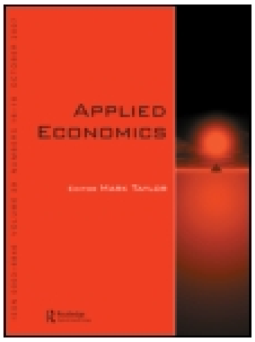

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Monetary policy in a dual currency environment

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We develop a small open economy general equilibrium model with sticky prices and partial dollarization – a situation where both domestic and foreign currencies coexist. We derive a tractable representation of the model in terms of domestic inflation and the output gap in which a trade-off, which depends on the degree of dollarization, arises endogenously due to the presence of foreign interest rate shocks. We use this framework to show analytically how higher degrees of dollarization induce larger volatilities of the output gap and inflation, thus hampering a central bank's effectiveness to stabilize the economy. Our impulse response functions show that the transmission of such shocks has a positive (negative) effect on inflation and negative (positive) effect on the output gap when money aggregates and consumption are complements (substitutes).

Keywords: dollarization; currency substitution; policy trade-off; staggered price setting; open economy

JEL Classification: E50; E52; F00; F30; F41

I. Introduction

One of the central issues in emerging economies is the idea of replacing the domestic currency with the US dollar. There is not a unique position in this debate with arguments found for and against dollarization. Recent research has focused on analysing extreme cases, either complete dollarization or an economy with only domestic currency (for example, Cooley and Quadrini, 2001; Schmitt-Grohe and Uribe, 2001; Chang and Velasco, 2002).

In some developing countries, foreign currency is legally used and it is difficult to persuade agents not to hold it. This is the case of a 'partially dollarized economy' where foreign currency can be demanded not only as a deposit of value but also as a medium of payment (commonly known as transaction dollarization or currency substitution). Some other forms of dollarization can be identified. For instance, it is common that transaction dollarization is accompanied by financial dollarization, price dollarization and vice versa, although to varying degrees.

The Peruvian economy is usually taken as a case study since it is by far one of the most highly dollarized economies among emerging market countries that target inflation. Armas *et al.* (2007) discuss the difficulties of implementing an independent monetary policy aimed at price stability through inflation targeting (IT) under financial, real and transaction dollarization.[1] Figure 1 below describes the importance of different types of dollarization for the Peruvian economy.

Dollarization of financial assets in Peru is by large the most important form of dollarization (50–60%). However, transaction dollarization is less strong but still very

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[1] See Armas *et al.* (2007) for a description of the main channels through which partial dollarization hampers the transmission mechanism of monetary policy.

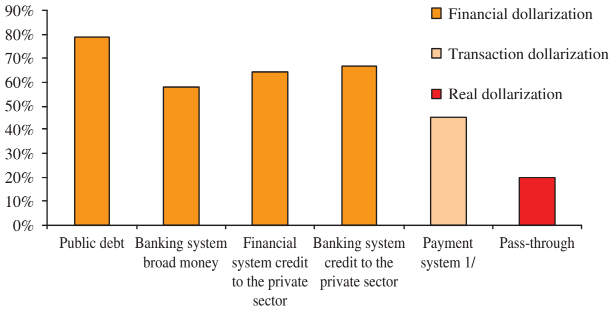

Fig. 1. Degree of transaction, real and financial dollarization in Peru, 2006 (From Armas *et al.*, 2006)

important (40%). Figure 1 shows percentages of dollar denominated private debt, the percentage of cash and check payments made in dollars (a proxy measure of transaction dollarization).[2] Given the nonnegligible importance of partial transaction dollarization, having an analytical framework to explore the effects of this form of dollarization will be useful for central bankers that implement monetary policy in such an environment.

Partial dollarization may not be an optimal regime but the costs of dollarizing or de-dollarizing can be substantial in the short run. It is therefore important for central banks to understand the constraints they face when operating in a dual currency environment. Policy makers are aware of some of the constraints faced by central banks when conducting monetary policy in these economies. But there is little analytical work in the literature to support how a central bank should behave in order to overcome such limitations.

Hence, this article seeks to provide such an analytical framework by studying monetary policy in a general equilibrium open economy model where both local and foreign currencies are demanded for transaction purposes in the home economy. We use this framework to assess qualitatively and quantitatively the extent to which higher degrees of dollarization rise the volatility of inflation and output, making the central bank less effective in stabilizing these variables. The article also seeks to explain the transmission mechanism of foreign interest rates shocks over domestic macro variables. The impact of foreign interest rate shocks are important in driving the business cycles in the Peruvian economy as reported by Castillo *et al.* (2012). For example, these authors find that foreign disturbances (Uncovered interest rate parity (UIP) and foreign interest rate shock) account for 34% of output fluctuations in the Peruvian economy.[3]

Hence, we develop a general equilibrium open economy framework in the spirit of Clarida *et al.* (2001, 2002), Gali and Monacelli (2005) and Benigno and Benigno (2006), allowing for partial degrees of currency substitution. We motivate partial dollarization by including a composite of both foreign and domestic currency in the utility function of the generic household in the home economy.[4] The composite is not separable from consumption. We derive a tractable linearized model that embeds the extreme cases of high dollarization – defined as a very high preference for foreign currency – and low dollarization in the domestic economy.

In order to obtain a tractable model, we employ a two-country specification. We explore the dynamic properties of the model in the limiting case when the size of the domestic economy is small. To illustrate the inner workings of the model, we evaluate analytically the unconditional volatility of its key variables following a foreign interest rate shock for various degrees of dollarization. The key insight is that treating money aggregates as a composite of consumption introduces a short-run trade-off between the stabilization of inflation and the output gap, thereby making the flexible price allocation no longer attainable. Moreover, such policy trade-off arises endogenously without considering an exogenous cost-push shock due to the presence of exogenous shocks to the foreign interest rate.[5] Therefore, the monetary authority cannot stabilize output and inflation at the same time. This trade-off is affected by the degree of dollarization.

Our simulations and analytical results of the canonical model show that macroeconomic volatility increases for higher degrees of dollarization, hence making it more difficult for central banks to stabilize the economy. Interestingly, the transmission mechanism of a positive foreign interest rate shocks might have a positive (negative) effect on inflation and a negative (positive) effect on output when money aggregates and consumption are complements (substitutes). The intuition for the case of complements is as follows: an increase in foreign interest rates reduces the demand for foreign currency, leading to a fall in demand of the overall money aggregates. If money and consumption are complements, the marginal utility of consumption (MUC) is increasing in real money balances. Then it follows that the MUC decreases, which, given a standard labour supply relation, increases real wages and inflation. Finally, given the trade-off, the output gap decreases.

[2] This percentage approximates the share of transactions in dollars relative to total transactions by combining data on (i) ATM dollar withdrawals; (ii) dollar checks; (iii) dollar interbank transfers and (iv) direct debits in dollars.

[3] Castillo *et al.* (2012) estimated a dynamic stochastic general equilibrium (DSGE) model with partial dollarization using Bayesian techniques and the Peruvian data.

[4] Partial dollarization can also be motivated using shopping time models. Castillo (2006) model currency substitution by adding transaction frictions in a cash-in-advance model.

[5] Clarida *et al.* (2001, 2002) assumed the presence of an exogenous cost-push shock in order to generate the trade-off between inflation and output gap.

Our model results in a formulation of the MUC that crucially affects the transmission mechanism of exogenous shocks in our set-up. The MUC depends not only on consumption but also on both foreign and domestic interest rates and their relative weights are sensitive to the ratio of foreign currency in the total money aggregates. The weights depend in turn on the degree of preference for foreign currency. In particular, any shock that causes an endogenous increase in the nominal interest rate increases the MUC, helping consumption and output to adjust. A higher degree of preference for foreign currency reduces the effect of the interest rate on the MUC and, therefore, consumption and output drop further in response to any shock. This reveals the fragility of monetary policy in a partially dollarized environment.[6]

The article is organized as follows. Section II outlines a general equilibrium model that allows for currency substitution in the domestic economy by introducing a composite of both domestic and foreign currency. We also describe how the optimality conditions and the international risk-sharing condition change. In Section III, we present the equilibrium; simulate the log-linear version of the model and calculate analytically the unconditional moments. Finally, Section IV concludes.

II. The Model

We consider a two-country open economy model with imperfect competition and nominal price rigidities along the lines of Obstfeld and Rogoff (1995), Clarida *et al.* (2002) and Benigno and Benigno (2006). In contrast, we give money a role in the model by introducing a money aggregate (composed of both local and foreign currency) as a composite of consumption for the home country.[7] We allow for tradable goods only, home bias and a complete asset market structure. In order to treat the home economy as small and open, we eliminate the effect of home variables on the foreign economy, as in Sutherland (2002).

Households

The world size is normalized to unity. There are two countries, home and foreign. The population in the home country lies in the interval [0, n], while in the foreign economy in the segment (n, 1]. A generic agent h belonging to the home economy is a consumer of all the goods produced in both countries H and F. Preferences of the generic household h in country H are given by

$$E_t \sum i \overset{\infty}{=} 0 \beta^i U^h \left[C_t^h, Z_t^h \left(\frac{M_t^h}{P_t}, \frac{D_t^h S_t}{P_t} \right), L_t^h \right] \tag{1}$$

$$U_{t+i}^h = \frac{1}{1-\sigma} \left\{ \left[\left(bC_{t+i}^h \frac{\omega-1}{\omega} + (1-b) Z_{t+i}^h \frac{\omega-1}{\omega} \right)^{\frac{\omega}{\omega-1}} \right]^{1-\sigma} - \frac{L_{t+i}^h (1+\upsilon)}{1+\upsilon} \right\} \tag{2}$$

where Z_{t+i}^h is a money aggregate defined as

$$Z_{t+i}^h = \left(v \left(\frac{M_{t+i}^h}{P_{t+i}} \right)^{\frac{\chi-1}{\chi}} + (1-v) \left(\frac{D_{t+i}^h S_{t+i}}{P_{t+i}} \right)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}} \tag{3}$$

where E_t denotes the expectation conditional on the information set at date t, and β is the inter-temporal discount factor, and σ and $\upsilon > 0$ represent the coefficient of risk aversion and the inverse of the elasticity of labour supply, respectively. $\omega > 0$ captures the degree of complementary or substitutability between the consumption and the overall money aggregate. This parameter will become important later on since it will capture the effect of money over the labour supply and consequently over the aggregate supply (AS) equation. To the extent that $\sigma = 1$, when $\omega > 1$, the MUC is decreasing in real money balances $U_{CZ}^h > 0$.[8] Therefore, higher interest rates along with the associated reduction in real balance holdings, increase the MUC, hence the overall money aggregate and the consumption are substitutes. On the other hand, when $0 < \omega < 1$, the MUC is increasing in real money balances $U_{CZ}^h > 0$ and therefore the overall money aggregate and the consumption are complements.[9] The parameter $0 < b < 1$ is the weight of consumption in the consumption–money aggregate; $\chi > 0$ represents the elasticity of

[6] There is now lengthy literature for small open economies that show that dollarization of liabilities affect the aggregate demand through balance sheets effect (debt denominated in either domestic or foreign currency). Some prominent examples include Gertler *et al.* (2007), Christiano *et al.* (2003) and Céspedes *et al.* (2004). In contrast with these studies, our set-up will affect both aggregate demand and aggregate supply by the effect of currency substitution over the MUC.

[7] As shown in Castillo *et al.* (2012) this formulation is supported by the Peruvian data.

[8] Note that $U_{CZ} = \left(\frac{1-\sigma\omega}{\omega}\right) b(1-b)(ZC)^{\frac{\omega-1}{\omega}-1}$, if $\left(\frac{1-\sigma\omega}{>\omega}\right) > 0$ consumption and money aggregates are complements. For the particular case when $\sigma = 1$, the condition simplifies to $\frac{1}{\omega} - 1 > 0$. Then if $0 > \omega < 1$ C and Z are complements, on the contrary if $\omega > 1$ consumption and the overall aggregate are substitutes $U_{CZ} < 0$.

[9] See Woodford (2003, chapter 2) for a brief discussion related to the consequences of nonseparable utility function and price determination.

substitution between domestic and foreign currency and $0 < v < 1$ is the preference for domestic currency within the overall money aggregate. Agents get utility from consumption C_t^h and from holding both domestic and foreign real money balances, $\frac{M_t^h}{P_t}$ and $\frac{S_t D_t^h}{P_t}$, respectively. The household also supplies hours of work, L_t^h.

The novelty in this formulation is the definition of the money aggregate Z_{t+i}^h as a constant elasticity of substitution (CES) composite of real domestic and foreign money balances.[10] When the weight of domestic money, v, equals 1, the model collapses to an open economy with no foreign currency used for transactions (no dollarization) in the home economy. Similarly, when $v = 0$, only foreign currency is used for transactions, implying full dollarization.[11] Standard models for open economies implicitly assume that there exist tight legal restrictions which prevent a country's resident from using foreign currency for domestic transactions. In economies that have experienced hyperinflations, however, it is difficult for the government to persuade their citizens to use only domestic currency for day-to-day transactions.[12] Moreover, agents often hold and use foreign currency even in the presence of legal restrictions. Obstfeld and Rogoff (1996) motivate the idea of partial dollarization by introducing a composite of foreign and domestic money-in-the utility function that tries to capture the existence of those legal restrictions.[13] We assume that there are no legal restrictions for holding foreign currency as it is the case in developing countries such as Peru, Bolivia and Uruguay. In these economies, there are no restrictions to hold foreign currency. Moreover, foreign currency is used for day-to-day transactions. Therefore, it seems plausible to include both foreign and domestic currencies as a CES aggregate in the utility function.[14] Furthermore, the advantage of considered specification is that it allows us to endogenously pin down the dollarization ratio in the steady state.

A generic household of the foreign country gets utility only from consumption C_t^* and supply labour L_t^*

$$E_t \sum_{i=0}^{\infty} \beta^i U^f \left[C_t^*, L_t^* \right] \tag{4}$$

$$U_{t+i}^f = \frac{{C_{t+i}^*}^{1-\sigma}}{1-\sigma} - \frac{L_{t+i}^*(1+\upsilon)}{1+\upsilon} \tag{5}$$

We define C_t as the consumption index in the home country

$$C_t \equiv \left[(1-\lambda)^{\frac{1}{\theta}} \left(C_{H,t}^h \right)^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} \left(C_{F,t}^h \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{6}$$

where θ is the elasticity of substitution between home- and foreign-tradable goods and $C_{H,t}$ and $C_{F,t}$ are the two sub-indices that refer to the consumption of home-produced and foreign goods. The parameter that determines home consumers' preferences for foreign goods, λ, is a function of the relative size of the foreign economy, $(1–n)$, and of the degree of openness, γ, That is, $\lambda = (1-n)\gamma$.

The corresponding consumption index for foreign households is given by:

$$C_t^* \equiv \left[(1-\lambda^*)^{\frac{1}{\theta}} (C_{H,t}^*)^{\frac{\theta-1}{\theta}} + \lambda^{*\frac{1}{\theta}} (C_{F,t}^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{7}$$

where $1 - \lambda^* = n\gamma$.

As in Sutherland (2002), $1-\lambda$ accounts for the degree of home bias in domestic consumption Notice that $\gamma = 0$ implies a completely closed economy. The indexes $C_{H,t}$, $(C_{F,t})$ and $C_{H,t}^*(C_{F,t}^*)$ are home (foreign) consumption of the differentiated products produced in countries H and F, respectively, which are defined as follows

$$\begin{aligned} C_{H,t} &\equiv \left\{ \left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \int_0^n \left[c_t(z) \right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \\ C_{F,t} &\equiv \left\{ \left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \int_n^1 \left[c_t(z) \right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}} \end{aligned} \tag{8}$$

[10] Notice that money shows up in both the budget constraint and in the utility function. In our model, the monetary distortion is taken to be nongnegligible or nontrivial. Moreover, nonseparability between money aggregates and consumption, guarantee a role for money even if monetary policy actions are defined in terms of an interest rate feedback rule.

[11] Note that when $b = 1$, the model is similar to the small open economy version presented by Clarida *et al.* (2001).

[12] Even with low inflation levels, agents still demand foreign currency not only as an mean of exchange but also as deposit of value.

[13] Obstfeld and Rogoff (1996) assumed a quadratic form as they consider an economy with legal restrictions in holding foreign currency. The form they consider is $a_0 \left(\frac{M_t}{P_t} \right) - \frac{a_1}{2} \left(\frac{D_t S_t}{P_t} \right)^2$, where the second term measures the evasion costs of the legal restrictions.

[14] A model with transaction technology with shopping time and real money balances in both foreign and domestic currencies would be another possibility at the cost of losing tractability. See Brock (1974) for an earlier use of shopping-time model to motivate a money-in-the-utility function approach. The advantage of the way we impose currency substitution is that it delivers a more tractable model. Other approaches to give rise to a valued role for money are suggested by Kiyotaki and Wright (1993) by imposing that direct exchange of commodities is assumed to be costly, but there is a fiat money that can be treated as costlessly for commodities.

$$C_{H,t}^{*} \equiv \left\{ \left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_{0}^{n} \left[c_{t}^{*}(z)\right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t}^{*} \equiv \left\{ \left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_{n}^{1} \left[c_{t}^{*}(z)\right]^{\frac{\varepsilon-1}{\varepsilon}} dz \right\}^{\frac{\varepsilon}{\varepsilon-1}} \tag{9}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within a country. In this context, the general price indexes that correspond to the previous specification are given by

$$P_t \equiv \left[(1-\lambda)(P_{H,t})^{1-\theta} + \lambda(P_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{10}$$

$$P_t^{*} \equiv \left[(1-\lambda^{*})(P_{H,t}^{*})^{1-\theta} + \lambda^{*}(P_{F,t}^{*})^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{11}$$

where $(1-\lambda)$ and λ^* are parameters that capture the degree of home bias in preferences in each country, respectively. As in Sutherland (2002), λ corresponds to the share of foreign goods in consumption basket of home agents and it will depend on the share of foreign goods in the total measure of goods in the world $(1-n)$ and on the degree of openness γ. $\gamma = 0$ implies a completely closed economy.[15] The previous limiting case could be interpreted as if the foreign currency have complete home bias. C_H^h and C_F^h are sub-indexes of consumption across the continuum of differentiated goods produced in country H and F, and are given by

$$C_{H,t}^{h} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}} \int_{0}^{n} c_{t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad C_{F,t}^{h} \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}} \int_{n}^{1} c_{t}(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{12}$$

where $\varepsilon > 1$ is the elasticity of substitution across goods produced within a country. In this context, the general price indexes that correspond to the previous specification are given by

$$P_t \equiv \left[(1-\gamma)\left(P_{H,t}\right)^{1-\theta} + \gamma\,\left(P_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{13}$$

$$P_t^{*} \equiv \left[(1-\gamma^{*})\left(P_{H,t}^{*}\right)^{1-\theta} + \gamma^{*}\left(P_{F,t}^{*}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{14}$$

where $P_{H,t}$ ($P_{H,t}^*$) is a price sub-index for home-produced goods expressed in domestic (foreign) currency and $P_{F,t}$ $\left(P_{F,t}^*\right)$ is the price sub-index for foreign-produced goods expressed in domestic (foreign) currency, respectively, as

$$P_{H,t} \equiv \left\{ \left(\frac{1}{n}\right) \int_{0}^{n} [p_t(z)]^{1-\varepsilon} dz \right\}^{\frac{1}{1-\varepsilon}}, \qquad P_{F,t} \equiv \left\{ \left(\frac{1}{1-n}\right) \int_{n}^{1} [p_t(z)]^{1-\varepsilon} dz \right\}^{\frac{1}{1-\varepsilon}}$$

$$P_{H,t}^{*} \equiv \left\{ \left(\frac{1}{n}\right) \int_{0}^{n} \left[p_t^{*}(z)\right]^{1-\varepsilon} dz \right\}^{\frac{1}{1-\varepsilon}}, \qquad P_{F,t}^{*} \equiv \left\{ \left(\frac{1}{1-n}\right) \int_{n}^{1} \left[p_t^{*}(z)\right]^{1-\varepsilon} dz \right\}^{\frac{1}{1-\varepsilon}}$$

Prices are set in producer currency. This assumption implies that the law of one price holds, $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$, where S_t denotes the nominal exchange rate (the price of foreign currency in terms of domestic currency). Note, however, that purchasing power parity (a constant real exchange rate) does not necessarily hold because of the presence of home bias in preferences. The home bias assumption allows generating real exchange rate dynamics in a model with only tradable goods. We define the terms of trade as the price of imported goods from abroad relative to the price of the exported goods abroad, such that $T_t = P_{F,t}/S_t P_{H,t}^* = P_{F,t}/P_{H,t}$.

Given the previous definitions, we can express the real exchange rate as a function of the terms of trade

$$Q_t = \frac{S_t P_t^*}{P_t} = \frac{\left[(1-\lambda^*)\left(S_t P_{H,t}^*\right)^{1-\theta} + \lambda^*\left(S_t P_{F,t}^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[(1-\lambda)\left(P_{H,t}\right)^{1-\theta} + \lambda\left(P_{F,t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}} \tag{15}$$

dividing both numerator and denominator of the above expression by $P_{H,t}$ and taking the limit when $n \to 0$, and considering that $(1-\lambda^*)\frac{(1-n)}{n} \to \gamma$, we get and expression that relates the real exchange rate with the terms of trade.

$$Q_t = \left[\frac{T_t^{1-\theta}}{(1-\gamma) + \gamma T_t^{1-\theta}}\right]^{\frac{1}{1-\theta}} \tag{16}$$

[15] Unlike Gali and Monacelli (2005) and Clarida *et al.* (2002), we rely on a complete general equilibrium structure. As you will see later, in steady state, γ will represent the share of domestic consumption allocated to imported goods, so it could be interpreted as a natural index of openness. So in this sense, $(1-\gamma)$ is interpreted as the degree of home bias and the larger this value (smaller γ) the closer to a closed economy counterpart.

Optimal consumption allocations and demand

The allocation of demands across each of the goods produced within a given country is given by

$$y_t^d(h) = c_t(h) + c_t^*(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\epsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\left[(1-\lambda)C_t + \frac{(1-\lambda^*)(1-n)}{n}C_t^*Q_t^\theta\right] \tag{17}$$

$$y_t^d(f) = c_t(f) + c_t^*(f) = \left(\frac{p_t(f)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{F,t}}{P_t}\right)^{-\theta}\left[\frac{\lambda n}{1-n}C_t + \lambda^* C_t^* Q_t^\theta\right] \tag{18}$$

To portray the small open economy, we use the definition of $(1-\lambda)$ and $(1-\lambda^*)$ and take the limit when $n \to 0$. We obtain

$$y_t^d(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta}\left[(1-\gamma)C_t + \gamma Q_t^\theta C_t^*\right] \tag{19}$$

and

$$y_t^d(f) = \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\varepsilon}\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\theta} C_t^* \tag{20}$$

Budget constraint and asset market structure

We assume that households have access to a complete set of state contingent nominal claims which are traded domestically and internationally.[16] We represent the asset structure by assuming a complete contingent one-period nominal bond denominated in home currency.[17] The household in the domestic economy with partial dollarization faces a sequence of inter-temporal budget constraint of the form

$$C_t^h = \frac{W_t}{P_t}L_t^h + \Pi_t - TR_t^h - \frac{M_t^h - M_{t-1}^h}{Pt} - \frac{E_t\{\xi_{t,t+1}B_{t+1}^h\} - B_t^h}{Pt} - \frac{D_t^h S_t - D_{t-1}^h S_t}{P_t} \tag{21}$$

B_{t+1}^h is a nominal random pay-offs of the portfolios purchased in domestic currency at t, with $\xi_{t,t+1}$ being the stochastic discount factor of nominal pay-offs.[18]

The government's budget constraint is balanced every period, so that total transfers are equal to seigniorage revenues

$$\int_0^n \left(M_t^h - M_{t-1}^h\right)dh = \int_0^n TR_t^h dh \tag{22}$$

Once we account for the optimal allocation of consumption and demands, together with the budget constraint, we can obtain the optimality condition by differentiating the objective function with respect to L_t^h, B_t^h, $\frac{M_t^h}{P_t}$ and $\frac{S_t D_t^h}{P_t}$, to obtain the following FOCs:

$$\frac{W_t}{P_t}U_{C,t} = \left(L_t^h\right)^\upsilon \tag{23}$$

$$\beta\frac{P_t}{P_{t+1}}\frac{U_{C,t+1}}{U_{C,t}} = \xi_{t,t+1} \tag{24}$$

$$U_{C,t} = (1+i_t)E_t\left\{\frac{P_t}{P_{t+1}}\beta U_{C,t+1}\right\} + U_{m,t} \tag{25}$$

$$U_{C,t} = \left(1+i_t^*\right)E_t\left\{\frac{P_t}{P_{t+1}}\frac{S_{t+1}}{S_t}\beta U_{C,t+1}\right\} + U_{d,t} \tag{26}$$

with the MUC given by

$$U_{C,t} = \Phi_t^{\frac{1}{\omega}-\sigma}C_t^{-\frac{1}{\omega}}b \tag{27}$$

with $\Phi_t \equiv \left(bC_{t+i}^{h\frac{\omega-1}{\omega}} + (1-b)Z_{t+i}^{h\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}$.

The marginal utility of domestic real balances can be written as

$$U_{m,t} = \Phi_t^{\frac{1}{\omega}-\sigma}(1-b)\nu Z_t^{h\left(\frac{1}{\chi}-\frac{1}{\omega}\right)}\left(\frac{M_t^h}{P_t}\right)^{-\frac{1}{\chi}} \tag{28}$$

and the marginal utility of foreign real balances as

$$U_{d,t} = \Phi_t^{\frac{1}{\omega}-\sigma}(1-b)(1-\nu)Z_t^{h\left(\frac{1}{\chi}-\frac{1}{\omega}\right)}\left(\frac{D_t^h S_t}{P_t}\right)^{-\frac{1}{\chi}} \tag{29}$$

Take conditional expectations to both sides of Equation 24 and let $(1 + i_t)$ denote the (gross) nominal yield of a one-

[16] Given this assumption, it is not necessary to characterize the current account dynamics in order to determine the equilibrium allocations.
[17] Given that markets are complete internationally, it does not matter the currency denomination of the securities.
[18] Therefore, $\xi_{t,t+1}$ is a price of one unit of nominal consumption of time $t+1$, expressed in units of nominal consumption at t, contingent on the state at $t+1$ being s_{t+1} and given any state s in t. If we define the value of the portfolio at the end of the period as A_t, the complete market assumption implies that there exists a unique discount factor $\xi_{t,t+1}$ of a portfolio with the property that the price in period t of the portfolio with random value B_{t+1} is $A_t = E_t\left[\xi_{t,t+1}B_{t+1}\right]$.

period risk-free discount bond in domestic currency (such that $E_t\{\xi_{t,t+1}\} = (1+i_t)^{-1}$ is the price of this bond), we can derive the inter-temporal home Euler equation[19]:

$$U_{C,t} = (1+i_t)E_t\left\{\frac{P_t}{P_{t+1}}\beta U_{C,t+1}\right\} \tag{30}$$

International risk sharing. Given that state contingent securities are tradable internationally,[20] the inter-temporal efficiency condition for the foreign economy is given by

$$\beta\frac{P_t^*}{P_{t+1}^*}\frac{S_t}{S_{t+1}}\frac{U_{C^*,t+1}}{U_{C^*,t}} = \xi_{t,t+1} \tag{31}$$

Then combining the above equation with Equation 24, we get

$$\frac{U_{C,t}}{P_t} = k_o\frac{U_{C^*,t}}{S_tP_t^*} \tag{32}$$

where k_o is a function of predetermined variables (see Chari *et al.* (2002) for details). Using the definition of the real exchange rate, the above expression can be written as

$$Q_t = k_o\frac{U_{C^*,t}}{U_{C,t}} \tag{33}$$

This equation gives us the relation between consumption at home and consumption abroad linked through the real exchange rate. It is worthwhile to mention that our model will deliver a different risk-sharing condition than standard two-country models. Notice that given the assumption of nonseparability between consumption and the money aggregate in the home economy, $U_{C,t}$ will be a function of domestic consumption and both domestic and foreign interest rates. In our model, both interest rates play a key role in explaining the real exchange rate dynamics and consequently the cross correlation between the relative consumption and the real exchange rate across countries.

As in the domestic economy, we define the foreign interest rate i_t^* as the price of the portfolio that delivers one unit of foreign currency in each contingent state next period. Therefore, given the complete markets assumption, we have

$$\frac{1}{1+i_t^*} \equiv E_t\left(\xi_{t,t+1}\frac{S_{t+1}}{S_t}\right) \tag{34}$$

Combining Equations 31 and 34, we can obtain the inter-temporal Euler equation for the foreign country

$$U_{C^*,t} = (1+i_t^*)E_t\left\{\frac{P_t^*}{P_{t+1}^*}\beta U_{C^*,t+1}\right\} \tag{35}$$

Notice that by combining both home and foreign Euler equations along with Equation 32, we obtain a version of the UIP condition,[21]

$$1 = E_t\left\{\xi_{t,t+1}\left[(1+i_t) - \left(1+i_t^*\right)\frac{S_{t+1}}{S_t}\right]\right\} \tag{36}$$

It is worthwhile to mention that the UIP holds given the complete asset market structure and it does not represent an additional equilibrium condition.[22]

Relative demand for foreign currency

By combining Equation 30 with Equation 25, we get an equation for the demand for domestic money balances

$$U_{m,t} = \frac{i_t}{1+i_t}U_{C,t} \tag{37}$$

On the other hand, by combining Equation 35 with Equations 26 and 32, we get the demand for foreign currency

$$U_{d,t} = \frac{i_t^*}{1+i_t^*}U_{C,t} \tag{38}$$

Note that we can derive a demand for foreign currency from Equation 38 and a demand for local currency from Equation 37. Using the last two equations, we obtain the relative demand of foreign currency with respect to local currency

$$RF_t \equiv \frac{\frac{D_t^hS_t}{P_t}}{\frac{M_t^h}{P_t}} = \left(\frac{i_t^*}{1+i_t^*}\frac{1+i_t}{i_t}\frac{\nu}{1-\nu}\right)^{-\chi} \tag{39}$$

[19] The interest rate at home is the price of the portfolio that delivers one unit of home currency in each contingency that occurs one period ahead.
[20] We assume complete markets for simplicity and tractability. For small open economy models with incomplete markets and stationary net foreign assets see Schmitt-Grohe and Uribe (2003) and Laxton and Pesenti (2003).
[21] Relaxing this assumption would give interesting results, however, since the main goal of the article is to derive a model for a partially dollarized economy, the assumption of completeness is reasonable to get a tractable model.
[22] The UIP holds even if we have deviations from purchasing power parity PPP. In an incomplete markets structure without financial frictions (also known as the 'bond economy'), the UIP will also hold in log-linear form. However, we can attain deviations from the UIP once financial frictions are taken into account.

Notice that $\frac{\partial RF_t}{\partial i_t} < 0$, which implies that if the opportunity cost of holding domestic currency increases then the relative demand for foreign currency increases, similarly $\frac{\partial RF_t}{\partial i_t^*} < 0$, which implies that if the opportunity cost of holding foreign currency increases then the relative demand for foreign currency decreases. Also note that $\frac{\partial RF_t}{\partial v} < 0$, the higher the preference for domestic currency the lower the relative demand for foreign currency. Manipulating expression 39, we derive the ratio of dollarization, the amount of real foreign currency as a proportion of total real money aggregates (local and foreign)

$$RD_t = \left[\left(\frac{i_t^*}{1+i_t^*}\frac{1+i_t}{i_t}\frac{v}{1-v}\right)^{\chi} + 1\right]^{-1} \tag{40}$$

Notice again that $\frac{\partial RD_t}{\partial i_t} > 0$, $\frac{\partial RD_t}{\partial i_t^*} < 0$, $\frac{\partial RD_t}{\partial v} < 0$.[23]

Price setting

The firms' price setting decision is modelled through a Calvo-type mechanism. We assume that prices are subject to changes at random intervals. In each period, a seller faces a fixed probability $(1-\alpha)$ of adjusting the price, independent of the length of the time period before the previous change. In this model, suppliers behave as monopolists in selling their products. The objective of a home firm selling traded goods is to maximize the expected discounted value of profits.[24] τ is an employment subsidy that eliminates the monopolistic distortion. Since all firms resetting prices in any given period will choose the same price, we henceforth drop the h subscript,

$$Max_{\widetilde{P}_{H,t}} E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \{\widetilde{P}_{H,t,t+k}\left(\widetilde{y}^d_{t,t+k} - MC^n_{t+k}\right)\} \tag{41}$$

subject to

$$y^d_{t+k}(h) = \left(\frac{\widetilde{P}_{H,t+k}}{P_{H,t+k}}\right)^{-\varepsilon}\left(\frac{P_{H,t+k}}{P_{t+k}}\right)^{-\theta} \left[(1-\gamma)C_{t+k} + \gamma C^*_{t+k}Q^{\theta}_{t+k}\right] \tag{42}$$

where $MC_t^n \equiv (1-\tau)\frac{W_t}{A_t^H}$. Each firm produces according to a linear technology,

$$y_t(h) = A_t^H L_t^h \tag{43}$$

where A_t^H is the country-specific productivity shock at time t.

The supplier maximizes Equation 41 with respect to $\widetilde{P}_t$ given the demand function and taking as given the sequences of prices $\left\{P^i_{H,t}, P^i_{F,t}, P^i_t, C^i_t\right\}$ for $i = H, F$.

The optimal choice of $_t$ is given by

$$\widetilde{P}_{H,t} = \sum_{k=0}^{\infty} \alpha^k E_t \xi_{t,t+k} Y_{t+k}\left[\widetilde{P}_{H,t} - \frac{\varepsilon}{\varepsilon-1}MC^n_{t+k}\right] \tag{44}$$

Finally, Calvo-price setting implies the following state equation for $P_{H,t}$:

$$P^{1-\varepsilon}_{H,t} = \alpha P^{1-\varepsilon}_{H,t-1} + (1-\alpha)\widetilde{P}^{1-\varepsilon}_{H,t} \tag{45}$$

An analogous expression can be derived for the foreign economy.

Monetary policy

For the specification of monetary policy, we could consider a rule that embeds different types of rules. The general form of the interest rate rule is given by

$$\frac{1+i_t}{1+i} = \Psi\left(\mathrm{F}, \xi_{\mathrm{t}}^{\mathrm{m}}\right) \tag{46}$$

where F is the set of target variables for the home country, and ξ_t^m is a pure monetary shock reflecting interest rate movements that do not correspond to the endogenous reaction of the monetary authority to instrumental variables.

The log-linear version in a limiting case: (n → 0)

In this section, we present a full log-linear version of the model. In what follows, a variable x_t represents the log-deviation of X_t with respect to its steady state, X.[25]

The following equations characterize the equilibrium of the domestic small open economy:

$$-u_{c,t} = -E_t u_{c,t+1} - i_t + (1-\gamma)E_t\pi_{H,t+1} + \gamma E_t \Delta S_{t+1} \tag{47}$$

$$\sigma c_t^* = \sigma E_t c^*_{t+1} - i_t^* \tag{48}$$

$$q_t = -u_{c,t} - \sigma c_t^* \tag{49}$$

[23] The derivative $\frac{\partial RF_t}{\partial \alpha}$ can be positive or negative depending on the size of $\left(\frac{i_t^*}{1+i_t^*}\frac{1+i_t}{i_t}\frac{\mathrm{v}}{1-\mathrm{v}}\right)$. If $\left(\frac{i_t^*}{1+i_t^*}\frac{1+i_t}{i_t}\frac{\mathrm{v}}{1-\mathrm{v}}\right) < 1$, then $\frac{\partial RF_t}{\partial \alpha} < 0$. If $\left(\frac{i_t^*}{1+i_t^*}\frac{1+i_t}{i_t}\frac{\mathrm{v}}{1-\mathrm{v}}\right) < 1$, then $\frac{\partial RF_t}{\partial \alpha} > 0$.

[24] $\xi_{t+k} = \beta^k \frac{U_C(C_{t+k})}{P_{t+k}}\frac{P_t}{U_C(C_t)}$ is the stochastic discount factor associated with the first-order condition for the recursive competitive equilibrium.

[25] In an appendix available upon request from the authors, we provide details on the derivation.

$$u_{c,t} = -\sigma c_t + \Psi\left[(1-\delta)i_t + \delta i_t^*\right] \tag{50}$$

where $\Psi \equiv \beta(\sigma\omega - 1)(1 - b_1)$
and $b_1 \equiv \frac{b}{b+(1-b)(A_2)^{\frac{\omega}{\omega-1}}}$ and $\delta = \frac{DS/P}{DS/P+M/P} = \frac{(v/(1-v))^{-\chi}}{1+(v/(1-v))^{-\chi}}$

$$\pi_{H,t} = \lambda mc_t + \beta E_t \pi_{H,t+1} \tag{51}$$

where $\lambda \equiv (1-\alpha)(1-\alpha\beta)/\alpha$

$$mc_t = \upsilon y_{H,t} + \sigma c_t - (1+\upsilon)a_t + \gamma t_t - \Psi\left[(1-\delta)i_t + \delta i_t^*\right] \tag{52}$$

$$q_t = (1-\gamma)t_t \tag{53}$$

$$t_t = t_{t-1} + \Delta S_t - \pi_{H,t} \tag{54}$$

$$y_{H,t} = \theta\gamma t_t + (1-\gamma)c_t + \gamma\left(c_t^* + \theta q_t\right) \tag{55}$$

Equations 47–55 along with the exogenous home-productivity process, the endogenous Taylor rule at home and an exogenous process for the foreign interest rate characterize the open economy with partial dollarization. The key equation is $u_{c,t} = -\sigma c_t + \Psi\left[(1-\delta)i_t + \delta i_t^*\right]$. Note that when $b = 1$, $b_1 = 1 \Rightarrow \Psi = 0$, the model collapses to a standard small open economy as in Clarida *et al.* (2001) and Gali and Monacelli (2005). The novelties in our set-up are the second and third terms on the right-hand side of the MUC. The MUC now depends on both the domestic and foreign interest rates in addition to consumption. For example, suppose that $\sigma = 1$, a positive foreign interest rate shock will reduce both the demand for real money balances denominated in foreign currency ($\downarrow D_t$) and as a consequence the overall money aggregate will fall ($\downarrow Z_t$). If $0 < \omega < 1$ (consumption and overall money aggregates are complements, $U_{CZ}<0$), after the shock we should observe a reduction in the MUC. Then, given the first-order condition for labour supply, Equation 23, which in log-linear form takes the following form $w_t - p_t = \upsilon l_t - u_{c,t}$, a decrease in the MUC induces an increase in real wages in equilibrium. This rises firms' marginal costs and inflation picks up. This in turn induces a stronger policy response of interest rates, driving down output and consumption. On the contrary, the MUC will rise if consumption and money aggregates are substitutes, leading to a fall in inflation following an increase in the foreign interest rate.

Note also that, in this economy, the risk-sharing condition will be affected by the presence of the foreign interest rate in the MUC, therefore, real exchange rate fluctuations in this particular economy will inherit the volatility and the persistence of the foreign interest rate.

Interest rate rule

We describe monetary policy as a variation of the Taylor (1993) rule, in which the nominal interest rate responds to expected movements in inflation, reflecting the aim of the monetary authority to stabilize future inflation rates.[26,27] This rule is followed by several central banks in emerging economies. It takes the following form:

$$i_t = r_t^n + \gamma_\pi E_t \pi_{H,t+1} + \gamma_x x_t \tag{56}$$

where r_t^n and x_t denote the unobservable natural real interest rate and the output gap, respectively, that will be defined in Section III.

III. A Tractable Representation of the Model

A tractable representation for a partially dollarized economy

In this section, we collapse our model to a representation similar to that in Clarida *et al.* (2001, 2002) and Gali and Monacelli (2004). The log-linearized equilibrium dynamics can be expressed in terms of the output gap and domestic inflation.

Let $y_{H,t}^n$ be the log of the natural level of output, defined as the level of output that arises with perfectly flexible prices and no cyclical distortions in the labour market (i.e. $mc_t = 0$). Therefore, $x_t = y_{H,t} - y_{H,t}^n$ is our measure of output gap.

Aggregate supply in gaps and the endogenous trade-off. In order to obtain the natural level of output, we combine the risk-sharing equation (Equation 49) with Equation 50 which leads to

$$c_t = c_t^* + \frac{1}{\sigma}q_t + \frac{\Psi}{\sigma}\left[(1-\delta)i_t + \delta i_t^*\right] \tag{57}$$

Combining the above expression with Equations 53 and 55, we obtain an expression for domestic output

$$y_{H,t} = \frac{1}{\sigma_\gamma}t_t + c_t^* + \frac{\Psi(1-\gamma)}{\sigma}\left[(1-\delta)i_t + \delta i_t^*\right] \tag{58}$$

where $\sigma_\gamma = \frac{\sigma}{[1+\gamma(2-\gamma)(\sigma\theta-1)]}$.

[26] Laxton and Pesenti (2003) find that inflation-forecast-based rules may perform better in small open economies than conventional Taylor rules.
[27] In the Peruvian economy, since the adoption of a full-fledge IT regime in 2002, the monetary policy has been conducting by targeting the inter-banking interest rate. Before that period, the central bank was implementing its monetary policy by targeting money aggregates. It has been observed a significant reduction of the mean in nominal variables in the more recent period as a consequence of the change in the instrument.

Combining the marginal cost equation (Equation 52) with Equations 57 and 58, we can express the real marginal cost in terms of home productivity, foreign consumption, domestic output and both domestic and foreign interest rates, where the degree of dollarization will play a role

$$mc_t \equiv (\upsilon+\sigma_\gamma)y_{H,t} + (\sigma-\sigma_\gamma)c_t^* - (1+\upsilon)a_t \\ -\frac{\sigma_\gamma\Psi(1-\gamma)}{\sigma}\left[(1-\delta)i_t+\delta i_t^*\right] \tag{59}$$

and the flexible and efficient levels of output can be attained by making $mc_t = 0$, conditional on the policy rule $i_t^n = 0$, and $i_t^{n*} = 0$ for all t as[28]:

$$y_{H,t}^n = a_1 a_t + a_2 c_t^* \tag{60}$$

where $a_1 \equiv \frac{(1+\upsilon)}{\upsilon+\sigma_\gamma}$, and $a_2 \equiv \frac{\sigma_\gamma-\sigma}{\upsilon+\sigma_\gamma}$. Notice that foreign consumption can be expressed in terms of the foreign interest rate by using Equation 48. Then, in our economy, the natural level of output will depend in addition to productivity shocks on the foreign nominal interest rate.

Then, combining Equation 60 with Equation 59, the real marginal cost can be expressed as

$$mc_t = (\upsilon+\sigma_\gamma)x_t - \frac{\sigma_\gamma\Psi(1-\gamma)}{\sigma}\left[(1-\delta)i_t+\delta i_t^*\right] \tag{61}$$

Finally, plugging equation (Equation 61) into the Phillips curve (Equation 51), we obtain

$$\pi_{H,t} = \kappa_x x_t + \beta E_t\pi_{H,t+1} - \kappa_i\left[(1-\delta)i_t+\delta i_t^*\right] \tag{62}$$

where $\kappa_x \equiv \lambda(\upsilon+\sigma_\gamma)$, and $\kappa_i \equiv \lambda\frac{\sigma_\gamma}{\sigma}\Psi(1-\gamma)$.

Equation 62 is a short-run AS curve that relates domestic inflation to the output gap. Unlike the baseline model with domestic money only, domestic inflation depends directly on deviations of the foreign interest rate. In particular, the presence of currency substitution allows the model to generate an endogenous cost-push shock in terms of the foreign interest rate. Any increase in the foreign interest rate affects directly the dynamics of inflation because there is a shift factor in the marginal cost. Note that the effectiveness of monetary policy is affected by δ. The larger the degree of dollarization (δ), the larger the effect of movements in the foreign interest rate shock on inflation. If the consumption and the aggregates are substitutes, i.e. $\omega > 1$, and $\kappa_i > 0$; hence, an increase in the foreign interest rate will decrease domestic inflation. In particular, following a positive shock in the foreign interest rate, the demand for foreign currency decreases and the MUC increases $(\uparrow u_{c,t})$. The increase in $u_{c,t}$ generates a reduction in real wages in equilibrium.

The higher the degree of dollarization (higher δ), the stronger the effect of foreign shocks over the aggregate supply, and consequently, the central bank is less effective in stabilizing inflation. The new transmission mechanism through which the central bank can affect inflation dynamics directly stems from the fact that money and consumption are complements. All else equal, an increase in either the domestic or foreign interest rates causes an increase in the marginal utility to consume (MUC). The previous mechanism arises because, in equilibrium, the MUC has to equate the disutility of work; therefore, an increase in MUC implies an increase in labour supply and consequently a reduction in domestic inflation.

Aggregate demand in gaps. Combining the risk-sharing condition (Equation 57) with Equation 58 and using the fact that $q_t = (1-\gamma)t_t$ leads to

$$c_t = a_3 y_{H,t} + (1-a_3)c_t^* \\ + [1-a_3(1-\gamma)]\frac{\Psi}{\sigma}\left[(1-\delta)i_t+\delta i_t^*\right] \tag{63}$$

where $a_3 \equiv \frac{\sigma_\gamma(1-\gamma)}{\sigma}$. Plugging in Equations 54 and 50 into the home Euler equation (Equation 47), we obtain

$$\sigma c_t = \sigma E_t c_{t+1} - i_t + E_t\pi_{H,t+1} + \gamma E_t\Delta t_{t+1} \\ - \Psi E_t\left[(1-\delta)\Delta i_{t+1} + \delta\Delta i_{t+1}^*\right] \tag{64}$$

From Equation 58, we obtain

$$\Delta t_{t+1} = \sigma_\gamma\Delta y_{H,t+1} - \sigma_\gamma\Delta c_{t+1}^* \\ - a_3\Psi E_t\left[(1-\Delta)\Delta i_{t+1} + \delta\Delta i_{t+1}^*\right] \tag{65}$$

Which combined with Equation 64 results in

$$\sigma c_t = \sigma E_t c_{t+1} - i_t + E_t\pi_{H,t+1} \\ + \gamma\sigma_\gamma\Delta E_t y_{H,t+1} - \gamma\sigma_\gamma E_t\Delta c_{t+1}^* \\ - \Psi(\gamma a_3+1)E_t\left[(1-\delta)\Delta i_{t+1} + \delta\Delta i_{t+1}^*\right] \tag{66}$$

Plugging Equation 63 into the above equation, we arrive at

[28] $i_t^n = 0$ corresponds to an interest rate peg in the flexible price allocation, not a zero nominal interest rate.

$$y_{H,t} = E_t y_{H,t+1} - \frac{1}{\sigma_\gamma}\left(i_t - E_t \pi_{H,t+1}\right) + \frac{\sigma - \sigma_\gamma}{\sigma_\gamma} E_t \Delta c^*_{t+1} - a_3 \frac{\Psi}{\sigma_\gamma}\left[(1-\delta)\Delta i_{t+1} + \delta \Delta i^*_{t+1}\right] \quad (67)$$

Finally, using the previous equation and the natural level of output, we obtain the IS equation in terms of output gap

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\gamma}\left[i_t - E_t \pi_{H,t+1} - r^n_t\right] + s_i\left[(1-\delta)E_t \Delta i_{t+1} + \delta E_t \Delta i^*_{t+1}\right] \quad (68)$$

where $s_i \equiv -a_3 \frac{\Psi}{\sigma_\gamma}$, and

$$r^n_t = \sigma_\gamma E_t \Delta y^n_{H,t+1} + \left(\sigma - \sigma_\gamma\right) E_t \Delta c^*_{t+1} \quad (69)$$

Equation 68 is an IS curve that relates the output gap inversely to the domestic interest rate and positively to the expected future output gap. In addition, the IS curve is also affected by the expected path of both domestic and foreign interest rates which corresponds to the third term of the right-hand side of the IS curve. The higher the degree of dollarization (higher δ), the smaller the effect of the domestic interest rate over the aggregate demand. This reveals the fragility of monetary policy in a partially dollarized economy. It is worth mentioning that when $\gamma = 0$, the IS curve for the partially dollarized economy boils down to the IS for the closed economy.

Finally, Equations 62 and 68, together with the Euler equation for the foreign economy (Equation 48) and the exogenous process for both productivity and foreign interest rate shocks characterize a partially dollarized economy.

Analytical solution and the transmission mechanism

Given the tractability of the model, we can obtain analytical solutions for the endogenous variable x_t and $\pi_{H,t}$. We obtain the rational expectations solution by implementing the undetermined coefficients method. We apply the previous-mentioned method to the aggregate supply equation (Equation 62), the aggregate demand (Equation 68) and the policy rule (Equation 56). The only source of shocks is a foreign interest rate shock, which follows a first order autoregressive (AR(1)) process:

$$i^*_t = \rho i^*_{t-1} + \varepsilon_t \quad (70)$$

We guess the following solutions for domestic inflation and output gap

$$\pi_{H,t} = \eta_{\pi i^*} i^*_t \quad (71)$$

$$x_t = \eta_{xi^*} i^*_t \quad (72)$$

where $\eta_{\pi i^*}$ and η_{xi^*} denote the partial elasticity of domestic inflation and the output gap with respect to the foreign interest rate shock, respectively. Plugging in the above possible solutions in the AS curve (Equation 62) we obtain

$$\pi_{H,t} = \frac{[\kappa_x - \kappa_i(1-\delta)\gamma_x]\eta_{xi^*} - \kappa_i \delta}{1 - [\beta - \kappa_i(1-\delta)\gamma_\pi]\rho} i^*_t$$

from the IS curve (Equation 68), we get

$$x_t = -\frac{1}{\sigma_\gamma}\frac{(\gamma_\pi - 1)\rho\eta_{\pi i^*}}{1 - C - \rho} i^*_t - \frac{s_i(1-\rho)}{1 - C - \rho}[(1-\delta)\rho\gamma_\pi\eta_{\pi i^*} + \delta] i^*_t$$

where $C = s_i(1-\delta)(\rho-1)\gamma_x$.

After some algebra manipulation, we can get the following two expressions:

$$\eta_{\pi i^*} = \frac{[\kappa_x - \kappa_i(1-\delta)\gamma_x]\eta_{xi^*} - \kappa_i \delta}{1 - [\beta - \kappa_i(1-\delta)\gamma_\pi]\rho} \quad (73)$$

$$\eta_{xi^*} = -\frac{1}{\sigma_\gamma}\frac{(\gamma_\pi - 1)\rho\eta_{\pi i^*}}{1 - C - \rho} - \frac{s_i(1-\rho)}{1 - C - \rho}[(1-\delta)\rho\gamma_\pi\eta_{\pi i^*} + \delta] \quad (74)$$

The above two equations represent a system of two equations and two unknowns $(\eta_{\pi i^*}, \eta_{xi^*})$ which can be solved. After intense algebra and some manipulations, we obtain the analytical solutions for domestic inflation and the output gap for the dual currency economy,[29]

$$\eta_{\pi i^*} = \frac{-\delta\sigma_\gamma(1-\rho)(\kappa_i + s_i\kappa_x)}{D} \quad (75)$$

$$\eta_{xi^*} = \frac{\delta\rho(\gamma_\pi - 1)\kappa_i - \delta s_i \sigma_\gamma (1-\rho)(1-\beta\rho)}{D} \quad (76)$$

where $D = (1-\rho)\sigma_\gamma(1-\beta\rho)[1 - s_i\gamma_x(1-\delta)] + (\gamma_\pi - 1)\rho[\kappa_x - \kappa_i\gamma_x(1-\delta)] + (1-\delta)(1-\rho)\sigma_\gamma\gamma_{\pi\rho}(\kappa_i + s_i\kappa_x)$

In order to gain further intuition, let us assume that the effect of overall real money balances over the aggregate demand is zero, $s_i = 0$, and that the central bank reacts only to expected inflation $\gamma_x = 0$, then the above solutions collapse to

[29] Details of the derivation are available in an appendix upon request from the authors.

$$\eta_{\pi i^*} = \frac{-\delta\sigma_\gamma(1-\rho)\kappa_i}{(1-\rho)\sigma_\gamma(1-\beta\rho)+(\gamma_\pi-1)\rho\kappa_x+(1-\delta)(1-\rho)\sigma_\gamma\gamma_\pi\rho\kappa_i}$$
$$\eta_{xi^*} = \frac{\delta\rho(\gamma_\pi-1)\kappa_i}{(1-\rho)\sigma_\gamma(1-\beta\rho)+(\gamma_\pi-1)\rho\kappa_x+(1-\delta)(1-\rho)\sigma_\gamma\gamma_\pi\rho\kappa_i}$$

If $\omega > 1$, which implies that $\Psi > 0$, $\kappa_i > 0$ and hence the denominator of the above expressions will be positive. Hence, the analytical solutions imply that after a positive foreign interest rate shock we should observe a decrease in inflation $\left(\downarrow \eta_{pii^*}\right)$ vis-a-vis an increase in the output gap $(\uparrow \eta_{xi^*})$, when money and consumption are substitutes. This result is captured by the term $\kappa_i\left[(1-\delta)i_t+\delta i_t^*\right]$ in the AS equation. A different pattern could emerge when money and consumption are complements.

Note also that the volatility of both domestic inflation and the output gap are increasing with respect to the implied degree of dollarization (δ). Figure 2 confirms our analytical findings with respect to the volatility. As expected, the volatility of both output and domestic inflation increases monotonically with the degree of dollarization, with the volatility of output rising by more. This result shows that the larger the implied degree of dollarization (larger δ) the larger the endogenous volatility of both domestic inflation and output gap.

Some simulated exercises

Parameterization. Our quantitative analysis intends to illustrate the transmission mechanism of the model, and in particular, the role of foreign currency in total money aggregates and the nonseparability of the latter from consumption. To do that, we calibrate the parameters taking the Peruvian economy as a reference, which, as illustrated

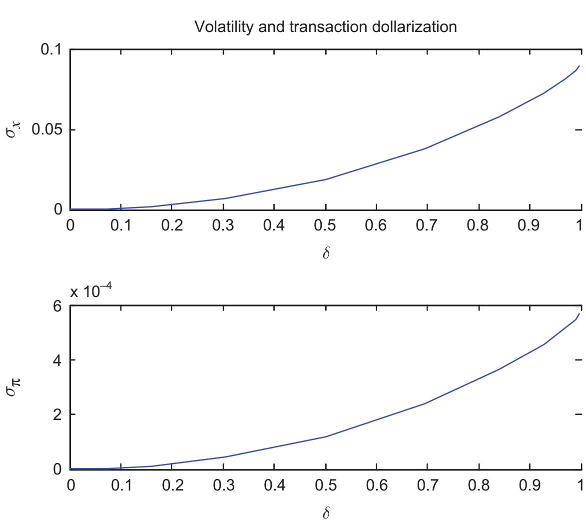

Fig. 2. $\omega = 2$

in Section I, is characterized by a dual currency environment. Some parameters were estimated using Peruvian data. The parameterization of the model seeks to characterize the qualitative behaviour of its main variables rather than to match the empirical data. The steady state equilibrium is derived in the Appendix.

We set a quarterly discount factor, β, equal to 0.99, which implies an annualized rate of interest of 4%. In the steady state, the gross foreign interest rate, assumed to be exogenous in the model, is also equal to $\beta^{-1} = 1.01$, which implies the same annualized interest rate. The share of foreign goods in consumption, $\gamma = 0.4$, which is close to the ratio of imports over aggregate consumption for the Peruvian economy. In order to isolate the role of foreign currency in total money aggregates and the nonseparability of the latter from consumption, we choose parameter values equal to 1 for the coefficient of risk aversion, σ, the inverse of the elasticity of labour supply, υ, and the elasticity of substitution between home and foreign goods, θ. We choose a degree of monopolistic competition, ε, equal to 7.66 following Rotemberg and Woodford (1997). This implies an average mark-up of 15%. For the monetary rule, we follow Taylor (1993) and set the coefficient on inflation, $\gamma_\pi = 1.5$., and the coefficient for output gap, $\gamma_x = 0.5$. As it is common in the literature on the Calvo (1983)- pricing technology, we let the probability of not adjusting prices, $\alpha = 0.75$.

Given that the main goal is to analyse the effect of foreign interest rate shocks, we calibrate this exogenous process. In order to calibrate this shock, we fit an AR(1) process to the FED funds rate (our proxy for foreign rates), by using quarterly data over the sample period 1955:01 to 2004:02. We obtain the following estimates: $\rho^* = 0.96$, and $var(\varepsilon_t^{i^*}) = (0.009)^2$. We set the elasticity of substitution between domestic and foreign money, χ, equal to 4.1 which is consistent with previous studies at the Central Bank of Perú. In our benchmark parameterization, we set ν equal to 0.5 that implies a steady state degree of dollarization of 70%. This value is roughly the average degree of dollarization from 1994 to 2005 for the Peruvian economy. We parameterize $b = 0.83$, which constitutes the share of consumption in the CES function (we use private consumption and overall money aggregate that is the sum of home currency liquidity and foreign currency liquidity). Finally, ω can take two values, $\omega = 0.9$, implying that consumption and the overall aggregate are complements and, $\omega = 2$, which implies that consumption and the overall aggregate are substitutes.

Impulse responses. Figures 3 and 4 display the impulse responses of the simulated model for three degrees of dollarization: high $(\nu = 0.33 \Rightarrow \delta = 0.96)$, medium $(\nu = 0.5 \Rightarrow \delta = 0.5)$ and low $(\nu = 0.67 \Rightarrow \delta = 0.05)$. We report the impulse responses under two possible scenarios regarding the parameter ω. The impulse responses confirm the analytical results. In Fig. 2, we depicts the responses when (ω = 2). Following the foreign interest shock and the given substitutability between the money aggregate and consumption, the demand for foreign currency decreases and the MUC increases. Consequently, real wages decrease in equilibrium and in turn induce a reduction in inflation, which is captured by the term $-\kappa_i \delta i_t^*$. It is useful to recall that for $\omega > 1$, $\kappa_i > 0$. Inflation decreases following the foreign interest rate shock. The limitations of the central bank under this environment are also clear. In particular, the larger the presence of foreign currency (larger δ), the larger the impact of the shock over domestic inflation. Given the reduction of inflation, the central bank must react by reducing its policy rate in order to contract the aggregate demand and finally to stabilize inflation. In Fig. 3, we observe that the policy rate decreases after the shock, triggering a contraction in the aggregate demand and consequently domestic inflation reaches its initial value. Again the central bank has a stronger response to the larger degree of dollarization, highlighting the limitations of a central bank in a partially dollarized economy.

In the cases where the consumption and the overall aggregates are complements, $(0 < \omega < 1)$, the impulse responses show the opposite pattern. Figure 4 depicts the results. Foreign interest rate shocks can generate persistent increase in inflation and a contraction in the output gap.

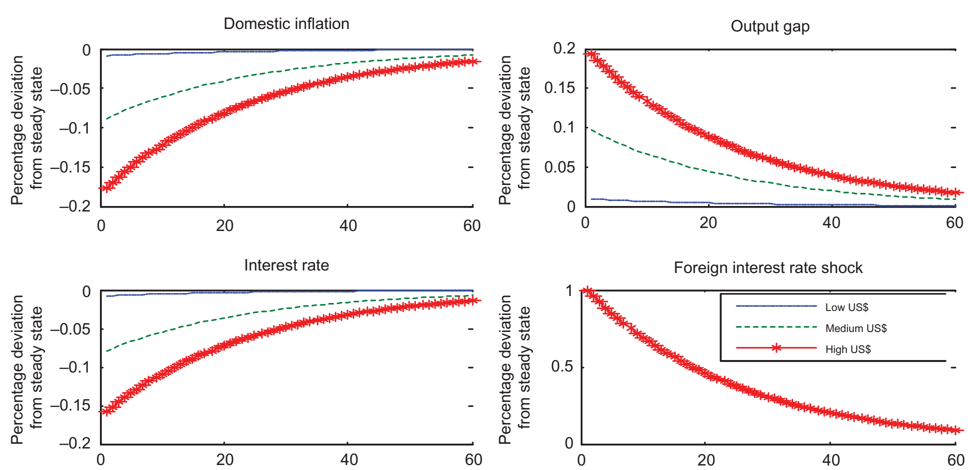

Fig. 3. **(High, $\delta = 0.95$, Medium, $\delta = 0.5$, Low, $\delta = 0.05$) and $\omega = 2.0$**

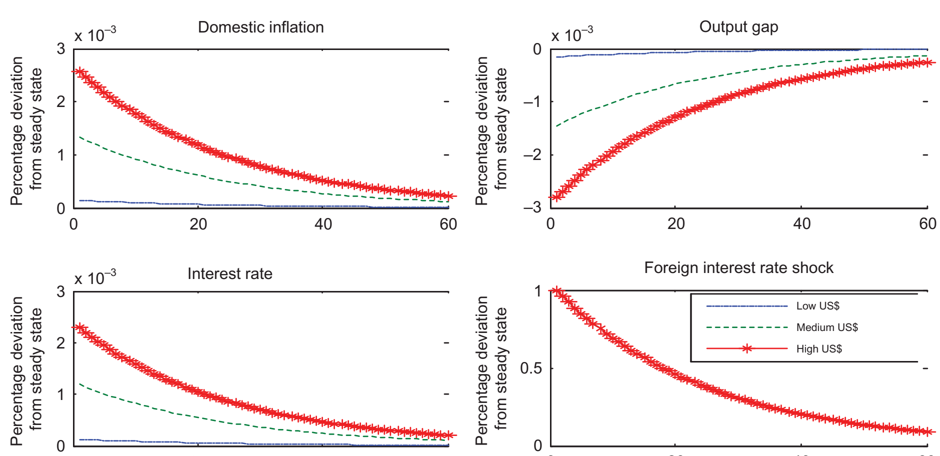

Fig. 4. **(High, $\delta = 0.95$, Medium, $\delta = 0.5$, Low, $\delta = 0.05$) and $\omega = 0.9$**

Table 1. Macroeconomic volatility and dollarization

Foreign interest rate shock (ω = 2.0)				Foreign interest rate shock (ω = 0.9)		
δ	$\sigma(\pi_H)$	$\sigma(x)$	$\sigma(i)$	$\sigma(\pi_H)$	$\sigma(x)$	$\sigma(i)$
0.20	1.00	1.00	1.00	1.000	1.000	1.000
0.30	1.56	1.56	1.56	1.530	1.529	1.529
0.40	2.10	2.10	2.10	2.018	2.018	2.018
0.50	2.58	2.60	2.60	2.529	2.528	2.529
0.70	3.87	3.78	3.76	3.553	3.554	3.554
0.95	5.42	5.43	5.49	4.866	4.865	4.865

How large and persistent the response of inflation to the foreign interest rate shock is, depends on how sensitive the AS is to the implied parameter δ.

Unconditional volatilities. In this section, we illustrate how higher levels of dollarization could generate greater macroeconomic volatility. We illustrate this issue in the context of our calibrated model. In particular, we compute the implied SD of domestic inflation, the output gap and nominal interest rate under various degrees of dollarization by varying parameter δ defined in the Appendix. Thus, given χ = 4.1 we change v, the preference for domestic currency, in order to obtain the implied degree of dollarization. The analysis is made conditional to a foreign interest rate shock. In addition, we also check for robustness by calculating the unconditional moments when the consumption and the overall aggregates are complements (ω = 0.9).

Table 1 reports the results. It shows how the SDs of domestic inflation, the output gap and the nominal interest rate vary in response to shifts in the degree of dollarization. The larger the degree of dollarization, the larger the SDs. The first three columns display the results for the benchmark parameterization with ω = 2.0. The last three columns reports the same exercise for a value of ω = 0.9. In order to facilitate the analysis, we normalize the SDs to unity corresponding to the calibration with v = 0.585 which implies a degree of dollarization of 20% (δ = 0.20).

The results make clear that the cyclical response of the economy to the foreign interest rate shock is quite sensitive to the degree of dollarization (δ). For example, as δ decreases from 0.70 to 0.40, the macroeconomic volatility reduces in almost halves. The implied reduction in δ can be obtained by a small increase in v, from 0.448 to 0.525. Thus, by a small increase of the parameter that captures the preference for domestic currency the model predicts a meaningful reduction in macroeconomic volatility. Finally, we note that the results we obtain in the context of the simple small open economy are robust to changes in ω which is the key parameter that generates the trade-off between stabilizing domestic inflation and the output gap. Therefore, the higher the degree of dollarization in the model economy is, the higher unconditional volatility of inflation and output gap than otherwise.

IV. Conclusions

This article has been motivated by the experiences of several developing economies, in particular the Peruvian economy, where both local and foreign currency coexist as a mean of transaction and as a deposit of value. The monetary authority faces the problem of managing the domestic currency component of the money aggregate in a situation where the foreign component can change significantly over time.

We develop a model that embeds a foreign and local currency money aggregate into a simple two-country open economy model as in Clarida *et al*. (2001, 2002) and Gali and Monacelli (2005). The resulting model yields a tractable formulation for the qualitative analysis of monetary policy in economies that face currency substitution as an equilibrium outcome. The results suggest that, given shocks to the foreign interest rate, inflation and output volatility increase when dollarization is high, meaning that the central bank's ability to reduce volatility is more limited in a partially dollarized economy. The transmission mechanism of these shocks can make inflation to fall and output increase when the overall money aggregate and consumption are substitutes. A novel result is worth highlighting. Our canonical model generates an endogenous trade-off between the stabilization of inflation and the output gap. The short-run trade-off, which depends on the degree of dollarization, arises due to the presence of a foreign interest rate shock, hence it cannot be evaluated in a closed economy environment.

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