Adding money to the open economy means that a number of additional variables and constraints need to be taken into account. If we have a domestic money, we need to assume a foreign money as well and an exchange rate between these two monies. We require conditions that will equalize the balance of payments and a rule to determine the exchange rate in each period. Since this is a "closed" open economy, we need to choose some restriction that will make foreign bond holdings defined in a stationary state. Note that in this model, we will only have monetary shocks that enter through transfers directly to households and that directly affect the household's cash-in-advance constraint.

13.4.1 The Open Economy Conditions

Now we add money to the open economy with capital adjustment costs and an interest rate on foreign debt and bonds that depends on the country's foreign indebtedness or wealth. Domestic money is added using a domestic cash-in-advance constraint for consumption purchases and for purchasing foreign money to use for paying foreign debt or buying foreign goods. The foreign bond is denominated in the foreign currency and pays interest (or if the bond is negative, the interest on the debt is paid) in the foreign currency. The period t exchange rate, measured in terms of units of domestic money per unit of foreign money, is e_t. There is a foreign market that in each period has a clearing condition of

$$B_t - (1 + r_{t-1}^f)B_{t-1} = P_t^* X_t,$$

where X_t is total net exports of the single good, B_t is the nominal quantity of foreign bonds, measured in the foreign currency, held at the end of period t, and P_t^* is the foreign price of the one good. The foreign interest rate is a function of the real (foreign) value of the stock of nominal foreign bonds (debt) held by the home country,

$$r_t^f = r^* - a\frac{B_t}{P_t^*}.$$

To keep things from being too simple-minded, we assume that the foreign price level follows a stochastic process of

$$P_t^* = 1 - \gamma^* + \gamma^* P_{t-1}^* + \varepsilon_t^*,$$

where $E_{t-1}\varepsilon_t^* = 0$ and ε_t^* is bounded below by $-(1-\gamma^*)$ and bounded above. We assume purchasing power parity, so the exchange rate, e_t, is defined in terms of units of the local currency per unit of the foreign currency as

$$e_t = \frac{P_t}{P_t^*}.$$

13.4.2 The Household

In an economy with indivisible labor and a cash-in-advance constraint, the household maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\ln c_{t+j}^i + B h_{t+j}^i \right]$$

subject to its budget constraints. The cash-in-advance condition for domestic household i in period t is

$$P_t c_t^i = m_{t-1}^i + (g_t - 1) M_{t-1}.$$

Households can receive lump sum money transfers or pay lump sum money taxes. The flow budget constraint for household i in period t is

$$c_t^i + \frac{m_t^i}{P_t} + \frac{e_t b_t^i}{P_t} + k_{t+1}^i = w_t h_t^i + r_t k_t^i + (1 - \delta) k_t^i - \frac{\kappa}{2} \left(k_{t+1}^i - k_t^i \right)^2$$
$$+ \frac{e_t (1 + r_{t-1}^f) b_{t-1}^i}{P_t} + \frac{m_{t-1}^i + (g_t - 1) M_{t-1}}{P_t},$$

which, after removing the elements from the cash-in-advance constraint, simplifies to

$$\frac{m_t^i}{P_t} + \frac{e_t b_t^i}{P_t} + k_{t+1}^i + \frac{\kappa}{2} \left(k_{t+1}^i - k_t^i \right)^2 = w_t h_t^i + r_t k_t^i + (1 - \delta) k_t^i$$
$$+ \frac{e_t (1 + r_{t-1}^f) b_{t-1}^i}{P_t}.$$

The term $\kappa/2 \left(k_{t+1} - k_t\right)^2$ is the capital adjustment costs the family must pay for changing the level of capital holdings. At the end of each period, the household's holdings of wealth are comprised of domestic money, foreign bonds, and physical capital.

In each period t, the household chooses c_t^i, k_{t+1}^i, b_t^i, m_t^i, and h_j^i to maximize its utility function subject to the budget constraints. The first-order conditions that come from this maximization are

$$0 = E_t \frac{e_t}{P_{t+1} c^i_{t+1}} - \beta E_t \frac{e_{t+1}(1 + r^f_t)}{P_{t+2} c^i_{t+2}},$$

$$0 = E_t \frac{P_t}{P_{t+1} c^i_{t+1}} \left[1 + \kappa \left(k^i_{t+1} - k^i_t\right)\right]$$

$$- \beta E_t \frac{P_{t+1}}{P_{t+2} c^i_{t+2}} \left(r_{t+1} + (1 - \delta) + \kappa \left(k^i_{t+2} - k^i_{t+1}\right)\right),$$

$$0 = \frac{B}{w_t} + \beta E_t \frac{P_t}{P_{t+1} c^i_{t+1}},$$

and the budget constraints are

$$0 = P_t c^i_t - m^i_{t-1} - (g_t - 1) M_{t-1},$$

and

$$0 = \frac{m^i_t}{P_t} + \frac{e_t b^i_t}{P_t} + k^i_{t+1} + \frac{\kappa}{2} \left(k^i_{t+1} - k^i_t\right)^2$$

$$- w_t h^i_t - r_t k^i_t - (1 - \delta) k^i_t - \frac{e_t (1 + r^f_{t-1}) b^i_{t-1}}{P_t}.$$

13.4.3 Firms

Domestic firms are completely competitive and have the standard Cobb-Douglas production function

$$Y_t = \lambda_t K^\theta_t H^{1-\theta}_t.$$

The equilibrium condition for the labor market is

$$w_t = (1 - \theta) \lambda_t K^\theta_t H^{-\theta}_t$$

and for the capital market is

$$r_t = \theta \lambda_t K^{\theta-1}_t H^{1-\theta}_t.$$

13.4.4 Equilibrium Conditions

The aggregate resource constraint for the domestic economy is

$$\lambda_t K^\theta_t H^{1-\theta}_t = C_t + K_{t+1} - (1 - \delta) K_t + X_t.$$

Domestic output can be used as consumption, domestic net capital accumulation (investment), or net exports. This is not an additional restriction since it is already incorporated in the aggregated form of the budget constraint of the household, the cash-in-advance constraint, and the balance of payments. Since the unit mass of households are identical, we have the aggregation conditions

$$C_t = c_t^i,$$

$$M_t = m_t^i,$$

$$B_t = b_t^i,$$

$$H_t = h_t^i,$$

and

$$K_{t+1} = k_{t+1}^i.$$

In addition, the money supply follows the rule

$$M_t = g_t M_{t-1}.$$

13.4.5 The Full Model

The full model is in the 11 variables, C_t, K_{t+1}, H_t, M_t, B_t, P_t, e_t, r_t, w_t, r_t^f, and X_t, and the stochastic processes, P_t^*, λ_t, and g_t. The full set of 11 equations of the model, written in aggregate terms, is

$$0 = E_t \frac{e_t}{P_{t+1}C_{t+1}} - \beta E_t \frac{e_{t+1}(1+r_t^f)}{P_{t+2}C_{t+2}},$$

$$0 = E_t \frac{P_t}{P_{t+1}C_{t+1}} \left[1 + \kappa \left(K_{t+1} - K_t\right)\right]$$
$$- \beta E_t \frac{P_{t+1}}{P_{t+2}C_{t+2}} \left(r_{t+1} + (1-\delta) + \kappa \left(K_{t+2} - K_{t+1}\right)\right),$$

$$0 = \frac{B}{w_t} + \beta E_t \frac{P_t}{P_{t+1}C_{t+1}},$$

$$0 = P_t C_t - M_t,$$

$$0 = \frac{M_t}{P_t} + \frac{e_t B_t}{P_t} + K_{t+1} + \frac{\kappa}{2}\left(K_{t+1} - K_t\right)^2$$

$$- w_t H_t - r_t K_t - (1-\delta)\,K_t - \frac{e_t(1 + r^f_{t-1})B_{t-1}}{P_t},$$

$$0 = w_t - (1-\theta)\,\lambda_t K_t^{\theta} H_t^{-\theta},$$

$$0 = r_t - \theta \lambda_t K_t^{\theta-1} H_t^{1-\theta},$$

$$0 = B_t - (1 + r^f_{t-1})B_{t-1} - P_t^* X_t,$$

$$0 = r_t^f - r^* + a\frac{B_t}{P_t^*},$$

$$0 = e_t - \frac{P_t}{P_t^*},$$

$$0 = M_t - g_t M_{t-1}.$$

In addition, there are the three equations that define the stochastic processes for P_t^*, λ_t, and g_t.

13.4.6 The Stationary State

Define $\pi = P_{t+1+j}/P_{t+j}$ as the stationary state rate of inflation. As usual, we assume a constant growth rate of money, $\bar{g}$, and look for a stationary state where the real variables of the economy are constant and ratios of nominal variables are constant. The foreign price level follows a stochastic process,

$$P_t^* = 1 - \gamma^* + \gamma^* P_{t-1}^* + \varepsilon_t^*,$$

so, in a stationary state, the foreign price level is $\bar{P}^* = 1$. Using the full model, some conditions for the stationary state are

$$\pi = \beta(1 + \bar{r}^f)\frac{e_{t+1}}{e_t}, \tag{13.16}$$

$$\frac{1}{\beta} = (\bar{r} + (1-\delta)),$$

$$-B\pi\bar{C} = \beta\bar{w}, \tag{13.17}$$

$$\bar{C} = \overline{M/P},$$

$$\overline{M/P} + \frac{e_t \bar{B}}{P_t} = \bar{w}\bar{H} + (\bar{r} - \delta)\,\bar{K} + \frac{e_t(1+\bar{r}^f)\bar{B}}{P_t}, \tag{13.18}$$

$$\bar{w} = (1-\theta)\,\bar{K}^{\theta}\bar{H}^{-\theta}, \tag{13.19}$$

$$\bar{r} = \theta\bar{K}^{\theta-1}\bar{H}^{1-\theta}, \tag{13.20}$$

$$-\bar{r}^f\bar{B} = \bar{X}, \tag{13.21}$$

$$\bar{r}^f = r^* - a\bar{B}, \tag{13.22}$$

$$\frac{e_t}{P_t} = 1, \tag{13.23}$$

$$M_t = \bar{g}M_{t-1}. \tag{13.24}$$

These conditions can be further simplified to find stationary state values of all the variables of the model as functions of the model's parameters. Notice that since we are dealing with stationary states, the capital adjustment costs, which are based on the changes in capital, do not appear in the above equations.

Using equation 13.23, equation 13.16 becomes

$$\pi = \beta(1+\bar{r}^f)\frac{P_{t+1}}{P_t} = \beta(1+r^f)\pi,$$

so

$$\bar{r}^f = \frac{1}{\beta} - 1.$$

Equation 13.22 then determines the stationary state foreign bond (debt) holdings as

$$\bar{B} = \frac{r^* + 1 - \frac{1}{\beta}}{a},$$

and $\bar{X}$ can be found from equation 13.21 as

$$\bar{X} = -\bar{r}^f\bar{B} = \frac{(1-\beta)^2 - (1-\beta)\,\beta r^*}{a\beta^2}.$$

As with earlier stationary states, since $\bar{r}$ is given, the conditions for competitive factor markets (equations 13.19 and 13.20) imply that

$$\bar{w} = (1-\theta)\left(\frac{\theta}{\bar{r}}\right)^{\frac{\theta}{1-\theta}},$$

and equation 13.17 gives us stationary state consumption as

$$\bar{C} = \frac{\beta \bar{w}}{-B\pi},$$

where the condition for the stationary state gross inflation rate, $\pi = \bar{g}$, is found using equation 13.24 and the argument

$$M_t = \bar{g} M_{t-1},$$

$$\frac{M_t}{P_t} = \bar{g}\frac{M_{t-1}}{P_t}\frac{P_{t-1}}{P_{t-1}} = \bar{g}\frac{M_{t-1}}{P_{t-1}}\frac{P_{t-1}}{P_t},$$

so

$$\overline{M/P} = \frac{\bar{g}\overline{M/P}}{\pi}.$$

To find $\bar{K}$, we use equation 13.18, to get

$$\overline{M/P} = \bar{w}\bar{H} + (\bar{r} - \delta)\,\bar{K} + \bar{r}^f \bar{B},$$

and substituting in the usual result (from the factor market conditions) that

$$\bar{H} = \frac{\bar{r}\,(1-\theta)}{\bar{w}\theta}\bar{K},$$

we get

$$\bar{K} = \frac{\theta\left(\overline{M/P} - \bar{r}^f \bar{B}\right)}{\bar{r} - \theta\delta}.$$

For the standard economy, the stationary state values are $\bar{r} = .0351$, $\bar{w} = 2.3706$, and $\bar{r}^f = .0101$ in all cases. The values for the other variables are shown in Table 13.3.

Notice that, as in the basic model with cash-in-advance money with transfers to the household, stationary states with higher money growth have lower stationary state consumption and production. Net foreign debt or savings does not change consumption in the stationary state but does change capital holding and the fraction of the population that is working in each period. Countries with foreign debt need to have higher production, capital, and employment to be able to meet interest rate payments and maintain consumption.

13.4.7 Log-Linear Version of Full Model

We use the now familiar method of Uhlig to find the log-linear version of the model around the stationary state found in the section above. We define the