1 Matching model for dynare example

1.1 Households

A household consists of a continuum of households, each of which is either employed or unemployed. Employed members get a wage rate w_t. Unemployed members are searching for a new job. For simplicity there are no unemployment benefits. There is no disutility of work or search and all unemployed workers accept a job if they get offered one and employment is determined by the demand for labor by firms. Households own the capital stock, which they rent out to firms.

The maximization problem of the household is then given by

$$\begin{aligned} v(k_{t-1};\tilde{n}_{t-1},z_t) &= \max_{k_t,c_t} \frac{c_t^{1-\eta}-1}{1-\eta} + \beta \mathrm{E}_t\left[v(k_t;\tilde{n}_t,z_{t+1}\right] \\ \text{s.t.} \quad c_t + k_t &= \tilde{n}_{t-1}w_t + r_t k_{t-1} + (1-\delta)k_{t-1} + f_t \end{aligned}$$

Here f_t stands for the profits the household receives from the firms. The other symbols have their typical interpretation and are summarized in the table below.

Note that the Dynare timing convention is used. For example, $\tilde{n}_{t-1}$, is the number of workers that has a job at the *beginning* of period t. Households take employment as given, since they are willing to supply whatever labor is demanded by firms. The tilde above a variable indicates it is chosen by the firm.

1.2 Firms

The production of the representative firm is given by

$$\tilde{y}_t = \tilde{k}_t^{\alpha}\tilde{n}_{t-1}^{1-\alpha}.$$

Capital is rented on a spot market. This means that for the individual firm capital is not predetermined. In equilibrium, we will get that the demand by the representative firm, $\tilde{k}_t$, is equal to the amount available to the household at the beginning of period t, i.e., k_{t-1}.[1]

A fixed fraction, ρ_x, of employment separates each period. Firms can find new workers by posting vacancies at cost ζ. The representative firm takes the probability of finding a worker, $\lambda_{f,t}$, as given. The firm's maximization problem is given by[2]

$$\begin{aligned} \tilde{v}(\tilde{n}_{t-1};z_t,k_{t-1}) &= \max_{\tilde{n}_t,\tilde{k}_t,v_t} c_t^{-\eta}\left(k_{t-1}^{\alpha}n_{t-1}^{1-\alpha} - wn_{t-1} - r_t k_{t-1} - \zeta v_t\right) + \beta \mathrm{E}_t\left[w(n_t,z_{t+1})\right] \\ \text{s.t.} \quad n_t &= (1-\rho_x)n_{t-1} + \lambda_t^f v_t \end{aligned}$$

[1] Note that the individual firm does not treat the capital stock as a predetermined variable, since it simply rents whatever it wants at the current spot price. Of course, the rental rate will be such that the firm will ask exactly what is available at the beginning of the period.

[2] Note that k_{t-1} is an argument of the firm's value function, because it affects the market prices and in particular the rental rate of capital. Since it is a state variable that the firm does not choose itself it is on the righthand side of the semicolon.

Note that the firm evaluates profits using the marginal utility of the household (its owner) which it takes as given. The idea is that the firm is small so that it's profits are small and can be evaluated using the marginal utility of the household.

An alternative way to write the firm's problem is as follows:

$$
\begin{aligned}
\tilde{v}(\tilde{n}_{t-1}; z_t, k_{t-1}) &= \max_{\tilde{n}_t, \tilde{k}_t, v_t} \left(\tilde{k}_t^{\alpha} \tilde{n}_{t-1}^{1-\alpha} - w_t \tilde{n}_{t-1} - r_t \tilde{k}_t - \zeta v_t \right) + \beta \mathrm{E}_t \left[\frac{c_{t+1}^{-\eta}}{c_t^{-\eta}} w(\tilde{n}_t; z_{t+1}, k_t) \right] \\
\text{s.t.} \quad \tilde{n}_t &= (1-\rho_x) n_{t-1} + \lambda_{f,t} \tilde{v}_t
\end{aligned}
$$

1.3 Matching and equilibrium

Matches are equal to

$$\phi_0 u_t^{\phi_1} \tilde{v}_t^{1-\phi_1},$$

where

$$u_t = 1 - n_{t-1}$$

The matching probability for the firm is, thus, given by

$$\lambda_t^f = \phi_0 \left(\frac{u_t}{v_t} \right)^{\phi_1}$$

1.4 Wages

Because of the matching friction, there are positive profits made by a firm that is matched with a worker. The question is how the revenues are divided between the worker and the owner of the firm. One possibility is Nash Bargaining. Here we simply assume that wages are given by

$$w_t = (1-\omega_0) \left(\begin{array}{c} \omega_1 (1-\alpha) z_t \tilde{k}_t^{\alpha} \tilde{n}_{t-1}^{-\alpha} \\ (1-\omega_1)\,(1-\alpha) \bar{k}^{\alpha} \bar{n}^{-\alpha} \end{array} \right)$$

where a bar on top of a variable indicates steady state value. If ω_1 is equal to 0, then wages are fully sticky. If ω_1 is equal to 1, then wages are proportionaly to the marginal product of labor.

The steady state value of the wage rate is (for any value of ω_1) equal to

$$(1-\omega_0)(1-\alpha)\bar{k}^{\alpha}\bar{n}^{1-\alpha}.$$

In the standard RBC model, steady state wages are equal to $(1-\alpha)\bar{k}^{\alpha}\bar{n}^{-\alpha}$. In this model total revenues have to be divided between those that work, those that rent out capital, *and* those that post vacancies, i.e., owners.

1.5 Definition of variables

$\tilde{n}_t$	end-of-period t employment
k_t	end-of-period t capital
p_t	marginal profit of one extra worker (mp-wage)
g_t	value of match
$\lambda_{f,t}$	matching probability for the firm
c_t	consumption
u_t	unemployed workers in period t
v_t	vacancies posted in t
r_t	rental rate of capital
y_t	output
z_t	productivity
e_t	innovation of capital

1.6 Equations of the model

$$
\begin{aligned}
u_t &= 1 - n_{t-1} \\
n_t &= (1-\rho_x)n_{t-1} + \lambda_{w,t}u_t \\
\lambda_{w,t} &= \phi_0 \left(\frac{u_t}{v_t}\right)^{\phi_1 - 1} \\
\zeta &= \phi_0 \left(\frac{u_t}{v_t}\right)^{\phi_1} g_t \\
g_t &= \mathrm{E}_t \left[\beta \frac{c_{t+1}^{-\eta}}{c_t^{-\eta}} \left(p_{t+1} + (1-\rho_x)g_{t+1}\right)\right] \\
p_t &= (1-\alpha)z_t \left(\frac{k_{t-1}}{n_{t-1}}\right)^{\alpha} - w_t \\
w_t &= (1-\omega_0)\begin{pmatrix} \omega_1(1-\alpha)z_t\tilde{k}_t^{\alpha}\tilde{n}_{t-1}^{-\alpha} \\ (1-\omega_1)(1-\alpha)\bar{k}^{\alpha}\bar{n}^{-\alpha} \end{pmatrix} \\
r_t &= \alpha z_t \left(\frac{k_{t-1}}{n_{t-1}}\right)^{\alpha - 1} \\
c_t^{-\eta} &= \beta \mathrm{E}_t \left[c_{t+1}^{-\eta}\left(r_{t+1} + 1 - \delta\right)\right] \\
y_t &= c_t + k_t - (1-\delta)k_{t-1} + \zeta v_t \\
y_t &= z_t k_{t-1}^{\alpha} n_{t-1}^{1-\alpha} \\
\ln(z_t) &= \psi \ln(z_{t-1}) + e_t
\end{aligned}
$$

1.7 Parameter values and steady state

Most structural parameters are simply given a "typical" value. The values are given by

$$
\begin{aligned}
\beta &= 0.99, \eta = 1 \\
\alpha &= 0.33, \delta = 0.025 \\
\psi &= 0.95, \sigma = 0.01 \\
\phi_0 &= 1, \phi_1 = 0.5
\end{aligned}
$$

The value of ω_0 and ω_1 are key parameters and you are asked to investigate how the properties of the model depend on them. The benchmark values are given by

$$\omega_0 = 0.5, \omega_1 = 1$$

The two remaining parameters are the vacancy cost, ζ, and the exogenous destruction rate. It is difficult to figure out what the right value for ζ is, especially since posting a vacancy should be thought of as more than simply the cost of posting an add in the newspaper. So instead we will choose the value of ζ such that it has sensible steady state implications. We do the same for the choice of ρ_x. That is, instead of giving the values of ζ and ρ_x, we give two pieces of information with which the values of these two parameters can be solved. The two pieces of information are that the steady state value of the matching probability for the worker should be equal to 0.7 and that the steady state value of the unemployment rate should be equal to 5%. Thus,

$$
\begin{aligned}
\bar{\lambda}_f &= 0.7 \\
\bar{u} &= 0.05
\end{aligned}
$$

In the parameter section of the Dynare program, ζ and ρ_x are still parameters. But the steady state values $\bar{\lambda}_f$ and $\bar{u}$ are parameters too. The parameter section contains the equations that determine the values of ζ and ρ_x as a function of $\bar{\lambda}_f$ and $\bar{u}$. To keep the program readable, I have introduced a bunch of intermediate steps. At these intermediate steps I calculate other steady state values, which are declared as parameters as well. These steady state values are not used in the model section, but they come in quite handy when giving initial values for the steady state in the "initval" section of the program.

1.8 Comments

- Variables known at the beginning of t must be dated $t-1$
- To get the dynare equations you only have to delete the conditional expectation. Dynare "knows" that there must be an expectational operator (and prediction error) in these equations because they have "$t+1$" terms

2 Exercises

Main exercise

1. Write down the model section. This is the only part of the program that you have to write yourself.

2. The options of the main Dynare command are such that the program calculates for employment and output the IRFs and standard deviations

3. Investigate the role of ω_0 and ω_1 for the volatility of employment relative to the volatility of output. Try to understand when this type of model can generate volatile employment?

Additional exercises

1. The program is written in levels. Rewrite the program in loglevels.

2. Program the calculation of IRFs and moments yourself. The analysis above is based on unfiltered data, but check how the results change if you HP-filter the data