the rate of growth of world consumption and output. The impact of world relative VAT and payroll taxes on world relative output can be explained in an analogous manner. We may rewrite these two equations systems in terms of inflation and output gaps

$$\pi_t^W = k((\phi+\sigma)\tilde{y}_t^W + t_{VAT,t}^W + t_{WAGE,t}^W) + \beta E_t \pi_{t+1}^W \quad (42)$$

$$\sigma E_t(\tilde{y}_{t+1}^W - \tilde{y}_t^W) = -E_t(t_{VAT,t+1}^W) - t_{VAT,t}^W) + E_t\left(r_t^W - E_t\pi_{t+1}^W - R_{N,t}^W\right) \quad (43)$$

$$\pi_t^R = k((\phi+\sigma_D)\tilde{y}_t^R + \zeta t_{VAT}^R - t_{WAGE}^R) + \beta E_t \pi_{t+1}^R \quad (44)$$

$$\sigma_D E_t(\tilde{y}_{t+1}^R - \tilde{y}_t^R) = -\zeta(E_t t_{VAT,t+1}^R - t_{VAT,t}^R) + E_t\left(r_t^R - \pi_{t+1}^R - R_{N,t}^R\right) \quad (45)$$

here $R_{N,t}^W = \rho + (1-\mu)\frac{\phi}{\sigma+\phi}\varepsilon_t^W$ is defined as the world average natural interest rate, and $R_{N,t}^R = (1-\mu)\zeta\frac{\phi}{\sigma_D+\phi}\varepsilon_t^R$ is the world relative natural real interest rate.

From these two equations, it is easy to see that the following combination of VAT and payroll tax changes can eliminate all gaps in a liquidity trap. The policy mix can be described by the following conditions:

$$t_{VAT}^{W} = \frac{\phi}{\sigma + \phi}\varepsilon_t^W \tag{46}$$

$$t_{WAGE}^{W} = -t_{VAT}^{W} \tag{47}$$

$$t_{VAT}^{R} = \frac{\phi}{\sigma_D + \phi}\varepsilon_t^R \tag{48}$$

$$t_{WAGE}^{R} = -\zeta t_{VAT}^{R} \tag{49}$$

In addition, these taxes are applied with the same persistence μ as the shock itself. This policy mix does the following. In terms of world averages, it combines a VAT tax cut with a payroll tax increase. The VAT tax cut takes on the same expected persistence as the negative demand shock, and so induces a fall in the expected real interest rate that cannot be achieved by a nominal interest rate cut. The VAT tax cut on its own however would be too expansionary, since with inflation and expected inflation maintained at zero, it would lead to a positive output gap. This must be offset by a payroll tax increase. In terms of world relatives, there is a relative VAT tax cut that is greater in the worst hit economy. That effectively achieves the relative price tilting that mimics the terms of trade depreciation that should take place in the fully efficient economy with fully flexible prices. Again, with $v > 1$ however, this would lead to a positive world relative output gap, and so must be countered by a rise in the relative payroll tax. The key feature of the solutions (46)-(49) is that they do not depend on the monetary rule. Thus, they are the same for the flexible exchange rate model and the single currency area. In both cases, they achieve the adjustment in world and relative output without any inflation adjustment, or nominal exchange rate adjustment. To see this, we note that the terms of trade (exclusive of VAT) in the presence of VAT changes can be written as

$$\hat{\tau}_t = 2\sigma_D \hat{y}_t^R - 2\zeta(\varepsilon_t^R - t_{VAT,t}^R) \tag{50}$$

It is easy to see from (44) and (50) that the adjustment of relative VAT rates obviates any movement in the terms of trade. Hence, if VAT rates are adjusted appropriately, neither domestic inflation or nominal exchange rate adjustment is necessary. But in contrast to the discussion on Fiscal Devaluations in the eurozone, we see that in a liquidity trap, the optimal fiscal policy mix is the same whether or not the region has a system of flexible exchange rates. Thus, the key constraint is the zero bound on interest rates, not the non-adjustability of the nominal exchange rate.

Appendix C. Terms of Trade with Persistent Shocks

Here we show that the impact of the savings shock on the terms of trade depends on the persistence of the shock. When the shock expires at the same time that the interest rate rises above the zero bound, then the terms of trade will always appreciate in response to a savings shock. But if the shock displays persistence beyond the application of the zero bound, and the response of policy is governed by a Taylor rule (which does not completely offset the shock), then the terms of trade may not appreciate.

The dynamics of the terms of trade may be obtained from (**??**) and (10), and may be written as

$$\tau_t = 2\sigma_D \tilde{y}_t + \tilde{\tau}_t \tag{51}$$

where $\tilde{y}_t$ is the output gap and $\tilde{\tau}_t$ means flexible price terms of trade Also, the relative natural real interest rate is

$$\tilde{r}_t^R = \frac{1}{2}(\tilde{\tau}_{t+1} - \tilde{\tau}_t) \tag{52}$$

The IS equation in relative terms is given by

$$\sigma_D(\tilde{y}_{t+1} - \tilde{y}_t) = i_t - \pi_{t+1} - \tilde{r}_t \tag{53}$$

Combining (51)-(53) we have

$$\tau_t = \tau_{t+1} - 2(i_t - \pi_{t+1})$$

Then combine this with the inflation equation:

$$\pi_t = \beta\pi_{t+1} + k(\sigma_D + \phi)\tilde{y}_t = \beta\pi_{t+1} + \frac{k(\sigma_D + \phi)}{2\sigma_D}(\tau_t - \tilde{\tau}_t)$$

If the liquidity trap ends at time $T+1$, and all gaps are zero when the shock ends (as in the paper), we have $\tau_{t+1} = \tilde{\tau}_{T+1} = 0$. If policy is set optimally with discretion, or if a Taylor rule applies but the shock is zero, then we have $\pi_{T+1} = 0$, so we must by the above condition have (given $i_T = 0$)

$$\tau_T = 0$$

Then by the inflation equation, if the shock is such that $\tilde{\tau}_T > 0$, we must have $\pi_T < 0$ (a negative output gap since the terms of trade doesn't depreciate enough).

Then it is easy to see that $\tau_t < 0$ for all $t < T$. So the terms of trade will move in the 'wrong direction'.

But if the shock continues after time $T+1$, so that $\tilde{\tau}_{T+1} > 0$, but we still assume policy discretion so that there is zero inflation at time $T+1$, we will have, in time T, via equation (52), $\tau_T = \tilde{\tau}_{T+1}$. Then we will still have

$$\tau_T - \tilde{\tau}_T < 0$$

where the shock is such that the flexible price terms of trade falls over time as the shock dissipates. Then $\pi_T < 0$ as before, but recursing backwards we can see that τ_t for $t < T$ may be positive. The dynamics of τ_t will depend on k, ϕ and other parameters.